Particle Simulations in Relaxed Taylor States

Daniel Dandurand[1], Vyacheslav Lukin[2], Mike Brown[1], Tim Gray[1], Xingyu Zhang[1]

[1]Department of Physics and Astronomy, Swarthmore College, Swarthmore, PA 19081
[2]Space Science Division, Naval Research Laboratory, Washington, DC 20375

Background and Motivation

The Swarthmore Spheromak Experiment (SSX) investigates plasma dynamics by injecting plasma plumes (in compact toroidal configurations called "spheromaks") into evacuated conducting target chambers of varying shapes and diagnostic capabilities. Currently, the SSX device is set up as a "wind tunnel," with a single plasma gun situated at one end of an L/R=13 cylinder. This setup has been used to study the relaxation of excited plasmas into force-free Taylor states and to experimentally calibrate the SSX Mach probe.

Here we investigate simulations of particle orbits in a Taylor state corresponding to the cylindrical SSX geometry. We also simulate drifting plasmas to obtain a numerical calibration for the SSX Mach probe. In both cases, simulation results are compared with experimental data from SSX.

Particle Pushing Code

- Particle Pushing Code (PPC) used in these simulations is an extension of the RMF code developed by A. Glasser of U. of Washington. [1]
- PPC numerically computes charged particle trajectories in analytically specified electromagnetic fields.
- The resulting modeled plasma is collisionless and the effects of self-consistent fields are ignored. This allows multiple PPC runs to be submitted independently and in parallel.
- Large simulations done on Teragrid computing cluster: 1000-fold increase in computing power allowed for tens of millions of particles per data point. [2]

Taylor State

- In conducting boundary, plasmas with non-zero resistivity "relax" to state of lowest magnetic energy while conserving global magnetic helicity.
- By minimizing magnetic energy *W* while conserving helicity *H*, with

$$W = \frac{1}{2\mu_0}\int B^2 dV \quad \text{and} \quad H = \int A \cdot B \, dV, \quad \text{where} \quad \nabla \times A = B,$$

  we arrive at the condition for a Taylor state: $\nabla \times B = \lambda B$, where λ is a scalar depending on the total initial magnetic energy and helicity.
- A truncated expansion in eigenfunctions of the curl was used as an approximate analytic model for the lowest-energy Taylor state corresponding to the current cylindrical SSX geometry. [3]
- VisIt visualization software lets us see the magnetic streamline configuration of this SSX Taylor state. [4]

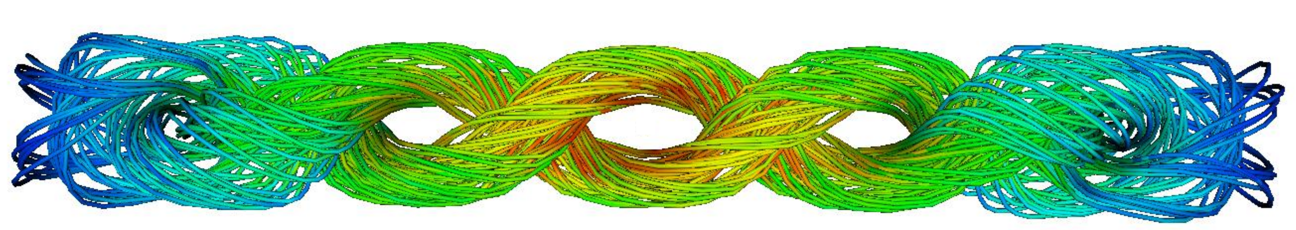

Figure 1. The minimum-energy Taylor state for the SSX L/R=13 cylinder.

Confinement

- SSX experiments confirm that plasmas injected in wind tunnel relax to predicted Taylor state. We use PPC to simulate charged particle orbits in this relaxed state to determine its particle confinement properties.
- PPC modeled cylindrical chamber of same dimensions as that of SSX. Taylor state was modeled using truncated eigenfunction expansion.
- To simulate plasma, a particle had initial coordinates drawn from a uniform spatial distribution inside the cylindrical chamber. The initial particle velocity components were drawn from Maxwellian distributions for a particular plasma temperature. Information about Taylor plasma state was gained by running millions of particles in this manner.
- Temperatures ranging from 1 to 50 eV were run. In each case, the proportion of particles that have yet to collide with cylinder walls was recorded over time.
- For all temperatures simulated, results show that a nonzero proportion of particles remains confined (i.e., bounded away from cylinder walls by magnetic fields of Taylor state) for upwards of 100 microseconds, the typical duration time of the Taylor state.
- VisIt gives 3-D displays of trajectories of confined particles. This allows us to determine the geometrical "bottle zone" of the Taylor state in which charged particles remain magnetically bounded away from chamber walls.

