Prices versus Quantities Revisited: The Case of a Transboundary Stock Pollutant*

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September 2021

Abstract

We study the relative merits of price versus quantity instruments in a dynamic model with two economies facing a transboundary stock pollutant like CO_2. The climate policy debate has emphasized the importance of the relative slopes of marginal damages and abatement costs for ranking taxes versus cap and trade under technological uncertainty. It finds that taxes dominate quotas as the social cost of carbon curve is extremely flat. Our model features persistent cost shocks that generate perfectly correlated shifts of these two curves; these are as important to the ranking as is the slope ratio. Gradual diffusion of innovations generally favors quotas by changing the relatives shifts. A larger size of an economy relative to the other also tend to favor quotas.

JEL Classification: Q50, H20, D80.

Keywords: prices versus quantities; tax vs quota; uncertainty; carbon price; international environmental agreement.

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*We thank Larry Karp and Victor Barranca for helpful advice. We are indebted to the generous support of Swarthmore College for this project.

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1 Introduction

Climate change is one of the greatest global challenges of our time. Global warming, driven primarily by anthropogenic greenhouse gas (GHG) emissions, threatens to roll back decades of global development progress and puts lives, livelihoods, and economic growth at risk. Recent reports from the Intergovernmental Panel on Climate Change (IPCC) (2014; 2018) renew calls to limit temperature increase to below 2°C, or even 1.5°C, relative to pre-industrial levels. To achieve this goal, the world needs immediate action and get to zero net GHG emissions by the end of the century (IPCC 2014). Setting a price for the emissions, commonly referred to as carbon pricing, is an essential part of the solution.

Explicit carbon pricing policies are enacted by a government mandate and impose a price based on carbon content. They are usually enacted through either a carbon tax or an emission trading system (ETS). According to a World Bank (2021) report, in 2021, there are 64 carbon pricing instruments (CPIs) in operation, covering 21.5% of global GHG emissions. These instruments differ greatly in their carbon prices and their coverage of emissions. A wide range of countries and subnational governments continue to move toward carbon pricing, in particular ETSs. Some of these have already scheduled start dates, while others are beginning with pilots. Other jurisdictions are in early stages of considering their policy options. One key decision for the various governments is whether to adopt a carbon tax, a price instrument, or an ETS, a quantity instrument.

In this paper, we revisit the classic question of "prices versus quantities," asked in Weitzman's (1974) seminal paper, in the case of carbon pricing in an international context. With full information, a price instrument and a quantity instrument to regulate pollution could be equivalent. When uncertainty is present, the ranking of the two market-based policies will depend on a number of factors. Weitzman's original analysis establishes that a tax is preferred to a quota if and only if the marginal abatement cost curve is steeper than the marginal damage curve. Following this reasoning, it is often inferred that the society would be better off using taxes to address climate change since the per-period marginal damage curve of GHG emissions tend to be very flat. However, Weitzman's work models a local flow pollutant, i.e., the pollutant does not have persistence or cumulative impact and there is only one relevant policy maker. This is not the case with climate change. The most persistent GHG in the atmosphere, carbon dioxide (CO_2) emissions, can affect the climate for hundreds or thousands of years. Additionally, GHGs are global pollutants and its abatement suffers from the free riding problem at the international level. Therefore, many scholars have attempted to extend the original analysis to consider stock pollutants, technology shocks, and/or strategic policy with multiple economies. Under certain scenarios, a quota can be better than a tax.

In this paper, we attempt to combine the intuitions of previous research and explore the conditions under which a tax or a quota is preferred by a regulator of a global stock pollutant facing technological uncertainty in a two-economy world. Section 2 reviews the related literature. Section 3 presents a two-economy model in which only one is actively setting a carbon pricing policy. Section 4 presents the full strategic interaction model where both economies pursue policies in response to each other. Section 5 calibrates the model based on widely-used estimates of abatement costs and climate-related damages, and presents numerical results on how the ranking of tax versus quota depends on key parameters. Section 6 concludes.

2 Literature Review

Weitzman (1974) models the choice of a price instrument versus a quantity instrument by a regulator facing uncertainty about firms' abatement costs. He shows that this uncertainty creates a smaller deadweight loss under a tax than under a quota if and only if the (absolute value) of the slope of the marginal abatement cost curve is larger than that of the marginal damage curve. This elegant result has underpinned much of the discussion about instrument choice in environmental policy in general and climate policy in particular. However, the logic was derived based on the analysis of a *local flow* pollutant and is flawed in the climate context. GHG emissions are a *stock* pollutant, with many gases persisting in the atmosphere, generating warming effect for many years. They are also a *global* pollutant, with the effect of climate change spanning across national borders regardless of the origin of the emissions. Therefore, many scholars have attempted to extend Weitzman's original analysis to make it more applicable to the climate context.

Much of this research focuses on the stock versus flow characteristic of the pollutant. Recent surveys on this strand of literature include Hepburn (2006); Aldy et al. (2010) and Goulder and Schien (2013). Under a tax, technological uncertainty creates uncertainty about emissions, and consequently about damages. Under a binding quota, such uncertainty does not alter emissions, but it creates cost uncertainty for firms. If the (aggregate) marginal abatement cost curve is steeper than the marginal damage curve, then uncertainty about abatement costs harms society more than uncertainty about damages. Hence, in the case of a flow pollutant, taxes are preferred if and only if the marginal abatement cost curve is steeper than the marginal damage curve. In a dynamic setting with stock pollution, in each period we have to compare current abatement costs against the discounted stream of damages occurring over an extended time horizon. Hereafter, in discussing stock pollutants, we refer to the discounted stream of marginal damages simply as "marginal damages." In the climate context, the relevant marginal damage is referred to as the social cost of carbon (SCC). Applying the logic described above to the climate context, one might be tempted to rank taxes versus quotas by comparing the slopes the marginal abatement cost curve and the SCC. Indeed, the literature often makes this leap (e.g. Nordhaus 2008; Aldy et al. 2010; Goulder and Schien 2013; Weitzman 2020).

However, as pointed out by Karp and Traeger (2019), this leap can be incorrect. They model the policy making process as a dynamic programming problem, where the regulator of an economy needs to balance the gains from current emissions and potential future damages due to the added stock. The model features serially correlated technological shocks and gradual diffusion of new technologies, under asymmetric information. In this dynamic setting, the abatement technology is a stochastic process. Every period gives rise to an innovation unknown to the policy maker at the onset of the regulation period. Because technology is highly persistent, the innovation affects not only the abatement cost in the present period, but also those in the future periods. Consequently, the innovation also changes future emissions, thus changing the future cumulative stock, or the future baseline concentration, of the pollutant. With convex damages, the marginal damage caused by additional emissions in a period depends on the pollutant's concurrent baseline concentration. As a result, the technology shock not only shifts the marginal abatement cost curve, but it simultaneously shifts the marginal damage curve. Hence, the policy ranking depends as much on this shift as it does on relative slopes. In some cases, this additional shift implies that a quota is the preferred instrument even if the marginal damage curve is much flatter than the abatement cost curve.

This mechanism was either not present in earlier models or was not recognized clearly. Hoel and Karp (2002) analyze prices versus quantities for a stock pollutant in a setting where current innovations have no impact on future technology, thereby ruling out the aforementioned result. Newell and Pizer (2003) rank the two policies when cost shocks are serially correlated in an open-loop setting, where the regulator chooses current and future policy levels at the initial time. They show that positively correlated cost shocks increase stock volatility under taxes, favoring quotas, although not by enough to tip the balance. Their intuition depends crucially on the open loop assumption that future policy makers do not respond when the realized technology and emission levels drift away from anticipated levels. In contrast, Karp and Traeger's result depends on the ability of the regulator to update the policy instruments after observing past technological innovations. Karp and Zhang (2005) compare the policy ranking across the open loop and feedback settings, with correlated shocks. They observe that the ranking of tax versus quota changes, but do not recognize the different mechanism in the feedback setting and its ability to make quota dominate tax in the climate context.

