MHD turbulence: observation and experiment

Michael Brown

Swarthmore College, NSF Center for Magnetic Self-Organization

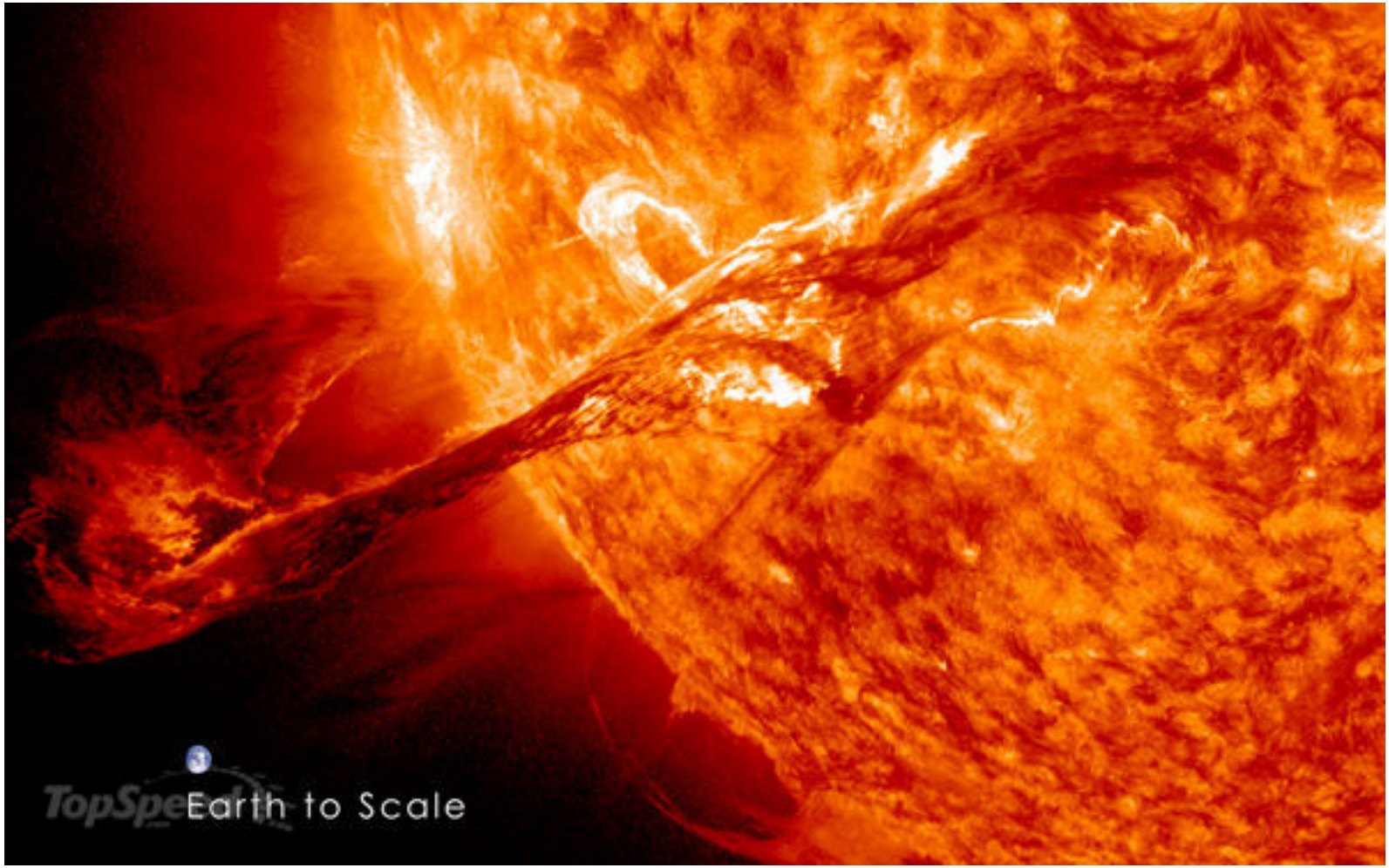
with contributions from

D. Schaffner, V. Lukin, P. Weck '14, A. Wan '14, R. Wicks

APS-DPP tutorial
October 29, 2014

Research supported by US DOE and NSF

Solar plume (CME) and wind (plasma)



