Food prices and inflation in a closed-economy IS/AS/MP model[1]

Stephen A. O'Connell[2]

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ABSTRACT: An adequate model of food prices and inflation requires at least two adjustments to the core DSGE model. The first is to disaggregate the food and non-food sectors in order to accommodate exogenous shocks to agricultural output. The relative price of food is determined mainly on the supply side and should play a key role in the short-run dynamics. The second adjustment is to incorporate some influence of international trade. In this note I focus on the first of these features by looking at food prices and inflation in a closed-economy IS/AS/MP model. I emphasize the potential importance of private grain storage in determining the dynamic response to food supply shocks.

[1] This note is an extended Appendix to my paper "Towards a Rule-Based Approach to Monetary Policy Evaluation in Sub-Saharan Africa" presented at the AERC Plenary Session on Central Banking in Africa, December 2009 (forthcoming in a special issue of *Journal of African Economies*).

[2] Eugene M. Lang Research Professor, Department of Economics, Swarthmore College; and International Growth Centre, Tanzania (steve_oconnell@swarthmore.edu).

1. Introduction

An adequate model of food prices and inflation requires at least two adjustments to the core DSGE model described in O'Connell (2010). The first is to disaggregate the food and non-food sectors, in order to accommodate exogenous shocks to agricultural output. The second is to incorporate some influence of international trade. In this note I focus on the first of these features. Starting from the simplest one-sector, closed-economy AS/IS/MP model, I incorporate a food sector in which supply is stochastic and demand obeys Engel's Law. Abstracting from supply-side interactions between the food and non-food sectors, I show how subsistence requirements in food consumption reduce the interest-elasticity of overall spending. I develop a simple model of private grain storage and emphasize the potential importance of this activity in smoothing food consumption over time. In O'Connell (2010) I show that a stronger smoothing response increases the persistence of food price inflation relative to that of non-food inflation, bringing this feature of the model into closer line with recent evidence on Ethiopia (Loening, Durevall and Birru 2009).

The model is extremely stripped-down, and a wide variety of extensions are possible.

2. The supply side and market clearing

I assume that food prices are flexible but that non-food prices are sticky. Non-food inflation follows a New Keynesian Phillips Curve

$$\pi_{Nt} - \bar{\pi} = E_t[\pi_{Nt+1} - \bar{\pi}] + \varphi(\tilde{Y}_{Nt} - \tilde{Y}_{Nt}^n), \tag{1}$$

where Y_{Nt} and Y_{Nt}^n are actual and natural output and where "~" denotes the difference between the log of a variable and the log of its value in the non-stochastic steady-state. We will treat natural output as a constant, so that $\tilde{Y}_{Nt}^n = 0$, this ignores any labor-market link between the food and non-food sectors and provides an interesting avenue for extensions.[3] Non-food output is demand-determined in the short run, so the market-clearing condition in the non-food sector is

[3] Matsuyama (1992) shows that when labor is mobile between sectors, the supply-side implications of Engel's Law are potentially very important in a closed economy. An adverse shock to the productivity of labor in the food sector, for example, draws labor *into* food production. This in turn means a supply-side contraction in the non-food sector. This general-equilibrium channel is absent in the present note, because I assume zero labor mobility between the two sectors.

$$\tilde{C}_{Nt} = \tilde{Y}_{Nt} \tag{2}$$

where $\tilde{C}_{Nt}$ is (the log deviation of) non-food consumption.

The food sector is a flex-price sector where output always equals the natural rate ($\tilde{Y}_{Ft} = \tilde{Y}_{Ft}^{n}$). The market-clearing condition in for food is therefore $C_{Ft} + S_t = Y_{Ft}^{n} + S_{t-1}$, where S_t is the end-of-period stock of food inventories held by the private sector (assuming no depreciation for simplicity). Equivalently,

$$\bar{C}_{Ft} + \frac{\gamma}{1-\gamma}\, \tilde{S}_t = \tilde{Y}_{Ft}^{n} + \frac{\gamma}{1-\gamma} \tilde{S}_{t-1}, \tag{3}$$

where $\gamma = \bar{S}/(\bar{C}_F + \bar{S})$ is the share of food inventories in the total supply (and demand) of food in the non-stochastic steady state.

