Particle Simulations in Relaxed Taylor States

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Background and Motivation

The Swarthmore Spheromak Experiment (SSX) investigates plasma dynamics by injecting plasma plumes (in compact toroidal configurations) into evacuated conducting target chambers of varying shapes and diagnostic capabilities. Currently, the SSX device is set up as a "wind tunnel," with a single plasma gun situated at one end of an L/R=13 cylinder. This setup has been used to study the relaxation of excited plasmas into force-free Taylor states and to experimentally calibrate the SSX Mach probe. Here we investigate simulations of particle orbits in a Taylor state corresponding to the cylindrical SSX geometry. We also simulate drifting probe to obtain a numerical calibration for the SSX Mach probe. In both cases, simulation results are compared with experimental data from SSX.

Particle Pushing Code

- Particle Pushing Code (PPC) used in these simulations is an extension of the RMP code developed by A. Glasser of U. of Washington. [1]
- PPC numerically computes charged particle trajectories in analytically specified electromagnetic fields.
- The resulting modeled plasma is collisionless and the effects of self-consistent fields are ignored. This allows multiple PPC runs to be submitted independently and in parallel.
- Large simulations done on Teragrid computing cluster: 1000-fold increase in computing power allowed for tens of millions of particles per data point. [2]

Taylor State

- In conducting boundary, plasmas with non-zero resistivity "relax" to state of lowest magnetic energy while conserving global magnetic helicity.
- By minimizing magnetic energy W while conserving helicity \( H \), with
  \[
  W = \frac{1}{2} \int \mathbf{B}^2 \, dV \quad \text{and} \quad H = \int \mathbf{A} \cdot d\mathbf{A},
  \]
  \( \mathbf{A} = \mathbf{B} \times \mathbf{R} \), where \( \mathbf{R} \) is a scalar depending on the total initial magnetic energy and helicity.
- A truncated expansion in eigenfunctions of the curl was used as an approximate analytic model for the lowest-energy Taylor state corresponding to the current cylindrical SSX geometry. [3]
- Visualizational software lets us see the magnetic streamtube configuration of this SSX Taylor state. [4]

Mach Probe Calibration

- SSX Mach probe is a directional Langmuir probe that determines local flow speed using current ratios of opposite-facing sensors.
- Flow speeds determined using model
  \[
  \log_{10} \frac{I_{1} - I_{2}}{I_{2}} = K W
  \]
  where \( \frac{I_{1}}{I_{2}} \) is plasma sound speed, \( K \) is the desired calibration constant.
- PPC modeled local probe volume using "test chamber" one third as long as current SSX chamber (with same cross-section). A replica of the SSX Mach probe was built into test chamber and particle hits on sensor faces were recorded. The number of hits on a sensor was taken to be proportional to the current density.
- Constant magnetic field parallel to bulk plasma flow was used as simple model for local field configuration.
- Using the current ratio model given above, simulations found calibration constant of \( K = 2.61 \pm 0.01 \).
- SSX experimental calibration constant using magnetic time-of-flight data is \( K = 2.0 \pm 0.5 \).

Confinement

- SSX experiments confirm that plasmas injected in wind tunnel relax to predicted Taylor state. We use PPC to simulate charged particle orbits in this relaxed state to determine its particle confinement properties.
- PPC modeled cylindrical chamber of same dimensions as that of SSX. Taylor state was modeled using truncated eigenfunction expansion.
- To simulate plasma, a particle had initial coordinates drawn from a uniform spatial distribution and a particle velocity component drawn from Maxwellian distributions for a particular plasma temperature. Information about Taylor plasma state was gained by running millions of particles in this manner.
- Temperatures ranging from 1 to 50 eV were run. In each case, the proportion of particles that have yet to collide with cylinder walls was recorded over time.
- For all temperatures simulated, results show that a nonzero proportion of particles remain confined (i.e., bounded away from cylinder walls by magnetic fields of Taylor state) for upwards of 100 microseconds, the typical duration time of the Taylor state.
- Visit gives 3-D displays of trajectories of confined particles. This allows us to determine the geometrical "bottle zone" of the Taylor state in which charged particles remain magnetically bounded away from chamber walls.

Future Work

- Investigate how different plasma temperatures, local magnetic field orientations and Mach probe geometries affect Mach probe calibration.
- Use probabilistic model to account for particle collisions while leaving code completely parallel.
- Investigate use of eigenfunctions of curl to model time-dependent plasma dynamics.

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References