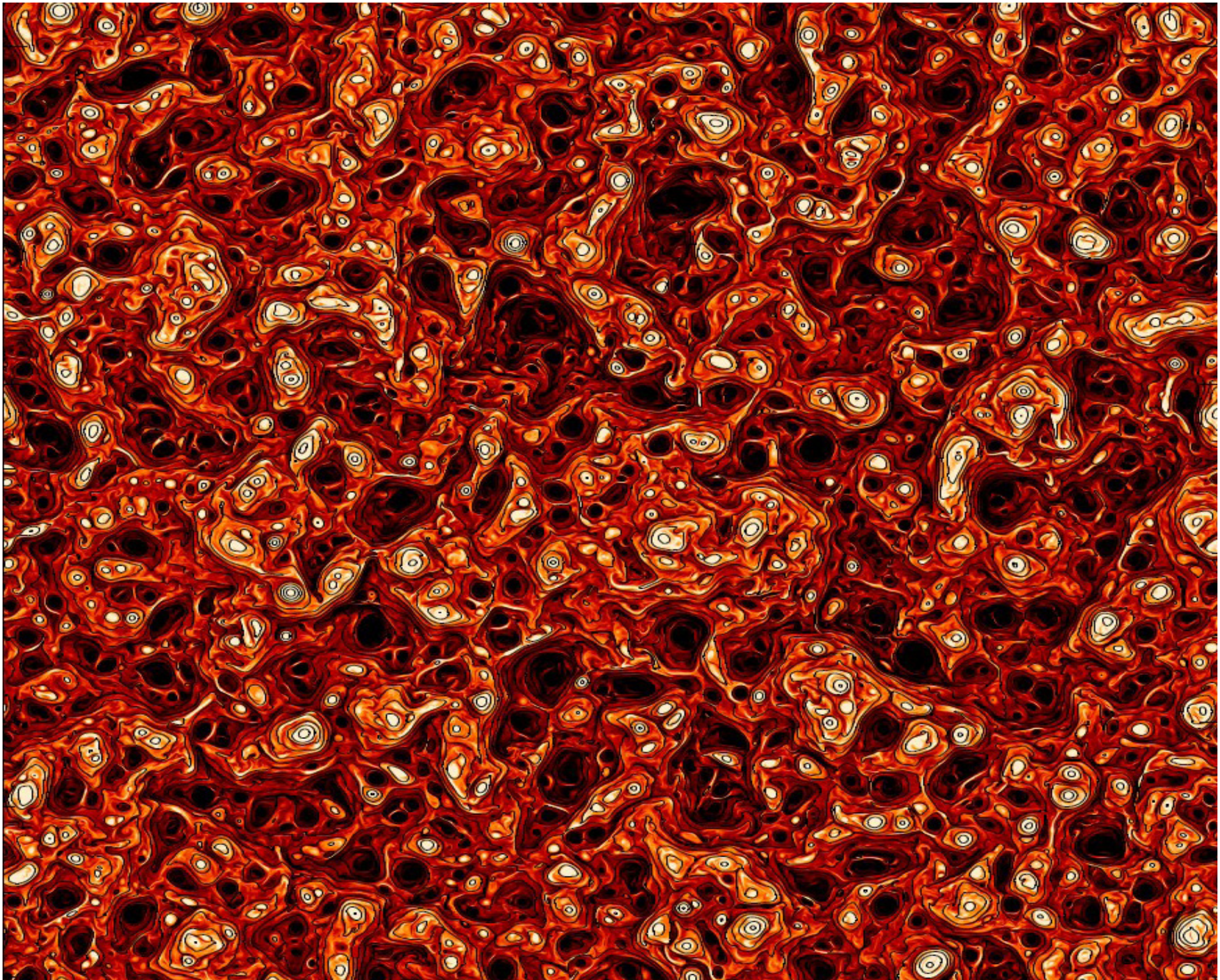
400 km/s plasma with entrained dynamical magnetic fields
(SDO, 8/12)

Outline

- Turbulence primer
- SSX plasma wind tunnel configuration
- Four statistical measures for MHD turbulence

Turbulence Primer

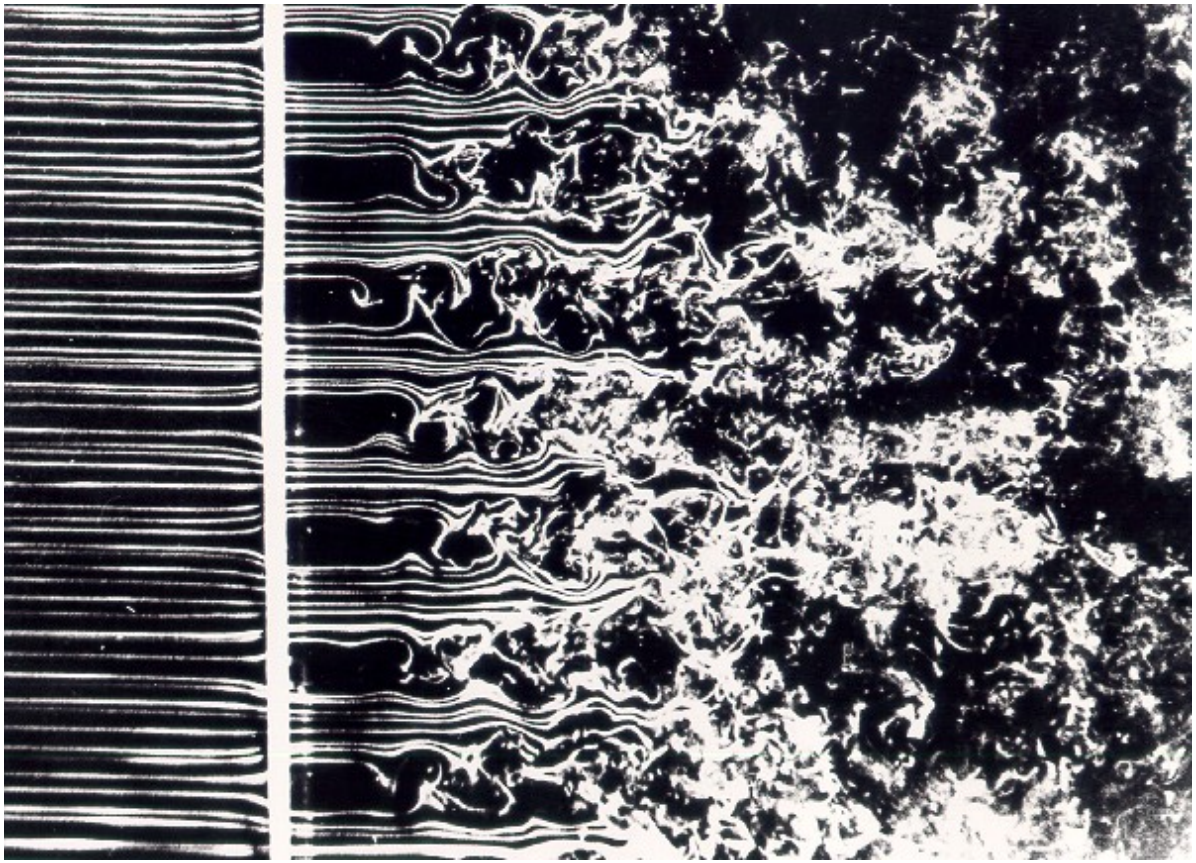
- Cascade from large to small scales
- Homogeneous, Isotropic, Stationary
- Universality in conventional fluids



