Notes for Retarding Grid Energy Analyzer (RGEA)
M. Brown, August 2008

The way the RGEA works is to regulate ion flux to the collector by adjusting the voltage on a retarding grid. At high positive retarding voltage, very few ions have enough energy to overcome the barrier to be collected as current. The current collected by the Faraday cup at the back in what should be a conservative underestimate is given by (MKS units, assuming just 1 eV ions for now and only1 mm^2 collector area):

$$I = nevA \cong 10^{20}\ 10^{-19}\ 10^{4}\ 10^{-6} \cong 0.1\ A$$

So we might predict lots of signal... even more if the ions have more energy or if the density is higher. The problem with this estimate is that the densities of directed ion flux are much lower (I think). Let's do a little analysis to see what we might expect.

Analysis of ion flux to RGEA collector: The probability of an ion having a vector velocity is given by the Boltzmann factor:

$$P(\mathbf{v}) \propto e^{-mv^2/2kT}$$

We can convert this to a density and make it an equality like this:

$$n(\mathbf{v}) = C_0\ n_0\ e^{-m(v_x^2+v_y^2+v_z^2)/2kT}$$

What I have in mind here is that $\hat{\mathbf{z}}$ is the direction of the probe axis (with positive z going into the entrance hole... there might even be a magnetic field aligned with the hole). To be completely general, I can have different temperatures in the different directions (this might happen if the probe were along a magnetic field):

$$n(\mathbf{v}) = C_1\ n_0\ e^{-m(v_x^2/2kT_\perp+v_y^2/2kT_\perp+v_z^2/2kT_\parallel)}$$

For now, let's assume the temperatures are the same in all directions (isotropic) and that there's no net flow. We can find the normalization constant by noting that if we integrate the distribution of velocities over all possibilities, we should get all the particles (this is essentially what Schroeder does in section 6.4). So we require that:

$$n_0 = \int_{-\infty}^{+\infty} n(\mathbf{v}) d^3v$$

We find (I think) that the normalization constant gives us (see Reif chapter 7):

$$n(\mathbf{v}) = \left(\frac{m}{2\pi kT}\right)^{3/2}\ n_0\ e^{-m(v_x^2+v_y^2+v_z^2)/2kT}$$

The point here is that if n_0 is the density right in front of the ion discriminator grid, then this function tells us how the velocities are distributed (again no drift yet). Imagine a sphere of arrows with their bases at the origin and all the tips facing out. The density of long arrows drops rapidly (like the Boltzmann factor).

If we were to do the RGEA analysis for this case (worth doing as a test), we need to come up with $I = nev_zA$ like we did for our back-of-the-envelope estimate above. This time, we need to worry about just those arrows in a narrow cone around the $+\hat{\mathbf{z}}$ axis. The acceptance angle θ is given by the geometry of the RGEA. The fraction of ions in the cone is easy in this case, its just:

$$\frac{2\pi \int d(cos\theta)}{4\pi}$$

So if the cone angle is $\pi/2$, we get the top hemisphere and get 1/2 of the ions. Finally, we want the mean z-component of those vectors, weighted by the Maxwellian (this is called the ion flux or first moment of the distribution):

$$n\bar{v_z} = \int v_z n(\mathbf{v}) d^3v$$

Drifting distribution of ions: Now let's assume the ions are all drifting towards the RGEA with a velocity v_0 in the $+\hat{\mathbf{z}}$ (still the same temperature in all directions). This is the same as adding a constant v_0 to all the arrows on the sphere so they all shift up. Also, since we haven't changed the density, the normalization constant is the same. I think that must be true just on physical grounds. Also, we could change variables to $V = v_z - v_0$ and $dV = dv_z$ and get the same mathematical problem as before.

So I think we have (worth checking):

$$n(\mathbf{v}) = \left(\frac{m}{2\pi kT}\right)^{3/2} \; n_0 \; e^{-m(v_\perp^2 + (v_z - v_0)^2)/2kT}$$

Because of the drift velocity, more of the ions are in this acceptance cone but its a harder integral. Once again, we want the mean z-component of those vectors, weighted by the Maxwellian (the ion flux). There will be a part due to $v_\perp$ which we could call:

$$G_\perp = \int C_\perp e^{-mv_\perp^2/2kT_\perp} dv_x dv_y$$

that amounts to another constant that you just carry through the calculation. The ion current to the collector at a particular setting of the retarding potential would be something like:

$$I_{ion} = eA \int_{v_r}^{\infty} v_z F(v_z) G_\perp dv_z$$

where v_r is the velocity corresponding to ions at the energy given by the retarding voltage (ie 10 km/s protons correspond to 1 eV of energy ie 1 volt of retarding potential will stop them).

$$v_r = \left(\frac{2eV_r}{m}\right)^{1/2}$$

Details: We should do more analysis but I realize now that we should expect much smaller signal than our back-of-the-envelope calculation. First of all, from the geometric considerations (ie the acceptance angle), we might be getting less than 0.1 of the ions near the entrance hole. Secondly, the screens only pass 0.3 each so 3 screens account for another factor of 0.05. In the end, we might expect well less than 1 mA of ion current (which is what we're measuring in the present configuration).

Simple case (Reif): Reif does a simple (but important) case in his section 7.11. He calculates the flux of particles hitting a wall (no entrance hole, no drift). The flux is given by (see his equation 7.11.9):

$$\Phi_0 = n\bar{v_z} = \int n(\mathbf{v}) v_z d^3v = \int_{v_z>0} n(v) v cos\theta \ v^2 dv \ sin\theta \ d\theta d\phi$$

where we note that $n(\mathbf{v}) = n(v)$ is an isotropic function of the magnitude of v. In this case, spherical coordinates make sense. The integral over ϕ gives 2π and the θ integral in the upper half-plane gives 1/2 so we get (7.11.10):

$$\Phi_0 = \pi \int_0^\infty v^3 n(v) dv$$

The integral is proportional to the mean speed for an isotropic, Maxwellian distribution (see Reif 7.10.13):

$$\bar{v} = \frac{4\pi}{n} \int_0^\infty v^3 n(v) dv = \sqrt{\frac{8kT}{\pi m}}$$

So evidently,

$$\Phi_0 = \frac{1}{4} n\bar{v}$$

cheers, mb