The consumer price index is

$$P = P_{Ft}^{\theta} P_{Nt}^{1-\theta},$$

where θ is the steady-state share of food consumption in total spending. Headline inflation is therefore given by

$$\pi_t = \pi_{Nt} + \theta(\tilde{p}_t - \tilde{p}_{t-1}), \tag{4}$$

where $p_t = P_{Ft}/P_{Nt}$ is the relative price of food. Total real GDP in this economy is

$$Y_t = \frac{P_{Ft}Y_{Ft} + P_{Nt}Y_{Nt}}{P_t}.$$

3. Aggregate demand

Aggregate demand is generated by households and, in the food sector, a competitive storage sector. We begin with households. To generate Engel's Law we assume that food consumption is subject to a subsistence floor of $Z > 0$. Households therefore maximize the discounted utility of the Cobb-Douglas consumption index $C = (C_F - Z)^{\xi}\ C_N^{1-\xi}$. This generates constant shares in above-subsistence spending within each period, so the demand function for food takes the form

$$P_{Ft}(C_{Ft} - Z) = \xi \cdot [P_{Ft}(C_{Ft} - Z) + P_{Nt}C_{Nt}] = \xi \cdot [P_t Y_t - P_{Ft}Z], \tag{5}$$

where I have imposed the market-clearing conditions in the food and non-food sectors. Equivalently,

$$\frac{P_{Ft}Z}{P_t Y_t} = \frac{\theta_t - \xi}{1 - \xi}.$$

where $\theta_t = P_{Ft}C_{Ft}/Y_t$ is the share of food in total output (or spending). This expression is important for calibration purposes. In the non-stochastic steady state, $\theta_t = \theta$, and we are free to choose the value of total output measured in terms of food. It follows that the three parameters Z, θ and ξ cannot be chosen independently – any two of them implies the third. If food consumption is half of total consumption and ¼ of above-subsistence consumption, for example ($\theta = 0.5, \xi = 0.25$), then the subsistence requirement must be 1/3 of GDP and 2/3 of food consumption.

To derive the household's intertemporal behavior, we start with the intertemporal budget constraint. Denoting end-of-period assets by A_t, this can be written

$$A_t = Y_t - P_{Ft}Z - P_{Ft}(C_{Ft} - Z) - P_{Nt}C_{Nt} + (1 + i_{t-1})A_{t-1}$$

Taking advantage of the linearly homogeneous form of the above-subsistence consumption index *C*, we can define the ideal price index $Q = P_{Ft}^{\xi} P_{Nt}^{1-\xi}$ and rewrite this as

$$A_t = Y_t - P_{Ft} \cdot Z - Q_t \cdot C_t + (1 + i_{t-1})A_{t-1},$$

where *C* is real above-subsistence spending $C_t = \frac{P_{Ft}(C_{Ft}-Z)+P_{Nt}C_{Nt}}{Q}$. This reduces to the standard intertemporal budget constraint when there is no subsistence requirement, because setting $Z = 0$ eliminates the second term on the right-hand side, sets $Q_t = P_t$ in the third term, and equates *C* with aggregate consumption. But it is clear that $Z > 0$ modifies the consumer's problem. The appropriate concept of income now excludes the cost of subsistence food consumption, and the price of the consumption basket relevant for the consumer's marginal utility is not 1 but Q. More fundamentally, the household is concerned with smoothing the marginal utility of above-subsistence consumption, *C*, not total consumption.