A related line of literature studies international environmental agreements (IEAs). Aldy and Stavins (2007); Aldy et al. (2010) and Marrouch and Chaudhuri (2016) provide recent surveys on this research that focuses on international policy design for transboundary environmental problems. The cross-border externality creates a strategic interdependence between involved countries. Since GHG emissions are a global pollutant, its regulation suffers from the free rider problem at the global level. To successfully address the issue, there needs to widespread international cooperation, at least among all major emitters of GHGs. Proposed architectures for international emissions control regimes can be loosely classified into those based on bottom-up versus top-down (i.e., internationally negotiated) approaches and cap-and-trade systems versus systems of emissions taxes.

Jørgensen et al. (2010), Van Long (2012) and Calvo and Rubio (2012) offer recent surveys of dynamic models of IEAs focusing on stock pollutants. A limitation revealed in these surveys is that, in the analysis of transboundary pollution problems, almost none of the models consider the uncertainty or asymmetric information issue discussed previously. Wirl (2007) considers uncertainty in the sense that global warming follows a stochastic process, but considers only the price instrument.

In this paper, we extend the model of Karp and Traeger (2019) to incorporate the international aspect. Our model features a non-cooperative dynamic game between two economies, each strategically pursuing its carbon pricing policy in response to the other. Our main contributions to the literature are (i) to show that a simple and intuitive ranking criterion similar to that in Karp and Traeger (2019) extends to the case in which an economy unilaterally pursues carbon pricing policy, and (ii) to show that in the case of strategic interaction between two economies, the rankings of tax versus quota depends crucially on the sizes of the economies along with other parameters.

3 Dynamic Model with Unilateral policy

We present a two-economy model in which only one is actively setting a carbon pricing policy, which we call home bloc. The other economy, the foreign bloc, does not set any carbon pricing policy, which is equivalent to setting a "zero carbon tax". Firms in foreign bloc maximize their emission benefits each period.

Our analysis focuses on two sources of asymmetry that cause the non-equivalence of taxes and quotas for home bloc: the relative size of the two economies and technology-related costs. Asymmetry arises because of the difference in relative size of home bloc and foreign bloc, which in turn determines the relative size of emissions. Asymmetry also arises because technology-related costs are private information when firms choose emissions (Karp and Traeger 2019).

3.1 Description of the Model

The model contains two state variables: a stock pollutant S_t and a technology level θ_t. The equation of motion for the stock pollutant is $S_t = \delta S_t + E_t$, such that E_t is the annual flow of total emission of the two blocs, and $1 - \delta_t \in (0, 1)$ is the decay factor of the harmful pollution. The annual damage caused by the stock pollutant is $\frac{b}{2}S_t^2$. The abatement technology θ_t is assumed to be symmetric for each bloc, and is described by a deterministic trend and a stochastic deviation from this trend.

The equation of motion for θ_t is $\theta_t = \rho\theta_{t-1} + \epsilon_t$, with $\rho > 0, \epsilon_t \sim iid(0, \sigma^2)$, and $\mathbb{E}_t(\epsilon_t) = 0$. Firms adopt only a fraction $\alpha \in (0, 1]$ of the technological innovations, such that $\hat{\theta}_t = \rho\theta_{t-1} + \alpha\epsilon_t$.

We added a size scalar k to the model of Karp and Traeger 2019, where the size of foreign bloc is k times that of home bloc. Home bloc and foreign bloc have different speeds of technological diffusion $\alpha_H \in (0, 1]$ and $\alpha_F \in (0, 1]$, respectively. Firms in home bloc and foreign bloc also have different slope of the marginal abatement cost curve f_H and f_F.

The foreign bloc does not actively implement any carbon pricing policies. The firms in foreign bloc have emission benefits $\hat{\theta}_t E_{F,t} - \frac{f_F}{2} E_{F,t}^2$, implying the first order condition $E_{F,t} = \frac{\hat{\theta}_t}{f_F}$. Therefore, the foreign firms' emission decision is $k(\frac{\phi}{f_F}(\rho\theta_{t-1}) + \frac{\alpha_F}{f_F}\epsilon)$.

For the home bloc, the firm's payoff under a tax τ_t is $\hat{\theta}_t E_{F,t} - \frac{f_F}{2} E_{F,t}^2 - \tau_t E_t$. Firms maximize their payoff resulting in the decision rule

$$E_t^T = e_t^T + \alpha_H \frac{\epsilon_t}{f_H} \text{ with } e_t^T \equiv \frac{\rho\theta_{t-1} - \tau_t}{f_H}$$

.

Under a binding quota, the regulator chooses the acutal emissions level E_T^Q with expected flow net benefit being

$$\rho\theta_{t-1} E_t^Q - \frac{f_H}{2}(E_t^Q)^2 - \frac{b}{2} S_t^2. \tag{1}$$

To formulate the home bloc's regulator's problem, we introduce a indicator function

$$\Phi = \begin{cases} 1 & \text{if tax} \\ 0 & \text{if quota} \end{cases}.$$

The policy bloc's regulator's problem for $i \in \{T, Q\}$ is

$$\max \mathbf{E}_t \Sigma_{\tau=0}^{\infty} \beta^\tau [\rho\theta_{t+\tau-1}) x_{H,t+\tau}^i - \frac{f_H}{2}(x_{H,t+\tau}^i)^2 + \Phi \frac{\alpha_H^2}{2 f_H}\sigma^2 - \frac{b}{2} S_{t+\tau}^2]\phi$$

$$\text{subject to } S_{t+\tau+1} = \delta S_{t+\tau} + \phi \frac{k}{f_F}(\rho\theta_{t-1}) + \phi x_{H,t+\tau}^i + \Phi\phi\alpha_H \frac{\epsilon_t}{f_H} + k\phi\alpha_F \frac{\epsilon_t}{f_F} \text{ and } \theta_t = \rho\theta_{t-1} + \epsilon_t$$

Here we take into consideration the relative size of the foreign bloc and include its annual emissions into the update rule for the stock pollutant.

We define the state vector as $Y_t = (S_t, \theta_{t-1})'$ and the following matrices:

$$Q = \begin{bmatrix} -b & 0 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} \delta & \phi\frac{k\rho}{f_F} \\ 0 & \rho \end{bmatrix}, W = \begin{bmatrix} 0 & \rho \end{bmatrix}, B = \begin{bmatrix} \phi \\ 0 \end{bmatrix}, C = \begin{bmatrix} k\phi\frac{\alpha_F}{f_F} + \Phi\phi\frac{\alpha_H}{f} \\ 1 \end{bmatrix}.$$

Then the net flow payoff and equation of motion for the generic problem for the home bloc are:

$$[-\frac{f_H}{2}x_{H,t}^2 + \frac{1}{2}Y_t'QY_t + WY_t x_{H,t} + \Phi\frac{\alpha_H^2}{2f}\sigma^2]\phi \tag{2}$$

$$Y_{t+1} = AY_t + Bx_{1,t} + C\epsilon_t \tag{3}$$

3.2 Policy ranking

The Social Cost of Carbon, which is the marginal damage of the stock pollutant, is

$$SCC_t = \chi_t + \lambda S_t + \mu\theta_{t-1}$$

We define $R \equiv \frac{\phi\lambda}{f_H}$, which relates the convexity of stock damages to that of abatement costs, and arrive at the following lemma.

Lemma 1. Under both taxes and quotas, the slope of the SCC of the policy bloc with respect to the stock of carbon, relative to the slope of marginal abatement cost is

$$R \equiv \frac{\lambda}{f}\phi = \frac{1}{2\beta}(-(1 - \beta\delta^2 - \beta\frac{b}{f_H}\phi^2) + \sqrt{(1 - \beta\delta^2 - \beta\frac{b}{f_H}\phi^2)^2 + \frac{4\beta\phi^2 b}{f_H}}). \tag{4}$$

The following proposition provides a simple and intuitive ranking criterion for taxes and quotas, which depends on the relative sizes of the economies along with other parameters.

Proposition 1. Given $\rho < 1/\beta\delta$, taxes dominate quotas if and only if

$$R < \frac{1}{\beta} - \frac{2\mu}{\alpha_H} - 2\frac{\alpha_F \lambda \phi}{f_F \alpha_H} \iff R < R^{crit} \equiv -\frac{1}{2}\kappa_1 + \frac{1}{2}\sqrt{\kappa_1^2 + 4\kappa_0} \tag{5}$$

with $\kappa_1 = \dfrac{(1-\beta\delta\rho)(\alpha_H f_F + 2\alpha_F f_H) + 2\beta^2\delta\rho(f_F + f_H k) - \alpha_H f_F \beta}{(\alpha_H f_F + 2\alpha_F f_H)\beta^2}$ and $\kappa_0 = \dfrac{f_F \alpha_H (1-\beta\delta\rho)}{(\alpha_H f_F + 2\alpha_F f_H)\beta^2}$

Note: for the R^{crit} to be real root, we need to make sure the discriminant is positive.