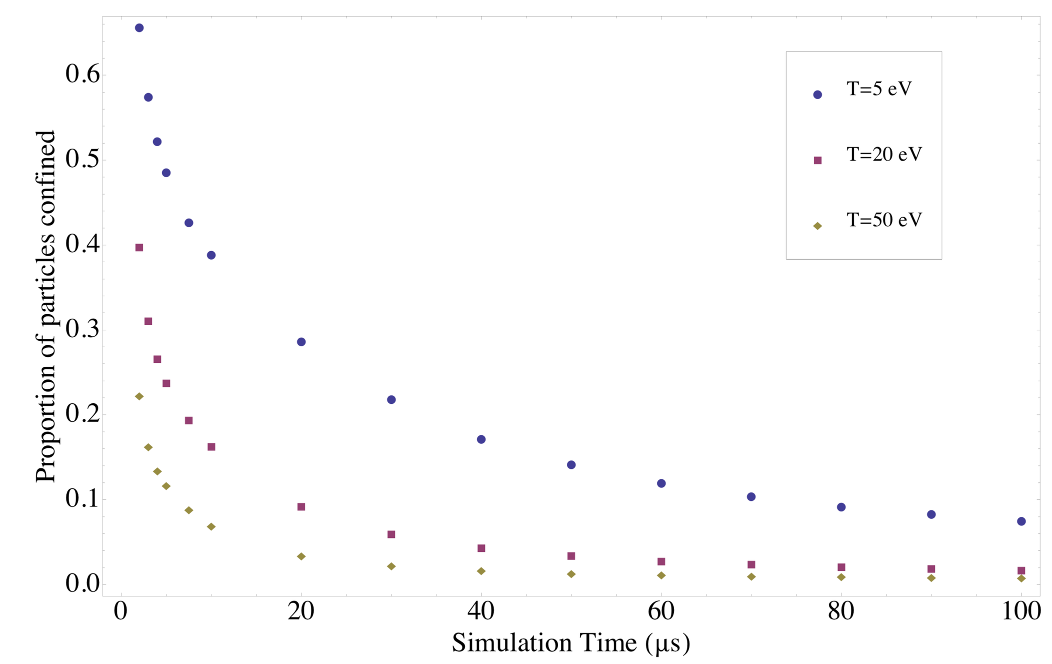

Figure 2. The proportion of particles confined over time. The proportion tapers off at around 80 microseconds, indicating the existence of a "plasma bottle" in the relaxed Taylor state. 20 eV is typical of SSX plasmas.

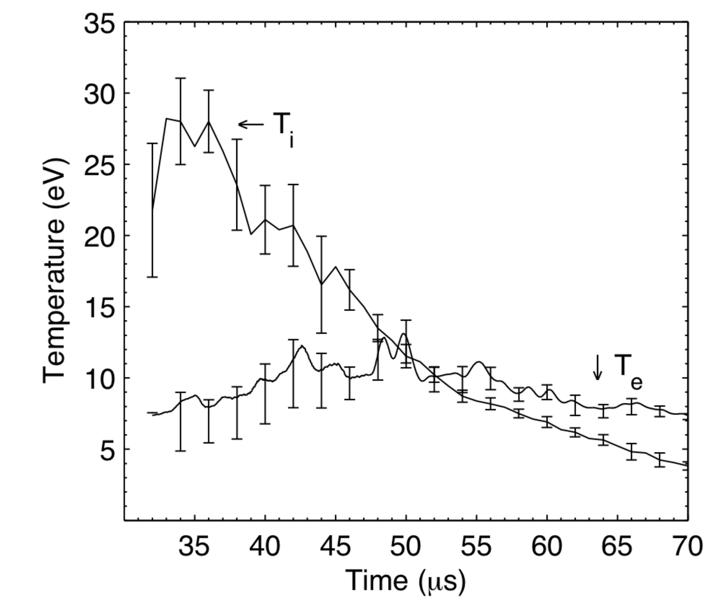

Figure 3. The simulation results compare favorably with these experimental temperature results. Simulations indicate that most "hot" particles are lost to cylinder walls within first 50 microseconds, explaining the cooling that is found experimentally.

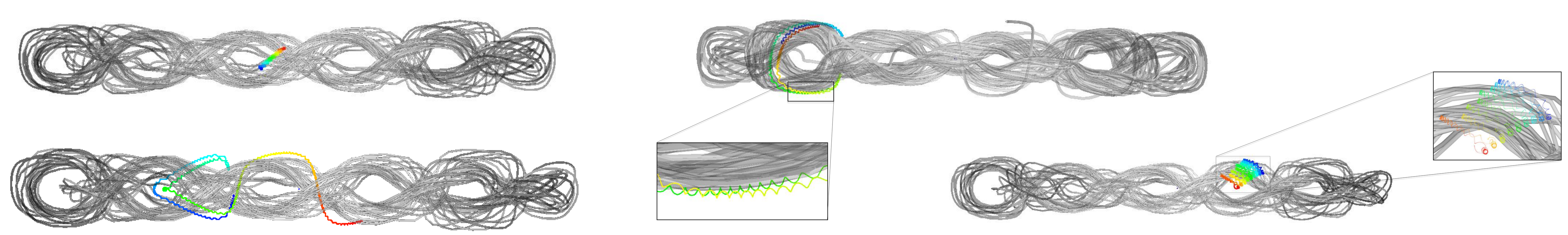

Figure 4. VisIt images of particle orbits in the Taylor state. Magnetic field lines are translucent gray, particle orbits are colored. The well-confined particles take a variety of orbits through the Taylor magnetic fields.

