Scaling for ion heating: α

An interesting parameter mentioned in Qin, et al (2001) as well as some of my other notes (all on my website) is:

$$\alpha \equiv \tau_{Alf}\omega_{ci}$$

In words, this is the number of orbits an ion executes in a characteristic dynamical time (the time it takes an Alfvénic disturbance to move across the SSX plasma). I want to be careful to separate out ion orbit motion for different charge-to-mass ratios while keeping the MHD motions regulated by the proton mass. The Alfvén speed for a proton plasma is:

$$v_{Alf} = \frac{B}{\sqrt{\mu_0 m_p n}}$$

So we get:

$$\alpha = \frac{\ell}{v_{Alf}}\frac{eB}{M_i} = \frac{\ell e\sqrt{\mu_0 m_p n}}{M_i}$$

Note that the magnetic field dependence dropped out. Now, let's note that:

$$c^2 = \frac{1}{\mu_0\epsilon_0} \quad \omega_{pi}^2 = \frac{ne^2}{\epsilon_0 M_i}$$

where the ion plasma frequency is a characteristic oscillation frequency of unmagnetized ions in a plasma. If all the ions in the plasma are protons ($M_i = m_p$), then we get the easy-to-remember result that the number of ion orbits in a dynamical time is the same as the number ion inertial lengths in a characteristic length:

$$\alpha = \ell e\sqrt{\frac{\mu_0 n}{m_p}} = \frac{\ell\omega_{pi}}{c} \equiv \frac{\ell}{\delta_i}$$

where δ_i is the ion inertial length, the characteristic scale an ion needs to change direction due to its mass. If the ions aren't the majority species (ie not protons), then we get:

$$\alpha = \frac{\ell e\sqrt{\mu_0 m_p n}}{M_i}\sqrt{\frac{m_p}{m_p}} = \ell e\sqrt{\frac{\mu_0 n}{m_p}}\frac{m_p}{M_i} = \frac{\ell}{\delta_i}\frac{m_p}{M_i}$$

What this means is that the number of orbits per dynamical time is reduced by the mass ratio. In SSX, $\alpha \cong 10$ for protons (see Qin, et al), but only 2-3 for helium ions. Carbon ions have $\alpha \leq 1$ and argon ions have $\alpha \ll 1$. Interestingly, we find that helium ions are heated pretty well by reconnection/turbulent dynamics, carbon less so, and argon very little.

In the ApJ paper on ion acceleration (Brown 2002), we talk about "vBL" scaling meaning that if we have a reconnection electric field with magnitude $E \sim vB$ then the maximum electromotive force (voltage) a charged particle can pick up is $\xi = \int E \cdot dl \sim vBL$. In SSX, the peak inflow speed is $v \sim 10^5\ m/s$, the magnetic field is $B \sim 0.1\ T$, and a typical macro scale is $L \sim 0.1\ m$, so $vBL \leq 1000\ eV$. The energies and temperatures we see are substantially less than this (some ions measured at 200 V and the hottest ion temperatures of 80 eV). A more realistic (typical) reconnection inflow speed is $10^4\ m/s$ which gives 100 eV. In the Qin, et al paper, we showed that ions picked up energy only if $\alpha \geq 10$ or so.

The particle simulations should be illuminating since we don't expect ions to pick up the full "vBL" of energy. Another limit is to consider that the reconnection event is limited in temporal extent (only "on" for 10 μs or so. In that case, $Ma = qE$ so

$$\Delta v = \frac{q}{M} E \Delta t \cong 10^5\ m/s$$

for protons (and using $E = 100\ V/m$ for vB). A $10^5\ m/s$ proton has an energy (temperature) of 50 eV (less for helium, carbon, and argon). I think the answer for us will be some combination of the α effect (not enough ion orbits during the dynamical time) and the finite scale of the reconnection zone.

$$m \frac{d\mathbf{v}}{dt} = \frac{Z}{M} q \left[\mathbf{E} + \mathbf{v} \times \mathbf{B} \right]$$