Proposition 2. a) An increase in k favors quotas for home bloc, that is when the foreign bloc is larger, R_{crit} becomes smaller.
b) Given the range of parameters specified in Table 1, an increase in α_H favors taxes for home bloc, that is, a faster technology diffusion favors taxes.

Proof. To show that a reduction in k favors taxes, we note that R_{crit} is a differentiable function of k. Using the chain rule and the definitions of κ_0 and κ_1, we obtain

$$\begin{aligned}\frac{\partial R_{crit}}{\partial k} &= \frac{\delta f_H \rho}{\alpha_H f_F + 2\alpha_F f_H}\left(-1 + \frac{\kappa_1}{\sqrt{\kappa_1^2 + 4\kappa_0}}\right) \\ &= \frac{\delta f_H \rho}{\alpha_H f_F + 2\alpha_F f_H}\left(\frac{\kappa_1 - \sqrt{\kappa_1^2 + 4\kappa_0}}{\sqrt{\kappa_1^2 + 4\kappa_0}}\right)\end{aligned}$$

Thus an increase in k lowers the critical value R_{crit}. By definition, taxes dominate quotas if and only if $R < R_{crit}$. Therefore, an increase in k favors quotas. □

Proof. Again, R_{crit} is a differentiable function of α_H. Using the chain rule and the definitions of κ_0 and κ_1, we obtain

$$
\begin{aligned}
\frac{\partial R_{crit}}{\partial \alpha_H} &= -\frac{1}{2}\frac{\partial \kappa_1}{\partial \alpha_H} + \frac{1}{4}\frac{1}{\sqrt{\kappa_1^2+4\kappa_0}}\frac{\partial(\kappa_1^2+\kappa_0)}{\partial \alpha_H} \\
&= \frac{1}{2}(\frac{f_F}{\alpha_H f_F + 2\alpha_F f_H}\kappa_1 + \frac{f_F\beta}{\beta^2(\alpha_H f_F + 2\alpha_F f_H)} - \frac{\kappa_0}{\alpha_H}) + \\
&\frac{1}{2}\frac{1}{\sqrt{\kappa_1^2+4\kappa_0}}[\frac{\beta f_F}{\beta^4(\alpha_H f_F + 2\alpha_F f_H)^2} + \frac{f_F(1-\beta\delta\rho)}{\beta^2(\alpha_H f_F + 2\alpha_F f_H)}\kappa_1 - \\
&\frac{f_F}{\alpha_H f_F + 2\alpha_F f_H}\kappa_1^2 - \frac{2f_F}{\alpha_H f_F + 2\alpha_F f_H}\kappa_0 + \frac{2\kappa_0}{\alpha_H}]
\end{aligned}
$$

Let the parameter range be what is assumed in Table 1, we know that

$$
\frac{f_F}{\alpha_H f_F + 2\alpha_F f_H} > 0. \tag{6}
$$

Note that $\frac{f_F\beta}{\beta^2(\alpha_H f_F+2\alpha_F f_H)} - \frac{\kappa_0}{\alpha_H} = \frac{f_F}{(\alpha_H f_F+2\alpha_F f_H)}(\frac{1}{\beta} - \frac{1-\beta\delta\rho}{\beta^2})$. Since $(1-\beta\delta\rho) < 1$ and $\beta \in [0.8, 1]$, we have $(\frac{1}{\beta} - \frac{1-\beta\delta\rho}{\beta^2}) > 0$ and hence we obtain

$$
\frac{f_F\beta}{\beta^2(\alpha_H f_F + 2\alpha_F f_H)} - \frac{\kappa_0}{\alpha_H} > 0 \tag{7}
$$

.

We know that $\sqrt{\kappa_1^2+4\kappa_0} > \kappa_1$, since $\kappa_1 > 0$, and thus $\frac{\kappa_1^2}{\sqrt{\kappa_1^2+4\kappa_0}} < \kappa_1$. Hence, we have

$$
\frac{f_F}{\alpha_H f_F + 2\alpha_F f_H}\kappa_1 > \frac{1}{\sqrt{\kappa_1^2+4\kappa_0}}\frac{f_F}{\alpha_H f_F + 2\alpha_F f_H}\kappa_1^2 \tag{8}
$$

.

Similarly, we know that

$$
\frac{1}{\sqrt{\kappa_1^2+4\kappa_0}}\frac{2f_F}{\alpha_H f_F + 2\alpha_F f_H}\kappa_0 < \frac{1}{\sqrt{\kappa_1^2+4\kappa_0}}\frac{2\kappa_0}{\alpha_H}. \tag{9}
$$

All parameters are assumed to be positive in Table 1, and hence the other terms in $\frac{\partial R_{crit}}{\partial \alpha_H}$ are positive. Therefore, we obtain that $\frac{\partial R_{crit}}{\partial \alpha_H} > 0$, and an increase in α_H favors taxes. □

Note: we probably don't need to restrict ρ at all. https://www.overleaf.com/project/61125128610b3f3f0d7719d1

4 Strategic Interaction Model

We extend the non-strategic dynamic model to one in which each bloc assumes a fixed strategy profile of its counterpart. We define $C_H = (c_{1H}, c_{2H})$ and $C_F = (c_{1F}, c_{2F})$ to be strategies home bloc and foreign bloc choose respectively, and we define the state vector Y_t such that $Y_t = (S_t, \theta_{t-1}, 1)'$. Let k_H, k_F be the relatively sizes of the home bloc and foreign bloc such that $k_H + k_F = 1$. Home bloc assumes the strategy of foreign bloc will be fixed such that $c_F Y_t = c_{1F} S_t + c_{2F} \theta_{t-1}$. Foreign bloc assumes the strategy of home bloc will be fixed at $c_H Y_t = c_{1H} S_t + c_{2H} \theta_{t-1}$. We can find the Nash Equilibrium for the game by solving for C_H and C_F. An economy's optimal control would be a linear combination of the state variables and therefore its optimal policy depends on the strategies they take.

Lemma 2. The strategy profiles C_H and C_F are vectors in $\mathbf{R}^2$ and the control variables in the dynamic programming problem $x_{F,t+\tau} = C_F Y_t$ and $x_{H,t+\tau} = C_H Y_t$ have no constant terms.

To formulate the regulator's problem for both blocs, we introduce indicator functions Φ_1 and Φ_2 for home and foreign, respectively, such that:

$$\Phi_1 = \begin{cases} 1 & \text{if tax} \\ 0 & \text{if quota} \end{cases} \quad \text{and, } \Phi_2 = \begin{cases} 1 & \text{if tax} \\ 0 & \text{if quota} \end{cases}.$$

Then home bloc's regulator's problem for $i \in \{T, Q\}$ is :

$$
\max \mathbf{E}_t \Sigma_{\tau=0}^{\infty} \beta^{\tau} [\rho\theta_{t+\tau-1} x^i_{H,t+\tau} - \frac{f_H}{2}(x^i_{H,t+\tau})^2 + \Phi_1 \frac{\alpha_H^2}{2f_H}\sigma^2 - \frac{k_H b}{2} S^2_{t+\tau}]\phi
$$

$$
\text{subject to } S_{t+\tau+1} = \delta S_{t+\tau} + k_H \phi x^i_{H,t+\tau} + \phi k_F c_F Y_t + \Phi_1 k_H \phi \alpha_H \frac{\epsilon_t}{f_H} + \Phi_2 k_F \phi \alpha_F \frac{\epsilon_t}{f_F} \text{ and } \theta_t = \rho\theta_{t-1} + \epsilon_t \tag{10}
$$