As expected, more energetic particles are less likely to remain confined for more than a few microseconds. The level of confinement, however, is sensitive to a particle's starting position, indicating the importance of the geometry of the Taylor state.

Mach Probe Calibration

- SSX Mach probe is a directional Langmuir probe that determines local flow speed using current ratios of opposite-facing sensors.
- Flow speeds determined using model

$$Log[\frac{J_{up}}{J_{down}}] = KM, \qquad \text{where} \qquad M = \frac{v_{drift}}{c_s}$$

  and c_s is plasma sound speed. *K* is the desired calibration constant.
- PPC modeled local probe volume using "test chamber" one third as long as current SSX chamber (with same cross-section). A replica of the SSX Mach probe was built into test chamber and particle hits on sensor faces were recorded. The number of hits on a sensor was taken to be proportional to the current density.
- Constant magnetic field parallel to bulk plasma flow was used as simple model for local field configuration.
- Using the current ratio model given above, simulations found calibration constant of *K*=2.61+/-0.01.
- SSX experimental calibration constant using magnetic time-of-flight data is *K*=2.0+/-0.5.

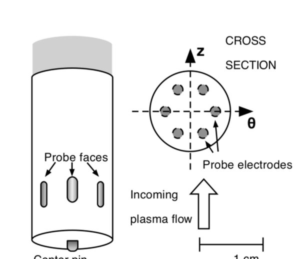

Figure 5. A diagram of the SSX Mach probe. The three sensor pairs measure only two independent directions, so a component analysis is done to extract the Mach number in the desired direction.

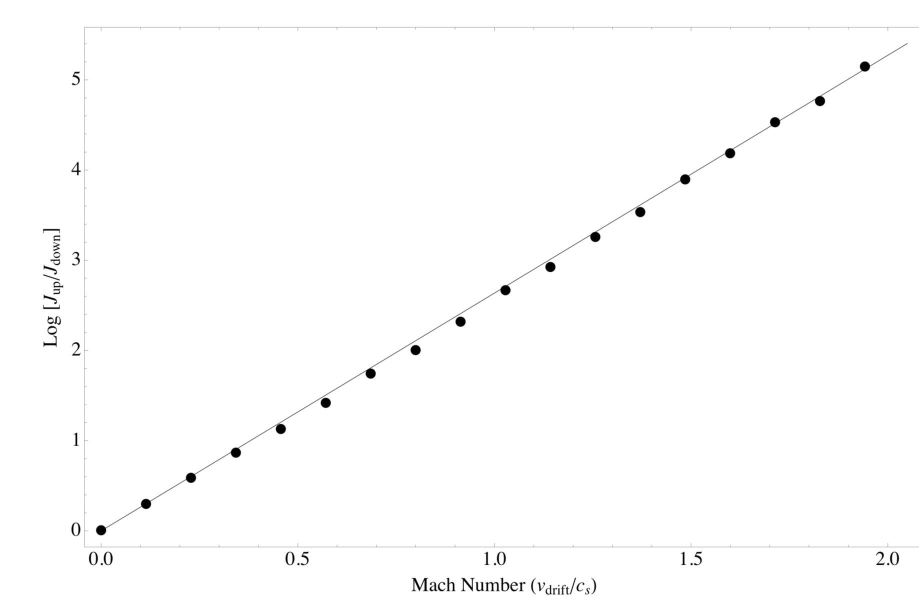

Figure 6. Simulation data used to extract calibration constant *K* via least squares. Each data point resulted from a 50 million particle run. The number of particle hits on a particular sensor was typically in the hundreds.

Future Work

- Investigate how different plasma temperatures, local magnetic field orientations and Mach probe geometries affect Mach probe calibration.
- Use probabilistic model to account for particle collisions while leaving code completely parallel.
- Investigate use of eigenfunctions of curl to model time-dependent plasma dynamics.

Acknowledgements

We thank the Department of Energy, the National Science Foundation and the Office of Naval Research for funding this work.

I thank Mike Brown and Tim Gray for their guidance and support over the past months. I thank Xingyu Zhang for his discussions and help with the experimental side of things. Thank you to Max Korein for his previous work on PPC. Thank you to Andrew Ruether for all the help with Teragrid. A special thanks goes to Slava Lukin for his help and advice during the course of this work.

References

[1] A. H. Glasser and S. A. Cohen, Phys. Plasmas 9, 2093 (2002).
[2] https://www.teragrid.org/
[3] E. Morse, J. Math. Phys. 46, 113511 (2005).
[4] https://wci.llnl.gov/codes/visit/