Energy cascade

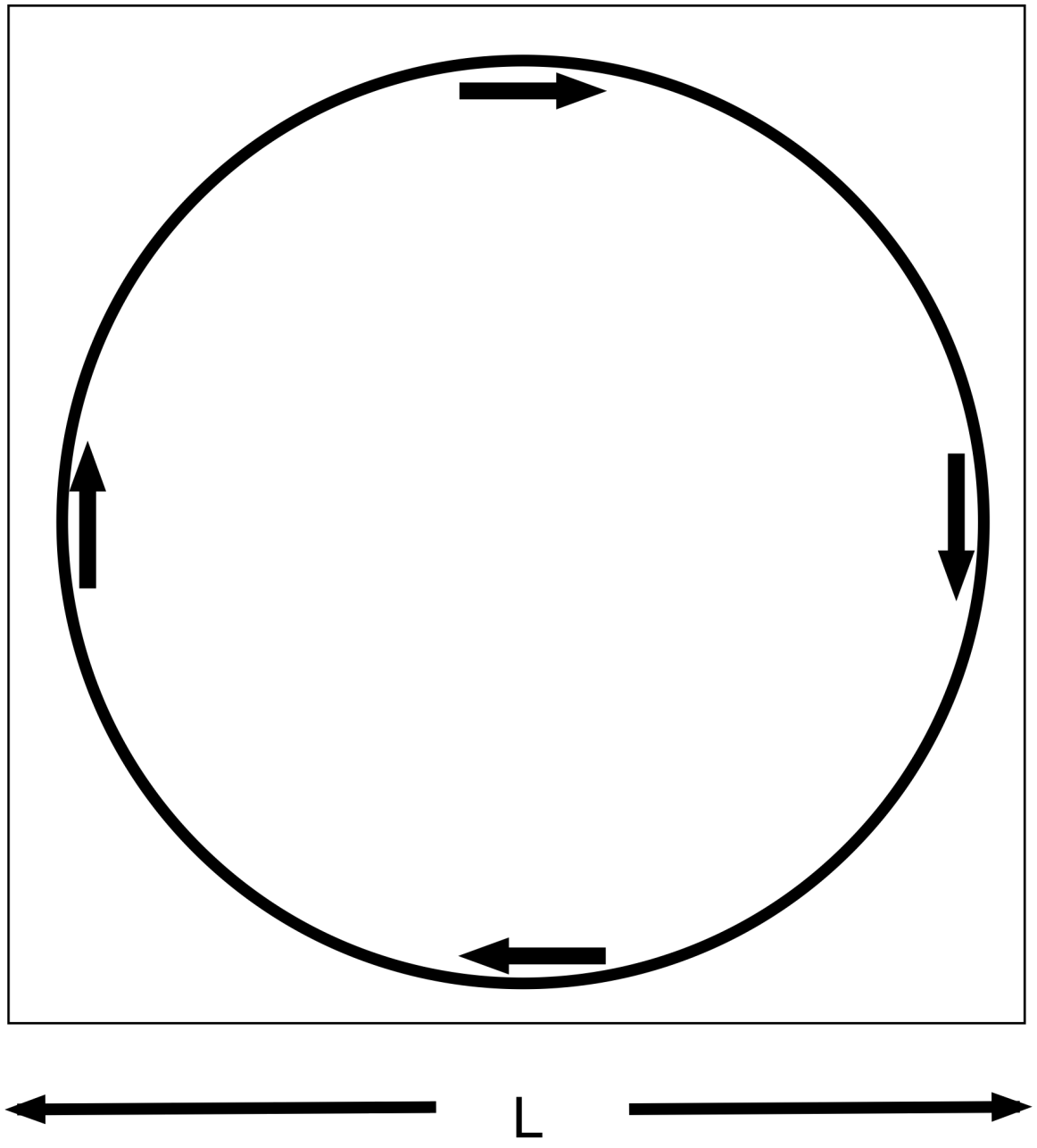
encyclopedia2.thefreedictionary.com

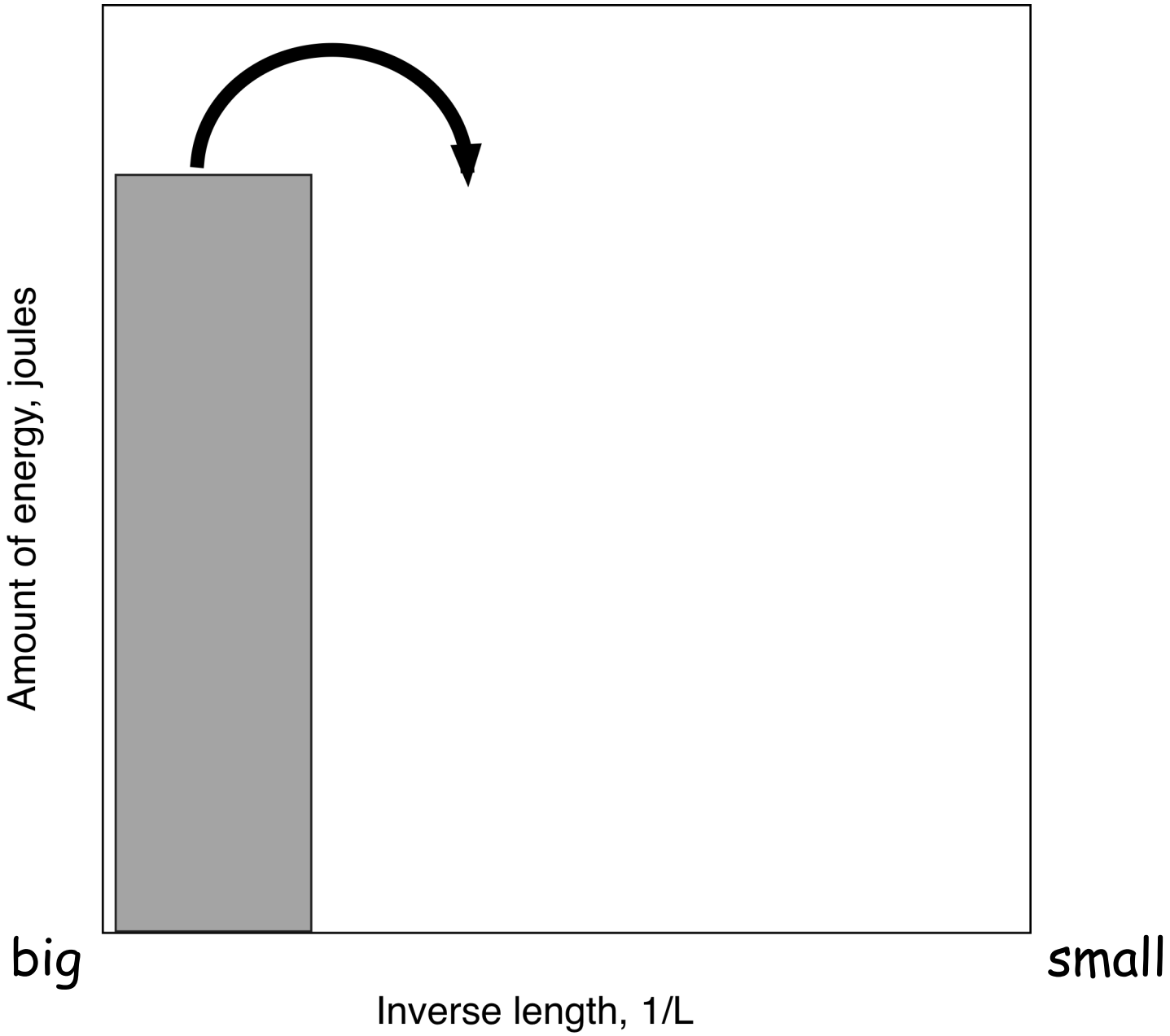
Grid turbulence

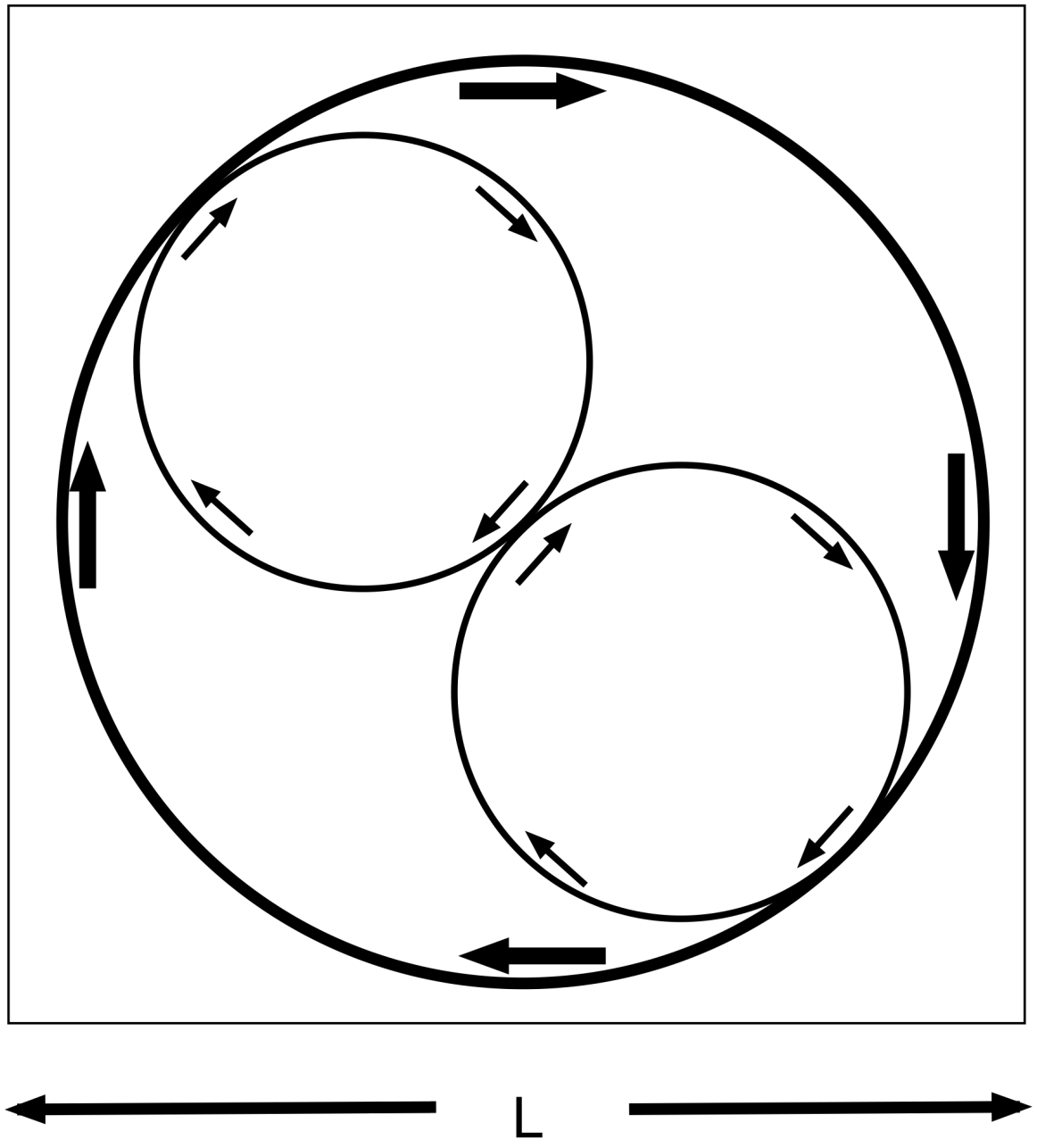


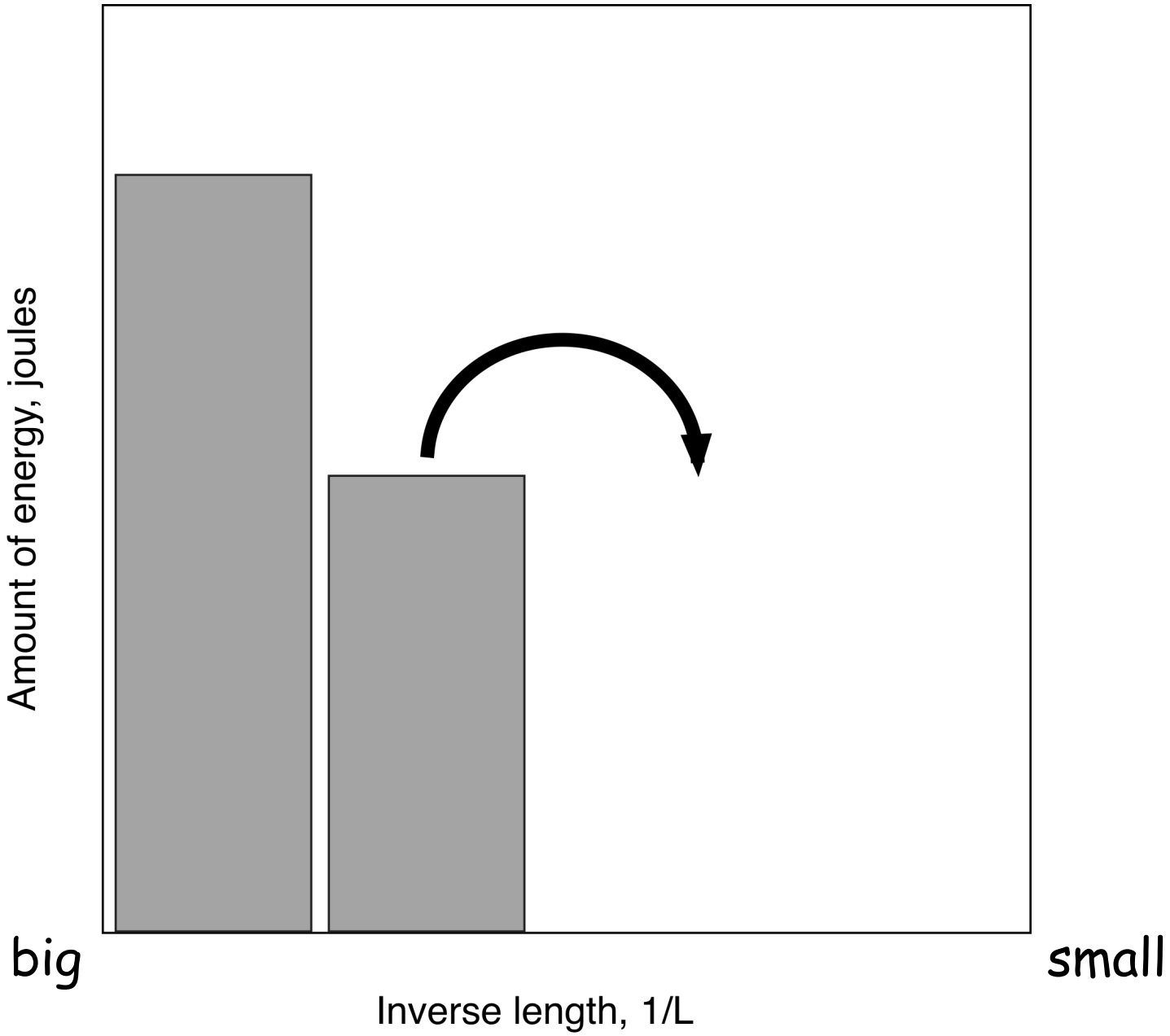