Similarly, the foreign bloc's regulator's problem for $i \in \{T, Q\}$ is :

$$
\max \mathbf{E}_t \Sigma_{\tau=0}^{\infty} \beta^{\tau} [\rho\theta_{t+\tau-1} x^i_{F,t+\tau} - \frac{f_F}{2}(x^i_{F,t+\tau})^2 + \Phi_2 \frac{\alpha_F^2}{2f_F}\sigma^2 - \frac{k_F b}{2} S^2_{t+\tau}]\phi
$$

$$
\text{subject to } S_{t+\tau+1} = \delta S_{t+\tau} + k_F \phi x^i_{F,t+\tau} + \phi k_H c_H Y_t + \Phi_1 k_H \phi \alpha_H \frac{\epsilon_t}{f_H} + \Phi_2 k_F \phi \alpha_F \frac{\epsilon_t}{f_F} \text{ and } \theta_t = \rho\theta_{t-1} + \epsilon_t \tag{11}
$$

We define the following matrices:

$$
Q_H = \begin{bmatrix} -k_H b_H & 0 \\ 0 & 0 \end{bmatrix}, A_H = \begin{bmatrix} \delta + \phi c_{1F} k_F & \phi c_{2F} k_F \\ 0 & \rho \end{bmatrix}, W_H = \begin{bmatrix} 0 \\ \rho \end{bmatrix}', B_H = \begin{bmatrix} \phi k_H \\ 0 \end{bmatrix}, C_H = \begin{bmatrix} \Phi_1 k_H \phi \frac{\alpha_H}{f_H} + \Phi_2 k_F \phi \frac{\alpha_F}{f_F} \\ 1 \end{bmatrix} \tag{12}
$$

With this notation, the net flow payoff and equation of motion for the linear quadratic problem of home bloc are:

$$
[-\frac{f_H}{2} x^2_{H,t} + \frac{1}{2} Y_t' Q_H Y_t + W_H Y_t x_{H,t} + \Phi_2 \frac{\alpha_H^2}{2f_H}\sigma^2]\phi \tag{13}
$$

$$
Y_{t+1} = A_H Y_t + B_H x_{H,t} + C_H \epsilon_t \tag{14}
$$

Since the linear quadratic problems for home bloc and foreign bloc are symmetric, we define the following matrices for the generic problem for foreign bloc:

$$Q_F = \begin{bmatrix} -k_F b_F & 0 \\ 0 & 0 \end{bmatrix}, A_F = \begin{bmatrix} \delta + \phi c_{1H} k_H & \phi c_{2H} k_H \\ 0 & \rho \end{bmatrix}, W_F = \begin{bmatrix} 0 \\ \rho \end{bmatrix}', B_F = \begin{bmatrix} \phi k_F \\ 0 \end{bmatrix}, C_F = \begin{bmatrix} \Phi_1 k_H \phi \frac{\alpha_H}{f_H} + \Phi_2 k_F \phi \frac{\alpha_F}{f_F} \\ 1 \end{bmatrix} \tag{15}$$

Then the net flow payoff and equation of motion for the linear quadratic problem of foreign bloc are:

$$[-\frac{f_F}{2} x_{F,t}^2 + \frac{1}{2} Y_t' Q_F Y_t + W_F Y_t x_{F,t} + \Phi_1 \frac{\alpha_F^2}{2 f_F} \sigma^2]\phi \tag{16}$$

$$Y_{t+1} = A_F Y_t + B_F x_{F,t} + C_F \epsilon_t \tag{17}$$

We use the same procedure as in appendix A.1 to derive the expressions for control variables by writing the right side of these dynamic programming problems in terms of the control variables. This results in the formula for the derivative of the SCC with respect to the carbon stock for home bloc:

$$\begin{aligned} \lambda_H = & \frac{f_H c_{1F}^2 k_F^2 \phi^2 + 2 f^H c_{1F} \delta k_F \phi + f_H \delta^2 + b k_H^3 \phi^2}{2 k_H^2 \phi} - \frac{f_H}{2 \beta k_H^2 \phi} \\ & \pm \frac{1}{2 \beta k_H^2 \phi} \sqrt{\begin{aligned} & \beta^2 b^2 k_H^4 \phi^4 + 2 \beta^2 b c_{1F}^2 f_H k_F^2 k_H^2 \phi^4 + 4 \beta^2 b c_{1F} \delta f_H k_F k_H^2 \phi^3 + 2 \beta b f_H k_H^2 \phi^2 + \beta^2 c_{1F}^4 \delta f_H^2 k_F^4 \phi^4 + \\ & 4 \beta^2 c_{1F}^3 \delta f_H^2 k_F^3 \phi^3 + 4 \beta^2 c_{1F}^2 \delta^2 f_H^2 k_F^2 \phi^2 - 2 \beta c_{1F}^2 f_H^2 k_F^2 \phi^2 - 4 \beta c_{1F} \delta f_H^2 k_F \phi + f_H^2 \end{aligned}} \end{aligned} \tag{18}$$

We also have the formula for the derivative of the SCC with respect to the technology realization for home bloc:

$$\mu = \frac{\beta \lambda \phi (k_H \rho + c_{2F} f_H k_F)(\delta + c_{1F} k_F \phi)}{\lambda \phi \beta k_H^2 + f_H - \delta f_H \rho \beta - c_{1F} f_H k_F \phi \rho \beta} \tag{19}$$

To solve for C_F and C_H, we substitute the formula for λ and μ into the control variables and we solve the system of nonlinear equations as follows:

$$\begin{aligned} c_{1H} &= \psi_{1H}(\lambda, \beta, k_H, k_F, \delta, c_{1F}, \phi, f_H), \\ c_{2H} &= \psi_{2H}(\lambda, \mu, \rho, \beta, k_H, k_F, c_{2F}, \phi, f_H), \\ c_{1F} &= \psi_{1F}(\lambda, \beta, k_H, k_F, \delta, c_{1H}, \phi, f_F), \\ c_{2F} &= \psi_{2F}(\lambda, \mu, \rho, \beta, k_H, k_F, c_{2H}, \phi, f_F). \end{aligned} \tag{20}$$

such that $x_{H,t} = \psi_{1H} S_t + \psi_{2H}\theta_{t-1}$ and $x_{F,t} = \psi_{1F} S_t + \psi_{2F}\theta_{t-1}$.

This dynamic programming problem is too complicated to produce analytic results with the introduction of strategy profiles above. We instead provide an accurate empirical description by solving the model numerically and select parameter values to provide an economically meaningful context.

5 Calibration and Numerical Results

The solutions for the strategic model describe in section 4 is many times more complicated than that of the unilateral model described in section 3. Therefore, we calibrate the dynamic model and present the numerical results.

5.1 Baseline Parameter Values

We adopt similar baseline parameter values used in , which calibrates their model to reasonable climate change scenarios using DICE 2013 (Nordhaus and Sztorc 2013) and IPCC (2014). Table 1 summarizes the baseline values for the key parameters in the model, the way they are calibrated and the range that they vary in our numerical calculation.

Marginal abatement cost f and marginal damage b are calibrated using DICE 2013. Setting abatement at 75% of the optimal level in DICE, we obtain values for marginal abatement costs during the period 2015 - 2050. Fitting the average of these values to the linear marginal abatement cost function we obtain f = 1.846 in units of billion dollars per billion tonnes of CO_2 (G\$/Gt CO_2).

To calculate the damage parameter b we set annual Gross World Product (GWP) to the IMF's 2016 estimate of 120 trillion dollars using purchasing power parity. In DICE's climate model, an increase of atmospheric carbon dioxide by 1270 Gt CO_2 over the preindustrial level implies a medium to long-run temperature increase of 2∘C. DICE assumes that this temperature increase lowers output by approximately 1%. This calibration assumption implies $b \approx 0.0015$ (G\$/Gt CO_2).

For α, we follow Karp and Traeger (2021), who estimate technological diffusion by regressing US CO_2 emissions in 1995-2010 on (stocks and flows of) green patents. Their preferred estimate is slightly above .25. We use $\alpha = .25$ as our baseline. We use the annual discount factor $\beta = 0.98$, consistent with the median 2% discount rate in the recent expert survey by Drupp et al. (2018). We assume that technological innovation is highly persistent and use $\rho = .99$. We calibrate the persistence of atmospheric carbon to Joos et al.'s (2013) model for carbon removal from the atmosphere, adopted in the 5th Assessment Report of the (IPCC 2014). A least square fit over 1000 years delivers an annual removal rate of 0.3%, or $\delta = 0.997$. In the results presented below, we use $\phi = 1$[1] and $\sigma^2 = 1$[2].