The standard perturbation argument leads to an Euler equation of the form

$$U'(C_t) = \beta E_t\left[\frac{1+i_t}{Q_{t+1}/Q_t} U'(C_{t+1})\right].$$

Using the period-by-period utility function $U(C_t) = C_t^{1-1/\sigma}/(1-1/\sigma)$, this can be expressed in log-differential form as

$$\tilde{C}_t = E_t[\tilde{C}_{t+1}] - \sigma(i_t - E_t[\pi_{t+1}] - \bar{r}) - \sigma(\theta - \xi)(E_t[\tilde{p}_{t+1}] - \tilde{p}_t). \quad (6)$$

where $r = (1+\bar{\imath})/(1+\bar{\pi}) - 1$ is the real interest rate in the non-stochastic steady state. Except for the final term, this looks like a conventional Euler equation. The final term translates the real interest rate into the rate relevant for trading above-subsistence consumption (and therefore marginal utility) across periods. If the real price of food is expected to rise, the conventional real interest rate understates the real yield on saving. The sign on this term is therefore negative.

To convert equation (6) into an IS curve, we have to express above-subsistence consumption in as a function of total income. Log-differentiating $Q_t C_t + P_{Ft} Z = P_t Y_t$, we get

$$\tilde{C}_t = \vartheta \cdot \tilde{Y}_t, \quad (7)$$

where $\vartheta = (1-\xi)/(1-\theta) = \bar{P}\bar{Y}/\bar{Q}\bar{C} > 1$. Substituting for *C* in (6) then yields

$$\tilde{Y}_t = E_t[\tilde{Y}_{t+1}] - \frac{\sigma}{\vartheta}(i_t - E_t[\pi_{t+1}] - \bar{r}) - \frac{\sigma(\theta-\xi)}{\vartheta}(E_t[\tilde{p}_{t+1}] - \tilde{p}_t). \quad (8)$$

The net result of incorporating subsistence consumption is therefore twofold: the Euler equation incorporates an additional term involving the expected change in the relative price of food, and the elasticity of total spending to the interest rate falls. The latter result occurs because the interest rate only acts directly on above-subsistence consumption, rather than total consumption.

Log-differentiating equation (5), food consumption satisfies

$$\tilde{C}_{Ft} = \frac{\xi}{\theta} \cdot \tilde{Y}_t - \frac{(1-\theta)\xi}{\theta} \cdot \tilde{p}_t. \tag{9}$$

The coefficient on income is below 1, as implied by Engel's Law. Our calibration assumes a preference parameter of $\xi = 0.25$ and a food share of $\theta = \frac{pC_F}{C} = 0.5$, yielding an income elasticity of food consumption of 0.5 and a price elasticity of 0.25.

4. Storage behavior

In a seminal paper on rational price expectations, Muth (1961) introduced a set of concepts and methods that continue to play a central role in the specification and solution of DSGE models. The substantive application developed in that paper emphasized the impact of storage in altering the time-series persistence of food-price shocks. Muth ignored stockouts, which present serious technical difficulties in a context that relies on first- or second-order approximations.[4] I will follow him in that respect, although this is a potentially important shortcoming given the prevalence of stockouts in the data (e.g., Osborne 2004). Most theories of storage behavior rely on stockouts to reproduce widely-observed features of commodity prices including their tendency to spike upwards at irregular intervals (Deaton and Laroque 1996).

A milder challenge still exists, in the need to generate a meaningful stationary state around which stocks can vary in the short run. Models of risk-neutral competitive storage tend to imply a zero stock in the stationary state, because expected capital gains are zero and the interest cost of stockholding is positive. To motivate steady-state storage I will assume that food stocks brought forward into period t+1, S_t, generate a convenience value of $v(S_t) \geq 0$ measured in terms of food, with $v(0) = 0, v' > 0$ and $v'' < 0$. Risk-neutral competitive storage then generates the arbitrage condition

$$\frac{P_{Ft+1}[1+v'(S_t)]}{P_{Ft}} = (1 + i_t) \tag{10}$$

where v' is the marginal value of storage. The demand for food stocks is therefore given by

$$S_t = f\left(\frac{\pi_{Ft+1} - i_t}{1+\pi_{Ft+1}}\right), \; f(0) > 0, \; f' > 0.$$

[4] DSGE models also routinely ignore the zero bound on nominal interest rates.

where $f = v'^{-1}$. A first-order Taylor expansion around the non-stochastic steady state then yields

$$\tilde{S}_t = \alpha \cdot [(1-\theta) \cdot (E_t[\tilde{p}_{Ft+1}] - \tilde{p}_{Ft}) + (E[\pi_{t+1}] - \bar{\pi}) - (i_t - \bar{\iota})]. \tag{11}$$

where $\alpha = \bar{f}'(1+\bar{r})/\bar{S}$.

