Permutation entropy analysis of dynamical turbulence in the SSX MHD wind tunnel and the solar wind

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Introduction

Turbulence is everywhere, from the surface of the sun to your coffee cup. It is well-understood in neutral fluids, but the ability of plasmas to conduct electricity adds complications to the study of plasma turbulence. A general understanding of turbulence in plasmas is vital to laboratory based efforts to study the solar wind, such as SSX, as well as efforts to make fusion a viable source of energy.

We make the first comparison between turbulence in the solar wind and laboratory plasma using the CH plane introduced in [1], which graphically represents a signal with a point whose horizontal position corresponds to its degree of "randomness" and its vertical position the degree of correlational structure.

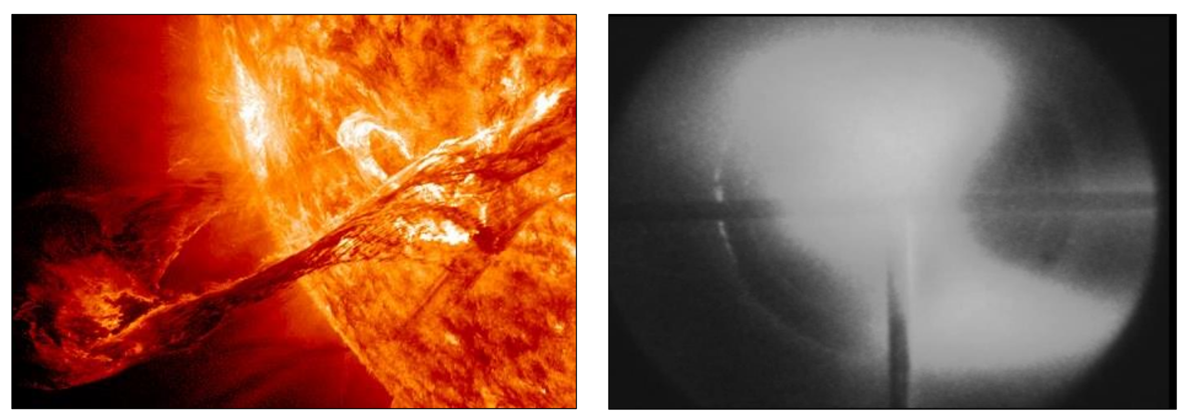

Figure 1: A coronal mass ejection event on the sun (left) and a plasma in the SSX wind tunnel device (right)

Datasets

The SSX MHD wind tunnel fires spheromaks of magnetized plasma into a cylindrical copper tube with no field guide. As the plasma evolves into a relaxed state, a high-resolution magnetic probe measures three-dimensional changes in $\dot{B}$ at 16 radial positions. B signals are obtained by integrating $\dot{B}$ data. Time series collected by the SSX device were compared to magnetic signals from the WIND spacecraft and the Large Plasma Device (LAPD) using the permutation entropy and Jensen-Shannon Complexity.

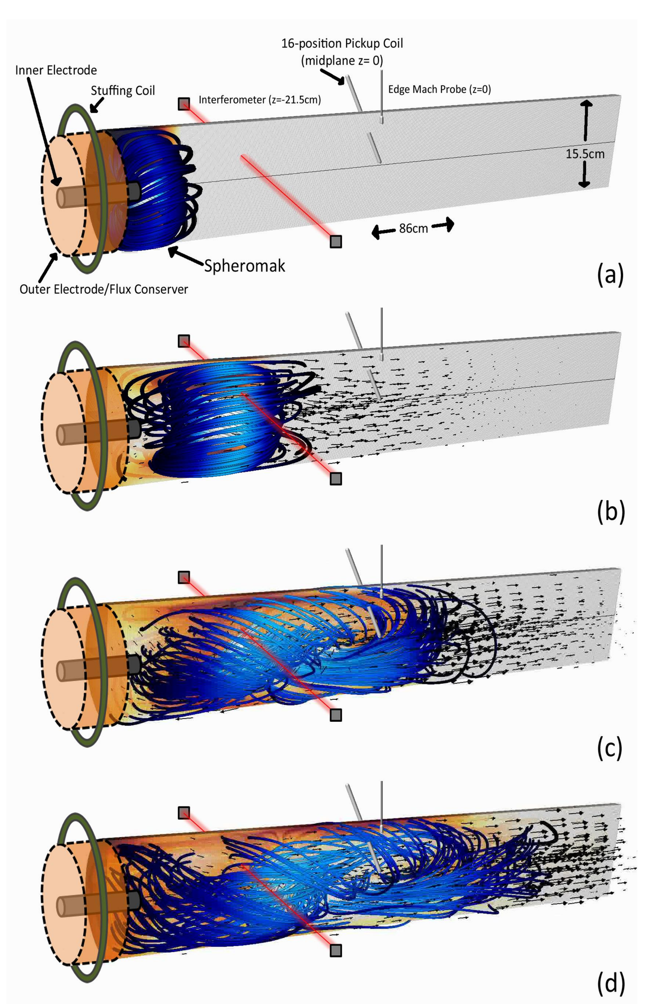

Figure 2: a sequence of simulated images of an evolving spheromak plasma in the SSX wind tunnel.