5.2 Numerical Results

To investigate how changes in the key parameters influence the equilibrium of the dynamic model, we solve the two-economy problem numerically varying each parameter while holding others at the baseline values,

The first set of results, presented in Figure 1, concern the size parameter k_H, with the other parameters at baseline values and identical for Home and Foreign. Panel (a) shows c_1's, the coefficients for Home and Foreign regulator's optimal response to cumulative emissions; Panel (b) shows c_2's, the response to technology level observed by the regulators at the onset of a regulatory period; Panel (c) shows the λ's, the marginal impact of cumulative emissions S_t on the SCC; and Panel (d) shows the μ's, the marginal impact of technology level θ_t on the SCC. Since the two economies have the same parameters except for their relative sizes, each set of coefficients are symmetric about $k_H = 0$, at which point the two economies are identical. As the relative size of an

[1]Using a different value of ϕ would not change the results qualitatively.

[2]For the set of results presented below, σ^2 only affects Δ. We can read the scale for Δ as in units of σ^2.

economy increases, its regulator becomes more responsive to cumulative emissions (i.e., c_1 becomes more negative) and less responsive to the observed technological level (i.e.,c_2 becomes less positive). Consistently, λ becomes more positive, as a larger economy suffers more from future damage of cumulative emissions. Meanwhile, μ initially becomes more positive, but then peaks and decreases.

Panels (e) and (f) show the value of Δ_H for $\alpha = .25$ and .50 respectively. The blue curves represent Δ_H when Foreign chooses quota, and the red curves represent Δ_H when Foreign chooses tax. When Foreign chooses tax, it introduces additional uncertainty in the cumulative emissions in future periods associated with the technology innovation, thereby favoring quota for Home[3]. We observe that when the economy is relatively small, tax is the preferred instrument. As the relative size becomes larger, Δ_H decreases and becomes negative, leaving quota the preferred instrument. At high values of k_H turns to increase with k_H. With some high α values, tax will be preferred by Home with small and large k_Hs, with quota the preferred instrument in an intermediate range of k_H.

The instrument preference for Foreign is not depicted directly in the figures. However, Δ_F and Δ_H must be symmetric about $k_H = .50$ in a similar fashion to the series in Panels (a) through (d). We can then use the information provided in Panel (e) to find the Nash equilibrium of the non-cooperative game between Home and Foreign when $\alpha = .25$. When k_H is small, Home prefers tax ($\Delta_H > 0$) and Foreign prefers quota ($\Delta_F < 0$) regardless of the instrument of choice of the other economy; therefore, Home choosing tax and Foreign choosing quota ($\Phi_H = 1$ and $\Phi_F = 0$, we denote it by T-Q) is a Nash equilibrium for the non-cooperative game. As k_H increases, Δ_Hs decrease and intersect with the zero line. Since Foreign still prefers quota (Δ_F ¡ 0 regardless of Φ_H) at the relevant values of k_H, the relevant series is the blue curve ($\Delta_{H,\Phi_F=0}$). After it hit zero, quota-quota (Q-Q) becomes the equilibrium. By symmetry, after ($\Delta_{F,\Phi_H=0}$) turns positive, Q-T becomes the equilibrium. To summarize, for small values of k_H, T-Q is the equilibrium; for intermediate values of k_H, Q-Q is the equilibrium; and for large values of k_H, Q-T is the equilibrium. This pattern remains for $\alpha = .50$, but the intermediate range with Q-Q being the equilibrium is now

[3] Equation **??** in Appendix A.2 shows the difference between $\Delta_{H,\Phi_F=0}$ and $\Delta_{H,\Phi_F=1}$. This difference increases with λ, which is increasing in k_H

smaller. Larger values of α will lead to T-T being the intermediate equilibrium. α close to 1 will have both economies preferred tax at all times.

The previous discussion suggest that a small α, or slow diffusion of technology, tends to favor quota. The next set of results concern in more detail with the effect of α on instrument preference. As shown in Appendix A.2, the αs do not factor into the solution to the dynamic problem, i.e., they do not affect c_1, c_2, λ or μ. Rather, they appear in the value function, and therefore the expressions for Δs. Figure 2 presents the numerical results when α_H and α_F vary together. Panels (a), (b) and (c) show the value of Δ_H when $k_H = .25, .50$ and $.75$ respectively. Same as previously, the blue curves represent Δ_H when Foreign chooses quota, and the red curves represent Δ_H when Foreign chooses tax. In each case, $\Delta_H = 0$ when $\alpha = 0$; decreases as α increases from 0; reaches a minimum and then increases as α increases further. The value of α where the minimum of Δ_H is attained seems to be larger with a larger k_H[4]. When $\alpha = 0$, there is in fact no uncertain adoption of technological innovation that is not observed by the regulator, therefore the price and quantity instruments are equivalent. Small values of α favor quota while larger values favor tax.

We can then characterize the equilibrium of the non-cooperative game between Home and Foreign in choosing pricing instruments. Using the same logic for symmetry discussed previously, we can view Panels (a), (b) and (c) as depicting Δ_F when $k_H = .75, .50$ and $.25$ respectively. When $k_H = .25$, Q-Q is the equilibrium with low values of α, as both Δ_H (Panel (a)) and Δ_F (Panel (c)) are negative; T-T is the equilibrium with high values of α. In the intermediate values of α between where the blue curve in Panel (a) crosses zero and where the red curve in Panel (c) crosses zero, T-Q is the equilibrium. This pattern applies to low values of k_H. With high values of k_H, the intermediate equilibrium is Q-T. When $k_H = .50$, the intermediate range is between the points at which the blue and red curves intersect the zero line in Panel (b). In this small range, there are multiple equilibria in Q-T and T-Q. Similar multiple equilibra exist when the segments of α between the crossing of the blue and red curves and the zero line for Home and Foreign overlap each other.

[4]This is not always the case when k_H is over a certain value. As shown in Panels (e) and (e) of Figure 1, Δ_H is not monotonic with respect to k_H

6 Conclusion

We present a model of two economies strategically pursuing regulation on a transboundary stock pollutant featuring correlated technology shocks. Our numerical results suggest that the relative sizes of the two economies and the speed of technological diffusion are key parameters affection the ranking between prices versus quantities instruments. Therefore, as countries around the world move to design their domestic policies to achieve their commitment under the Paris Agreement, policymakers may adopt different instruments for carbon pricing. It will be interesting to compare the instruments of choice of countries of varying sizes. Additionally, the Technology Mechanism in the Paris Agreement may help innovation and transfer of climate-friendly technology, and it may tip the scale of quantity versus price instruments under uncertainty. We also recognize that the model is also missing some important features of the world economy. In particular, there is no trade between countries, with the only connection through the global bad of emissions. Such issues need to be addressed in future research.

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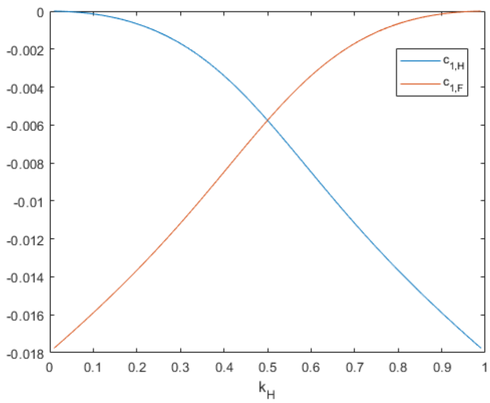
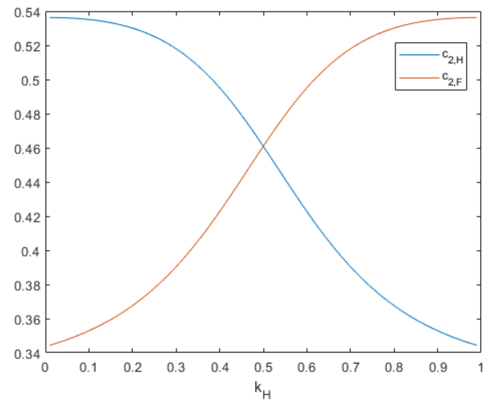
Figures and Tables