Fluid Dynamics Research Center

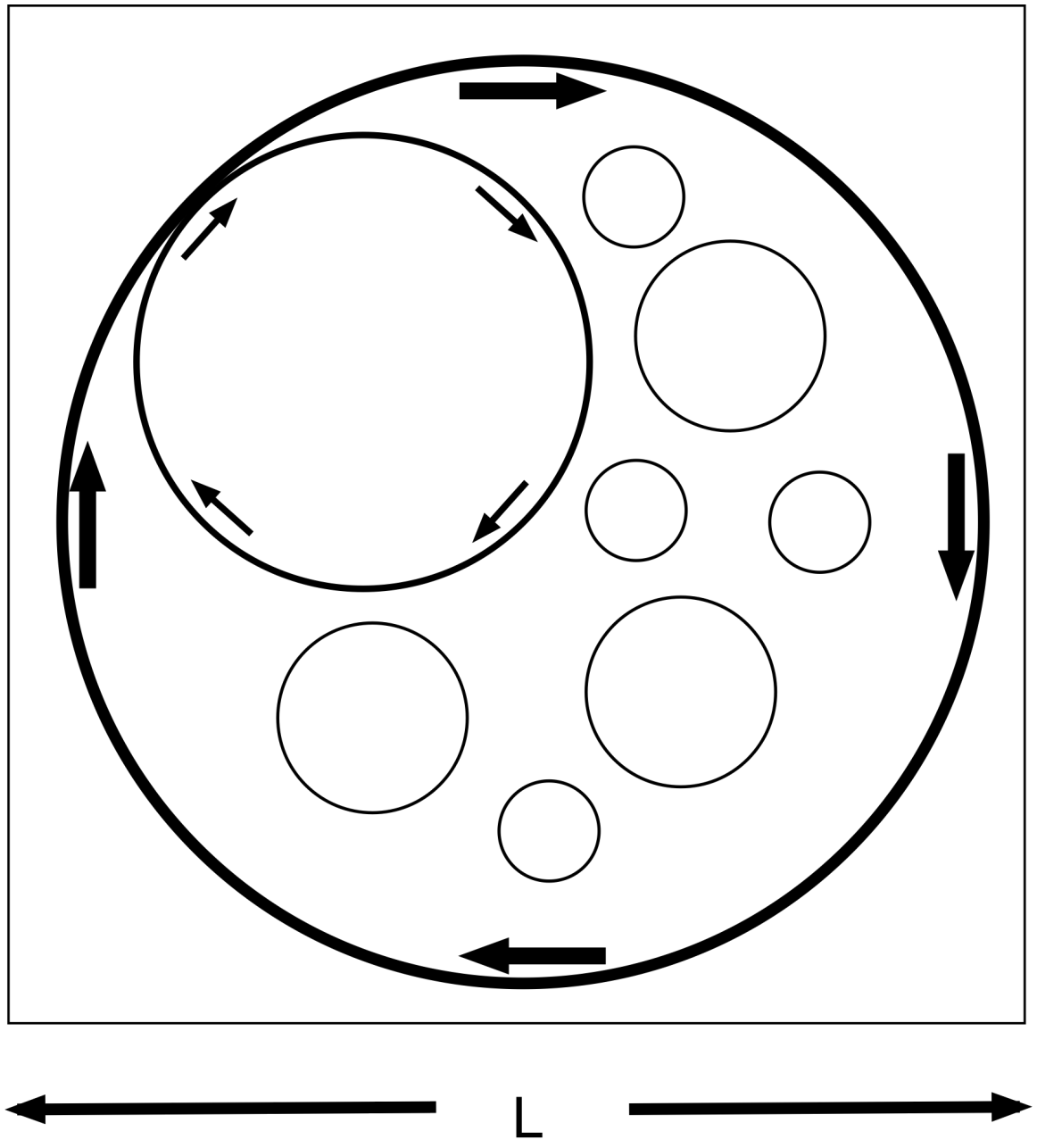
Note: Taylor hypothesis

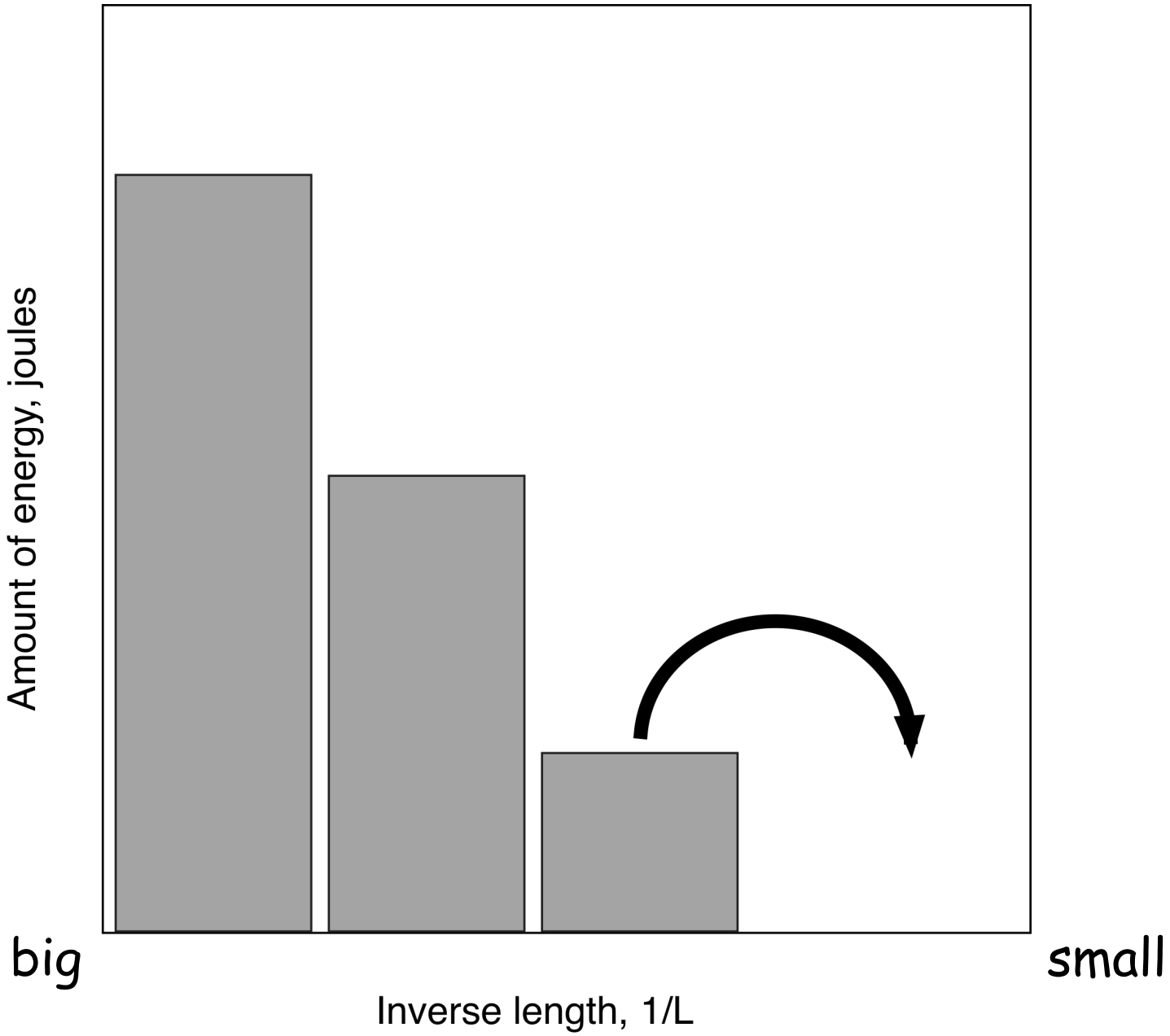

L


Amount of energy, joules
big
small
Inverse length, 1/L