5. Monetary policy

The model is closed, as in the standard IS/AS/MP case, by a monetary policy rule. Introduction of the food sector raises a pair of new questions, however, about the appropriate specification of the inflation target and about how the monetary authority interprets the GDP gap.

Woodford (2003) argues that there is no welfare case for stabilizing prices that are perfectly flexible. The appropriate target, from this perspective, is the inflation rate of non-food prices, rather than the headline inflation rate. The GDP gap, in turn, is always equal to the gap in the non-food sector, because food output is always equal to the natural level. The appropriate form for the Taylor Rule is therefore

$$i_t = \bar{r} + \pi_{Nt} + \phi_\pi(\pi_{Nt} - \bar{\pi}) + \phi_y\left(\tilde{Y}_t - \tilde{Y}_t^n\right). \tag{10}$$

where (as above) we are assuming $\tilde{Y}_t^n = 0$.

6. The log-linearized model

To tie down the nine endogenous variables

$$y = \left[\pi_N, \tilde{Y}_N, \tilde{Y}, i, \pi, \tilde{p}, \tilde{Y}_{Ft}, \tilde{C}_F, \tilde{S}\,\right],$$

we have the following linear equations:

$$\pi_{Nt} - \bar{\pi}_N = E_t[\pi_{Nt+1} - \bar{\pi}_N] + \varphi \tilde{Y}_{Nt} + \varepsilon_t^{\pi_N} \qquad \text{(Phillips curve for N-goods)}$$

$$\tilde{Y}_t = E_t[\tilde{Y}_{t+1}] - \frac{\sigma}{\vartheta}(i_t - E_t[\pi_{t+1}] - \bar{r}) + \frac{\sigma(\theta-\xi)}{\vartheta}(E_t[\tilde{p}_{t+1}] - \tilde{p}_t) + \varepsilon_t^Y \qquad \text{(IS curve)}$$

$$i_t = \bar{r} + \pi_{Nt} + \phi_\pi(\pi_{Nt} - \bar{\pi}) + \phi_y \tilde{Y}_{Nt} \qquad \text{(Monetary policy rule)}$$

$$\pi_t - \bar{\pi} = \pi_{Nt} - \bar{\pi}_N + \theta(\tilde{p}_t - \tilde{p}_{t-1}) \qquad \text{(Headline inflation)}$$

$$\tilde{Y}_t = \theta \tilde{Y}_{Ft} + (1-\theta)\tilde{Y}_{Nt} \qquad \text{(Definition of aggregate output)}$$

$$\tilde{C}_{Ft} = \frac{\xi}{\theta} \cdot \tilde{Y}_t - \frac{(1-\theta)\xi}{\theta} \cdot \tilde{p}_t \qquad \text{(Demand for food)}$$

$$\tilde{S}_t = \alpha \cdot [(1-\theta) \cdot (E_t[\tilde{p}_{Ft+1}] - \tilde{p}_{Ft}) + (E[\pi_{t+1}] - \bar{\pi}) - (i_t - \bar{\imath})]. \qquad \text{(Food storage behavior)}$$

$$\tilde{C}_{Ft} = \tilde{Y}_{Ft}^n - \frac{\gamma}{(1-\gamma)}[\tilde{S}_t - \tilde{S}_{t-1}] \qquad \text{(Food market clearing)}$$

$$\tilde{Y}_{Ft}^n = \varepsilon_{Ft} \qquad \text{(Stochastic process for food output)}$$

Notice that we have added shocks to the IS and AS curves as in the core DSGE model discussed in O'Connell (2010).

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