Table 1: Baseline Parameter Values and Range in Simulation

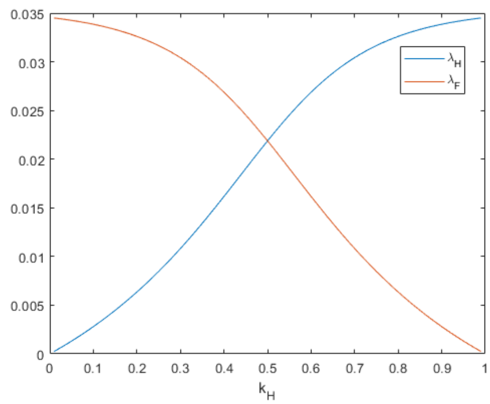
Parameter	Method/Source	Baseline Value	Range
f	DICE model	1.846	0.01-3
b	DICE model + IMF	0.0015	0.0003-0.0099
α	Karp and Traeger (2021)	0.25	0.01-0.99
β	Drupp et al. (2018)	0.98	0.90-0.99
δ	Joos et al. (2013)	0.997	0.01-0.99
ρ	by assumption	0.99	0.01-0.99
k_H	by assumption	0.5	0.01-0.99



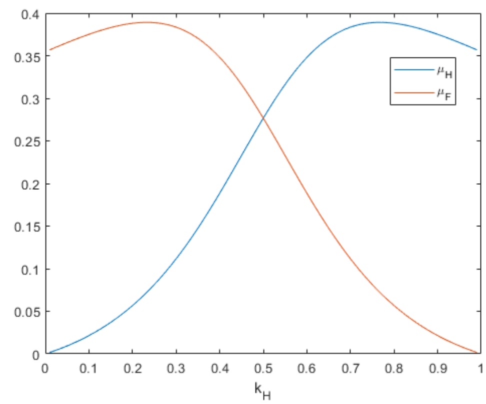
(a) $c_{1,H}$ and $c_{1,F}$



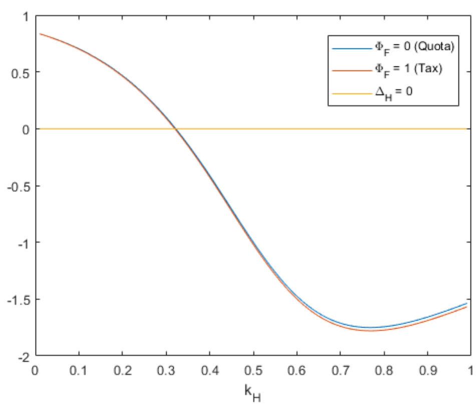
(b) $c_{2,H}$ and $c_{2,F}$



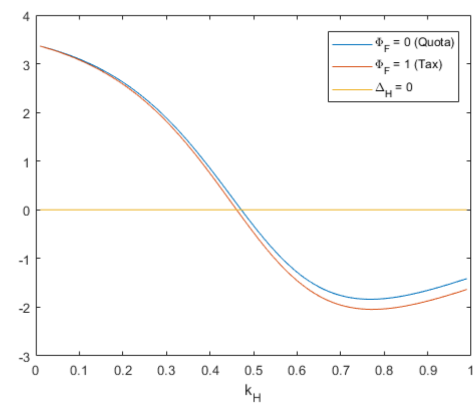
(c) λ_H and λ_F



(d) μ_H and μ_F



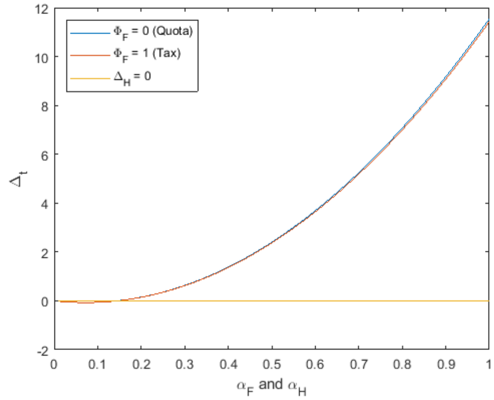
(e) $\Delta_H(\alpha = .25)$



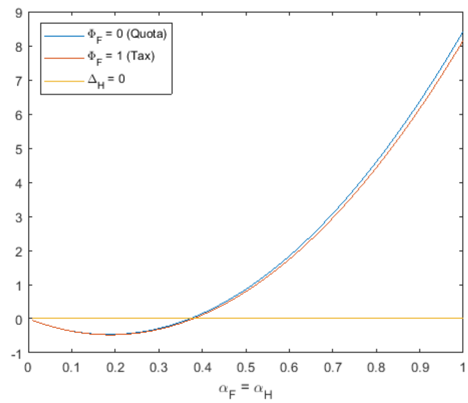
(f) $\Delta_H(\alpha = .5)$

Figure 1: Numerical Results as k_H Varies.

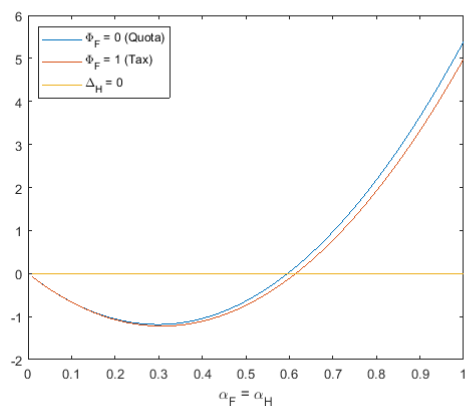
Note: Unless otherwise specified, all parameters take baseline values as listed in Table 1.
Source: Authors' calculation.



(a) Δ_H ($k_H = .25$)



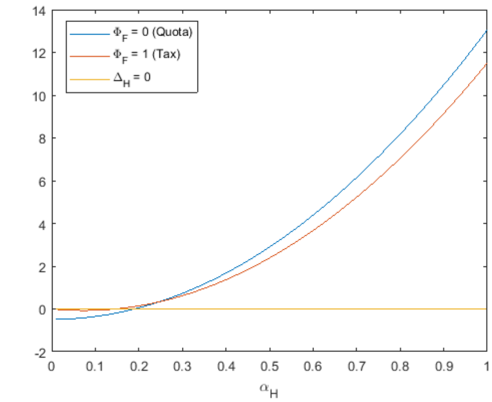
(b) Δ_H ($k_H = .5$)



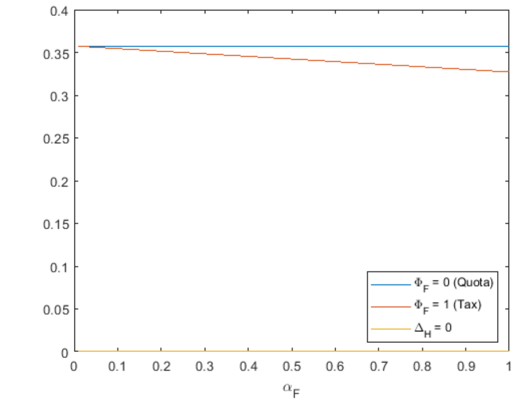
(c) Δ_H ($k_H = .75$)

Figure 2: Numerical Results as α Varies.

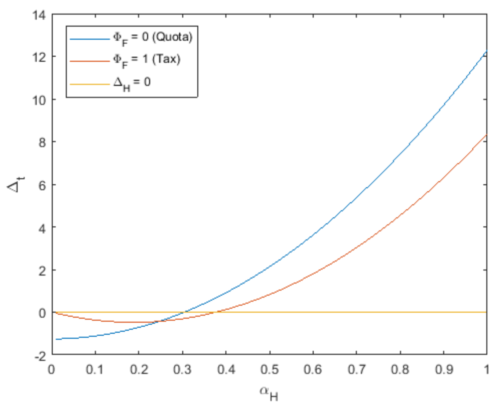
Note: Unless otherwise specified, all parameters take baseline values as listed in Table 1. α_H and α_F assume the same value in this set of calculation.
Source: Authors' calculation.