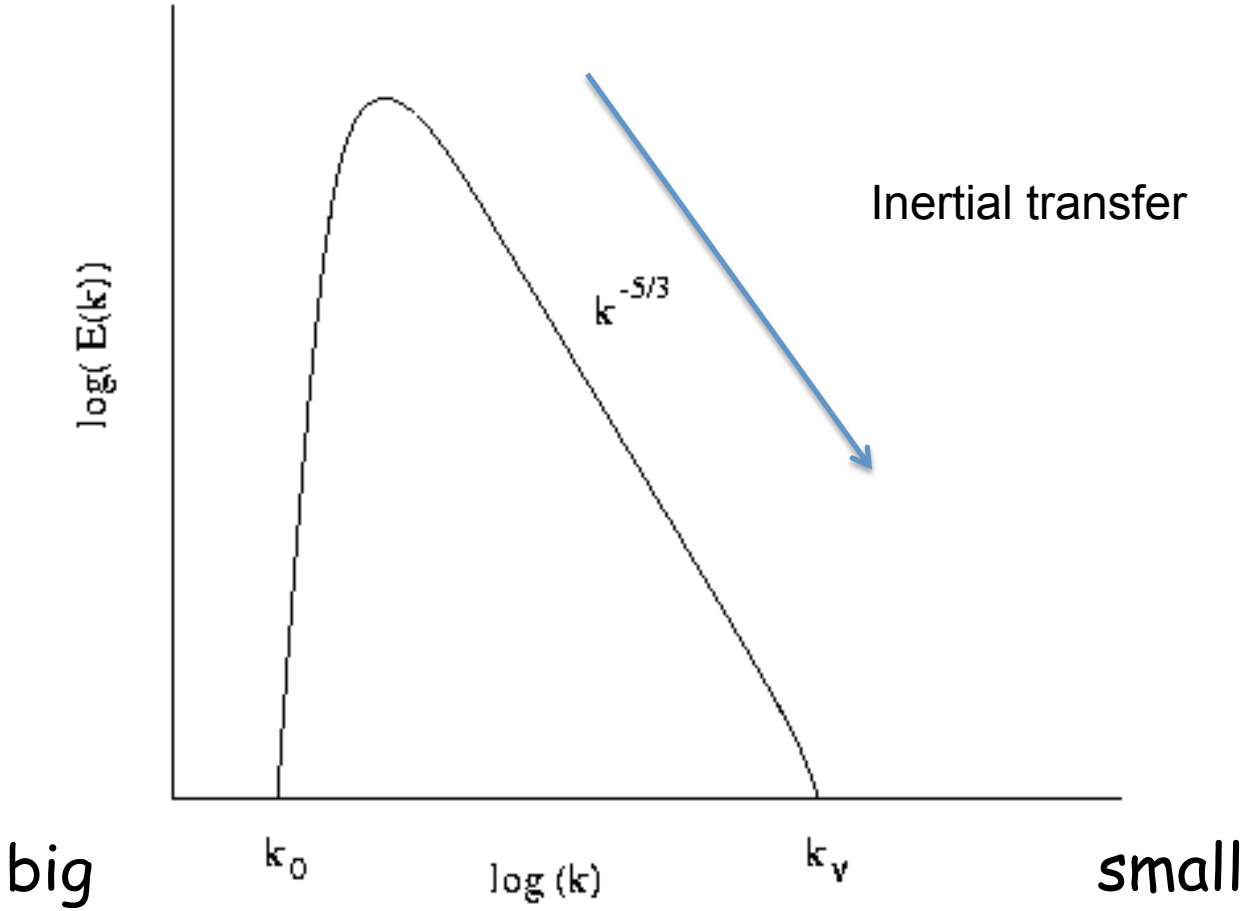

L


Amount of energy, joules
big
small
Inverse length, 1/L

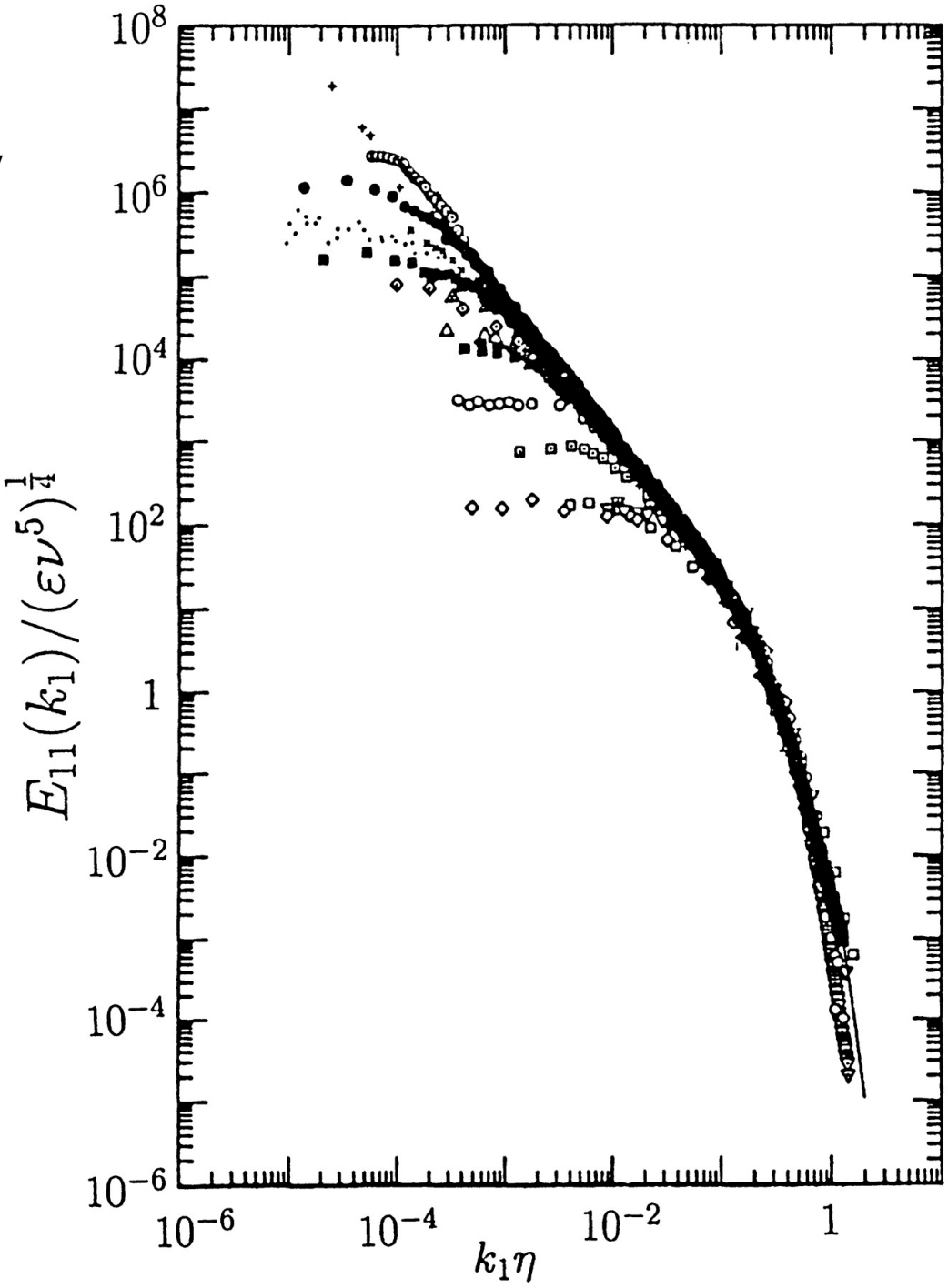

L


Amount of energy, joules
big
small
Inverse length, 1/L

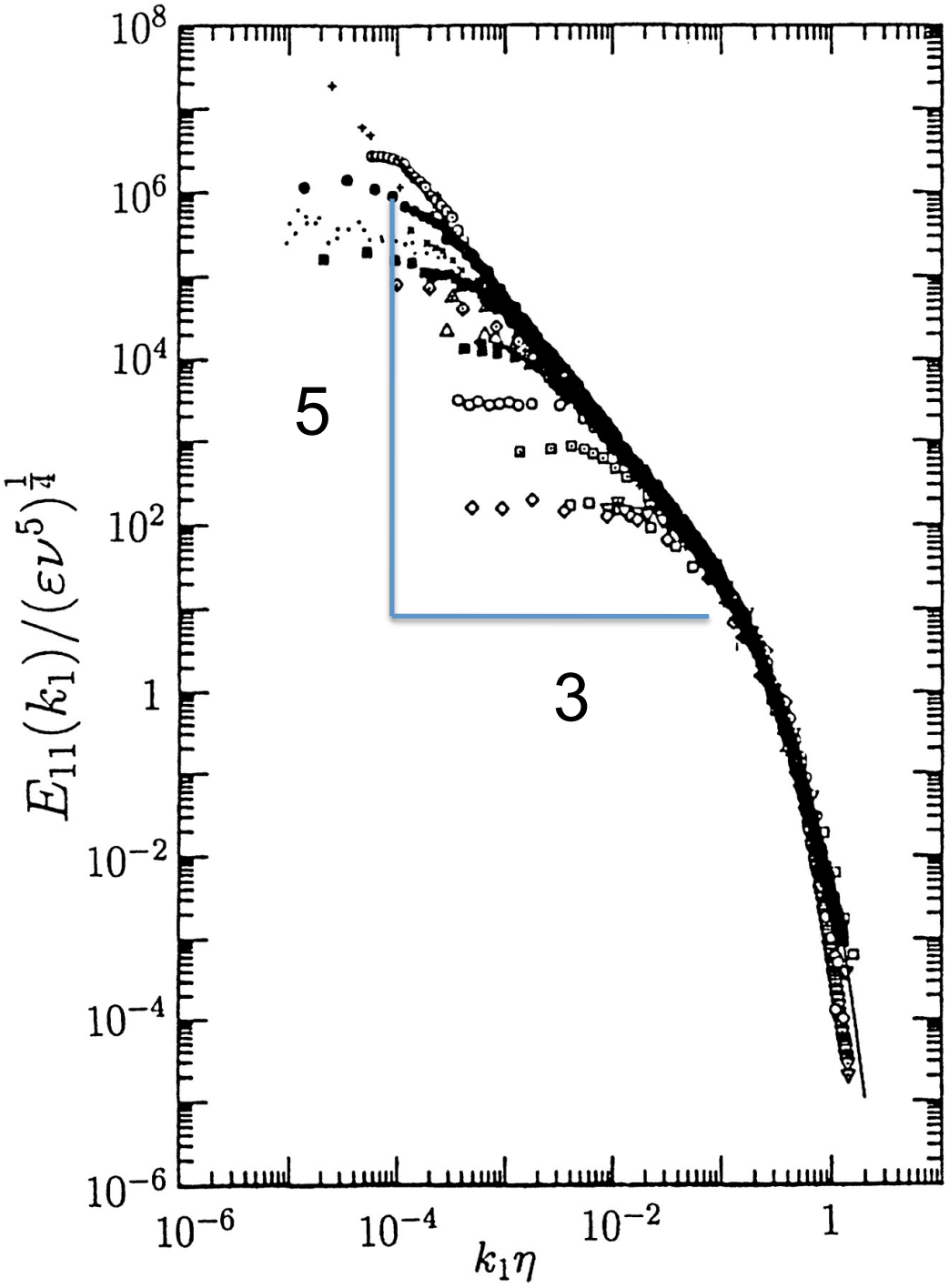