Permutation Entropy and the CH Plane

As a robust measure of randomness applicable to arbitrary time series [2], permutation entropy has been used to study everything from economic trends to the effects of anesthesia on the brain . The permutation entropy PE of a time series is defined in terms of a window length called the embedding dimension n. Given n, PE is calculated using the Shannon entropy equation:

$$PE = -\sum p(\pi)\log p(\pi) \qquad (1)$$

where $p(\pi)$ is the frequency of occurrence of ordinal pattern π in all length n segments of the time series and the sum runs over all $n!$ possible permutations. The logarithm is base 2.

Example: If $n = 3$ for the two time series shown in figure 3, the first will have $PE = -\frac{1}{2}\log\frac{1}{2} - \frac{1}{2}\log\frac{1}{2}$, since each of the ordinal patterns appearing in the two 3-value segments of the series are distinct ($2^{nd} < 1^{st} < 3^{rd}$ and $1^{st} < 2^{nd} < 3^{rd}$) . The second will have less entropy, $-1\log 1 = 0$, since only one ordinal pattern appears.

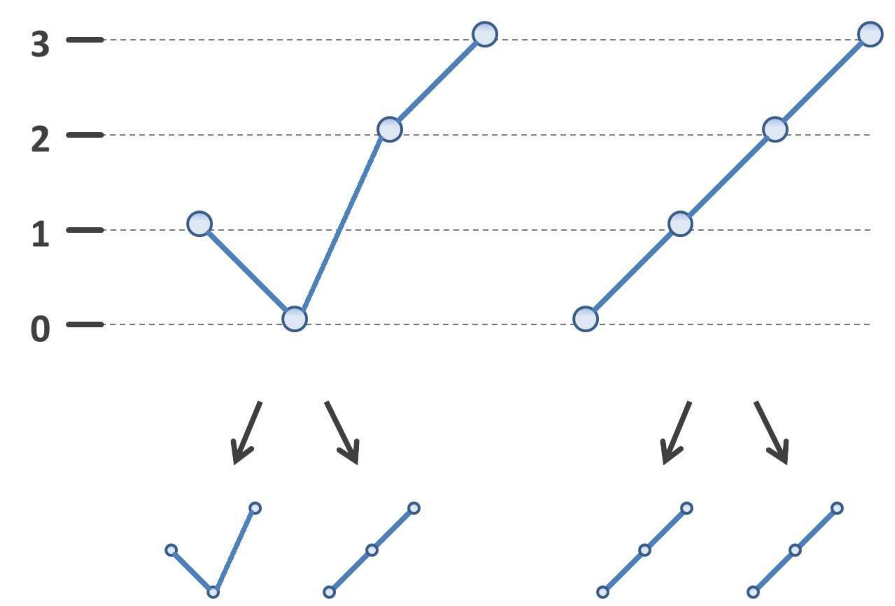

Figure 3: $n = 3$ ordinal patterns for two, 4-value time series.

While the permutation entropy quantifies randomness, a measure of statistical complexity such as the Jensen-Shannon complexity C_{JS} is required to quantify the degree of correlational structure. By comparing the positions of time series on the $\frac{PE}{\log n!} \times C_{JS}$ plane, commonly referred to as the CH plane, one can gain valuable information about the degree of stochastic, chaotic, or periodic dynamics in time series.

For example, while simple mathematical functions like the Sine function (downward facing triangle in Figure 4) occupy the lower left corner of the plane, chaotic maps (purple shapes) occupy the upper middle region of the plane. Stochastic systems, on the other hand, tend to have lower complexity and greater entropy, such as fractional Brownian motion, shown by the dotted black line in Figure 4.