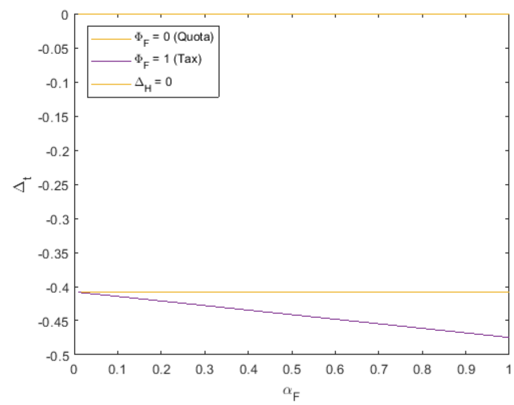
(a) Δ_H ($k_H = .25$) as α_H varies



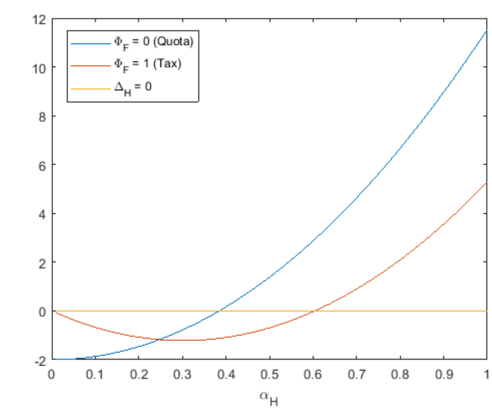
(b) Δ_H ($k_H = .25$) as α_F varies



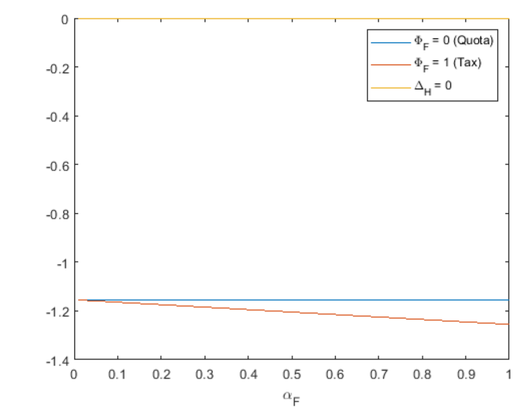
(c) Δ_H ($k_H = .5$) as α_H varies



(d) Δ_H ($k_H = .5$) as α_F varies



(e) Δ_H ($k_H = .75$) as α_H varies



(f) Δ_H ($k_H = .75$) as α_F varies

Figure 3: Numerical Results as α_H or α_F Varies.

Note: Unless otherwise specified, all parameters take baseline values as listed in Table 1. One α is fixed at .25 while the other varies.
Source: Authors' calculation.

A Appendix

A.1 Proofs for Section 3

Proof for Lemma 1. The dynamic programming equation for the generic problem is:

$$J_t(Y_t) = \max[-\frac{f_H}{2}x_{H,t}^2 + \frac{1}{2}Y_t'QY_t + WY_tx_{H,t} + \Phi\frac{\alpha_H^2}{2f_H}\sigma^2]\phi + \beta\mathbf{E}_tJ_{t+1}^i(Y_{t+1}) \tag{A1}$$

The value function for the LQ problem is

$$J_t^i(Y_t) = V_0^i + V_{1,t}'Y_t + \frac{1}{2}Y_t'V_2Y_t \tag{A2}$$

where

$$V_2 = -\begin{bmatrix} \lambda & \mu \\ \mu & \nu \end{bmatrix}. \tag{A3}$$

The right side of the DPE is

$$[-\frac{f_H}{2}x_{H,t}^2 + \frac{1}{2}Y_t'QY_t + WY_tx_{H,t} + \Phi\frac{\alpha_H^2}{2f_H}\sigma^2]\phi + \beta\mathbf{E}_t(V_{0,t+1} + V_{1,t+1}'Y_{t+1} + \frac{1}{2}Y_{t+1}'V_2Y_{t+1}) \tag{A4}$$

Substituting the equations of motion and take expectation, we have:

$$\begin{aligned}(-\frac{f_H}{2}x_{H,t}^2+\frac{1}{2}Y_t'QY_t + WY_tx_{H,t} + \Phi\frac{\alpha_H^2}{2f_H}\sigma^2)\phi + \beta[V_{0,t+1} + V_{1,t+1}'(AY_t + Bx_{H,t})+ \\ \frac{1}{2}(AY_t + Bx_{H,t})'V_2(AY_t + Bx_{H,t}) + \frac{1}{2}\sigma^2C'V_2C]\end{aligned} \tag{A5}$$

The FOC is:

$$\phi(-f_H x_{1,t} + WY_t) + \beta(V_1'B + B'V_2 B x_{H,t} + B'V_2 AY_t) \tag{A6}$$

$$\implies \text{(control rule) } x_t = \frac{1}{f_H\phi - \beta(B'V_2B)}(\beta V_1'B + (W\phi + \beta B'V_2A)Y_t) \tag{A7}$$

Rewrite x_t in terms of Z_{0t} and Z and Y_t such that

$$Z_0 = \frac{\beta V_1'B}{f_H\phi - \beta(B'V_2B)} \text{ and } Z = \frac{W\phi + \beta B'V_2A}{f_H\phi - \beta(B'V_2B)} \tag{A8}$$

Then, we have $x_t = Z_0 + ZY_t$.

Substituting the control rule into the expectation of the r.h.s of the DPE and getting the quadratic terms should be the same as in the Karp paper.

So we have

$$V_2 = [(Q - f_H Z'Z + W'Z + Z'W)\phi + \beta(A + BZ)'V_2(A + BZ)] \tag{A9}$$

Then, solving for λ, we obtain

$$\lambda = \frac{(b^2\beta^2\phi^4 + 2b\beta^2\delta^2 f_H\phi^2 + 2b\beta f_H\phi^2 + \beta^2\delta^4 f_H^2 - 2\beta\delta^2 f_H^2 + f_H^2)^{1/2} - f_H + \beta\delta^2 f_H + b\beta\phi^2}{(2\beta\phi)} \tag{A10}$$

(A11)

Let $\overline{\omega} \equiv f_H(1 - \beta\delta^2 - \beta\frac{b}{f_H}\phi^2)$, the positive root of is

$$\lambda = \frac{1}{2\beta\phi}(-\overline{\omega} + \sqrt{\overline{\omega^2} + 4\beta\phi^2 b f_H}). \tag{A12}$$

□

Similarly we can write μ as follows:

$$\mu = \beta\delta\frac{\rho}{f_H + \beta\lambda\phi}(f_H\mu + \lambda\phi + \frac{f_H}{f_F}k\lambda\phi) \implies \mu = \frac{(\beta\delta\lambda\phi\rho(f_F + f_H k))}{(f_F(f_H + \beta\lambda\phi - \beta\delta f_H \rho))} \tag{A13}$$

Hence we have

$$\mu = \beta\delta\rho\frac{\lambda}{f_H}\frac{\phi}{1 - \beta\delta\rho + \beta\frac{\lambda}{f_H}\phi}(1 + \frac{f_H}{f_F}k) \tag{A14}$$

Proof for Proposition 1. We know that

$$V_0^i = (-\frac{1}{2}f_F(Z_0)^2 + +\Phi\frac{\alpha_H^2}{2f_H}\sigma^2)\phi + \beta[V_0 + V_1'BZ_0 + \frac{1}{2}(BZ_0)'V_2(BZ_0) + \frac{1}{2}\sigma^2 C'V_2 C] \tag{A15}$$

Since V_t, Z_0 and V_2 are the same under taxes and quotas, we have

$$\Delta = V_0^T - V_0^Q \tag{A16}$$

$$= \frac{\alpha_H^2}{2f_H}\sigma^2\phi + \beta\Delta - \frac{1}{2}\beta\sigma^2(\begin{bmatrix} k\phi\frac{\alpha_F}{f_F} + \phi\frac{\alpha_H}{f} & 1 \end{bmatrix}\begin{bmatrix} \lambda & \mu \\ \mu & \nu \end{bmatrix}\begin{bmatrix} k\phi\frac{\alpha_F}{f_F} + \phi\frac{\alpha_H}{f} \\ 1 \end{bmatrix} - \begin{bmatrix} \phi\frac{\alpha_F}{f_F} & 1 \end{bmatrix}\begin{bmatrix} \lambda & \mu \\ \mu & \nu \end{bmatrix}\begin{bmatrix} \phi\frac{\alpha_F}{f_F} \\ 1 \end{bmatrix}) \tag{A17}$$

$$= \frac{\alpha_H^2}{2f_H}\sigma^2\phi + \beta\Delta - \frac{1}{2}\beta\sigma^2\phi\alpha_H\frac{2f_F f_H\mu + 2\alpha_F f_H\lambda\phi + \alpha_H f_F\lambda\phi}{f_F f_H^2} \tag{A18}$$

Therefore, the difference between taxes and quotas

$$\Delta = \frac{1}{1-\beta}\frac{\alpha_H\phi}{2f_H}\sigma^2(\alpha_H - \beta\frac{2f_F f_H\mu + 2\alpha_F f_H\lambda\phi + \alpha_H f_F\lambda\phi}{f_F f_H}). \tag{A19}$$

Using the definition $R \equiv \frac{\lambda}{f_H}\phi$, we obtain

$$\Delta = \frac{1}{1-\beta}\frac{\alpha_H \phi}{2 f_H}\sigma^2(\alpha_H - \beta(2\mu + 2\frac{\alpha_F \lambda \phi}{f_F} + \alpha_H R)). \tag{A20}$$

This equation implies taxes dominate quotas if and only if

$$\alpha_H - \beta(2\mu + 2\frac{\alpha_F \lambda \phi}{f_F} + \alpha_H R) > 0. \tag{A21}$$

So

$$R < \frac{1}{\beta} - \frac{2\mu}{\alpha_H} - 2\frac{\alpha_F \lambda \phi}{f_F \alpha_H}. \tag{A22}$$

Using the definition $R \equiv \frac{\lambda}{f_H}\phi$ and $\mu = \frac{\beta\delta\rho R}{(1-\beta\delta\rho)+\beta R}(1+\frac{f_H}{f_F}k)$, we have

$$\alpha_H - \beta(2\frac{\beta\delta\rho R}{(1-\beta\delta\rho)+\beta R}(1+\frac{f_H}{f_F}k) + 2\frac{\alpha_F \lambda \phi}{f_F} + \alpha_H R) > 0. \tag{A23}$$

Note that $\frac{\alpha_F \lambda \phi}{f_F} = \frac{\alpha_F R f_H}{f_F}$.

Then,

$$\begin{aligned} &\alpha_H - \beta(2\frac{\beta\delta\rho R}{(1-\beta\delta\rho)+\beta R}(1+\frac{f_H}{f_F}k) + 2\frac{\alpha_F f_H}{f_F}R + \alpha_H R) > 0 \\ \equiv\ & R^2 + \frac{(1-\beta\delta\rho)(\alpha_H f_F + 2\alpha_F f_H) + 2\beta^2\delta\rho(f_F + f_H k) - \alpha_H f_F \beta}{(\alpha_H f_F + 2\alpha_F f_H)\beta^2}R - \frac{f_F \alpha_H (1-\beta\delta\rho)}{(\alpha_H f_F + 2\alpha_F f_H)\beta^2} < 0 \\ \equiv\ & R^2 + \kappa_1 R - \kappa_0 < 0 \end{aligned} \tag{A24}$$

with $\kappa_1 = \frac{(1-\beta\delta\rho)(\alpha_H f_F + 2\alpha_F f_H) + 2\beta^2\delta\rho(f_F + f_H k) - \alpha_H f_F \beta}{(\alpha_H f_F + 2\alpha_F f_H)\beta^2}$ and $\kappa_0 = \frac{f_F \alpha_H (1-\beta\delta\rho)}{(\alpha_H f_F + 2\alpha_F f_H)\beta^2}$

□

A.2 Proofs for Section 4

Proof for Lemma 2. Assume there exists a constant term $c_{0H} \neq 0$ in home's control variable $x_{H,t+\tau}$ such that $x_{H,t+\tau} = c_{0H} + c_{1F}S_t + c_{2F}\theta_{t-1}$. We know that

$$V_1' = [-f(Z_0Z) + Z_0W]\phi + \beta[V_1'A + V_1'BZ + (BZ_0)'V_2(A + BZ)], \tag{A25}$$

and equating the 1,1 elements on both sides, we obtain the difference equation

$$\nu_{1t} = \frac{\beta f_H \nu_{1,t+1}(\delta + c_{1F}k_F\phi)}{\lambda\phi\beta k_H^2 + f_H}. \tag{A26}$$

Then we have

$$\nu_{1t} = \begin{cases} K & \text{if } f_H + \lambda\beta k_H^2 = \delta\beta f_H + c1_F k_F, \\ 0 & \text{otherwise,} \end{cases} \tag{A27}$$

such that $K \in \mathbf{R}$ is an arbitary constant.

$$\lambda_H = \frac{(\delta\beta - 1)f_H + c1_F k_F}{\beta k_H^2}$$

However, λ_H takes the form as follows:

$$\lambda_H = \frac{f_H c_{1F}^2 k_F^2 \phi^2 + 2f^H c_{1F}\delta k_F\phi + f_H\delta^2 + bk_H^3\phi^2}{2k_H^2\phi} - \frac{f_H}{2\beta k_H^2\phi}$$
$$\pm \frac{1}{2\beta k_H^2\phi}\sqrt{\begin{aligned}\beta^2 b^2 k_H^4\phi^4 + 2\beta^2 bc_{1F}^2 f_H k_F^2 k_H^2\phi^4 + 4\beta^2 bc_{1F}\delta f_H k_F k_H^2\phi^3 + 2\beta bf_H k_H^2\phi^2 + \beta^2 c_{1F}^4\delta f_H^2 k_F^4\phi^4 + \\ 4\beta^2 c_{1F}^3\delta f_H^2 k_F^3\phi^3 + 4\beta^2 c_{1F}^2\delta^2 f_H^2 k_F^2\phi^2 - 2\beta c_{1F}^2 f_H^2 k_F^2\phi^2 - 4\beta c_{1F}\delta f_H^2 k_F\phi + f_H^2\end{aligned}} \tag{A28}$$

Hence $\nu_{1t} = 0$. Note that $Z_0 = \frac{\beta V_1' B}{f_H \phi - \beta(B' V_2 B)}$ and $Z = \frac{W\phi + \beta B' V_2 A}{f_H \phi - \beta(B' V_2 B)}$. Since $\nu_{1t} = 0$, we have $Z_0 = 0$.

Then, we conclude that

$$\begin{aligned} x_{H,t+\tau} &= Z_0 + ZY_t \\ &= ZY_t \\ &= c_{0H} + c_{1F} S_t + c_{2F} \theta_{t-1} \end{aligned}$$

and $c_{0H} = 0$ since $x_{H,t+\tau} = Z_{1,1} S_t + Z_{1,2} \theta_{t-1}$. This leads to a contradiction. Therefore, the control variable $x_{H,t+\tau}$ is a linear combination of the state variables.

Since the foreign bloc's regulator's problem is symmetric to the home's problem, we have $c_{0F} = 0$ without loss of generality. □

With i and j varying between 0 and 1

The difference $V_{0,T} - V_{0,Q} = \Delta_t$ when $\Phi_2 = 1$ is:

$$\Delta_t = \beta \Delta_t - \frac{\alpha_H^2 \beta \lambda \phi^2 \sigma^2}{2 f_H^2} - \frac{\alpha_H \phi \sigma^2 (2\beta f_F \mu - \alpha_H f_F + 2\alpha_F \beta \lambda \phi)}{2 f_F f_H} \tag{A29}$$

Assuming there is no time difference, meaning that $\Delta_t = \Delta_{t+1}$, we get

$$\Delta_t = -\frac{\alpha_H^2 \beta \lambda \phi^2 \sigma^2}{2 f_H^2 (1-\beta)} - \frac{\alpha_H \phi \sigma^2 (2\beta f_F \mu - \alpha_H f_F + 2\alpha_F \beta \lambda \phi)}{2 f_F f_H (1-\beta)} \tag{A30}$$

As the steady state solution

Substituting R for $\phi \frac{\lambda}{f_H}$, we get

$$\Delta_t = -R \frac{\alpha_H^2 \beta \phi \sigma^2}{2 f_H (1-\beta)} - \frac{\alpha_H \phi \sigma^2 (2\beta f_F \mu - \alpha_H f_F)}{2 f_F f_H (1-\beta)} + R \frac{\alpha_H \phi \sigma^2 2 \alpha_F \beta}{2 f_F (1-\beta)} \tag{A31}$$