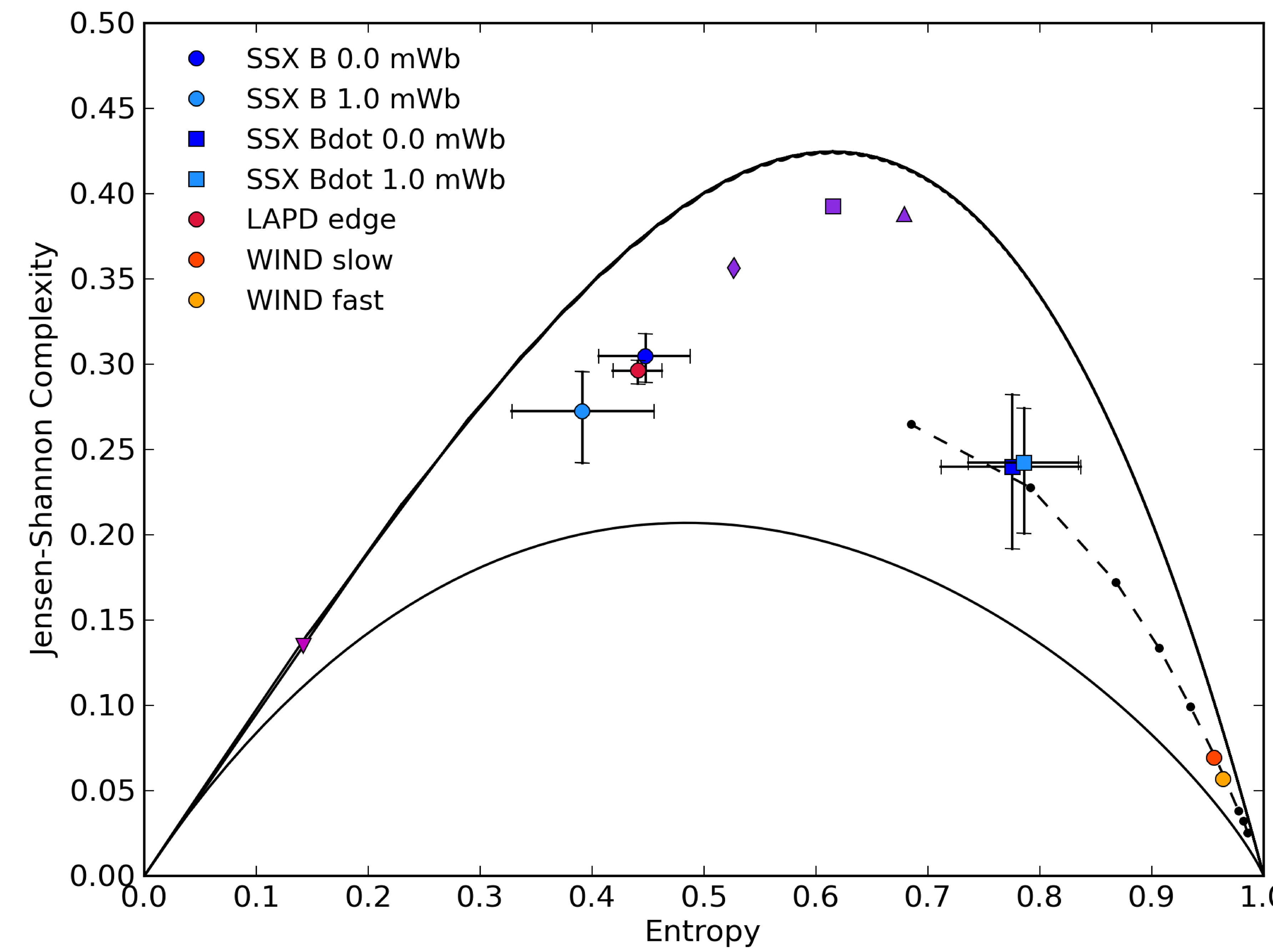

Figure 4: The $n = 5$ CH plane.

Results

Of the three plasma systems analyzed, the solar wind was the most stochastic. In fact, both Wind datasets exhibit more entropy and less complexity than classical Brownian motion.

Edge fluctuations in LAPD plasmas, on the other hand, are the most chaotic-like. These chaotic dynamics are likely associated with non-linear interactions of relatively few drift-wave modes.

SSX $\dot{B}$ signals appear to be more stochastic than LAPD, but not yet as turbulent as the solar wind. This limitation may be connected to the relatively short and confined nature of the experiment.

This novel application of the CH plane suggests that the method is capable of differentiating between different kinds of turbulence. The CH method also appears to highlight the number of degrees of freedom in the system in question, and comparisons with power spectra suggest close connections between a system's position on the CH plane and its spectral characteristics.

Literature cited

[1] O.A. Rosso, H.A. Larrondo, M.T. Martin et al. *Phys. Rev. Lett.* 99 154102 (2007).

[2] C. Bandt and B. Pompe. *Phys. Rev. Lett.* 88 174102 (2002).

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More Information

Please contact: pweck1@swarthmore.edu.
More about SSX can be found at: http://www.swarthmore.edu/ssx-lab