Kolmogorov turbulence spectrum (1941)



universality



Saddoughi and Veeravalli, JFM 1994



Saddoughi and Veeravalli, JFM 1994

Kolmogorov spectrum (K41)

$$E(k) = C\epsilon^{\alpha}k^{-\beta}$$ Depends only on transfer rate and k

$$E(k) \propto v^2/k$$ Energy per mass per wavenumber

$$\epsilon \propto kv^3$$ Energy per mass per time

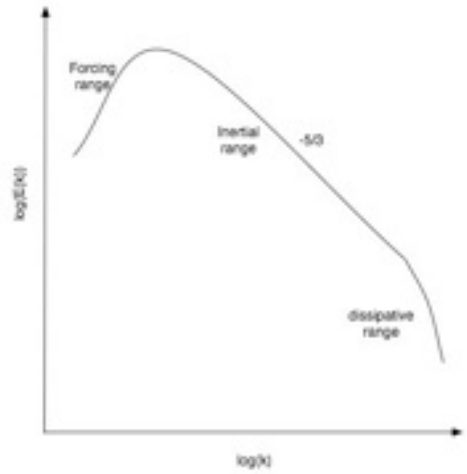
$$v^2k^{-1} \propto k^{\alpha}v^{3\alpha}k^{-\beta}$$ Dimensional analysis

$$2 = 3\alpha$$

$$-1 = \alpha - \beta$$

$$\alpha = 2/3$$

$$\beta = 5/3$$



Turbulence Summary

- Turbulence in conventional fluids usually begins at large scales and "cascades" to small

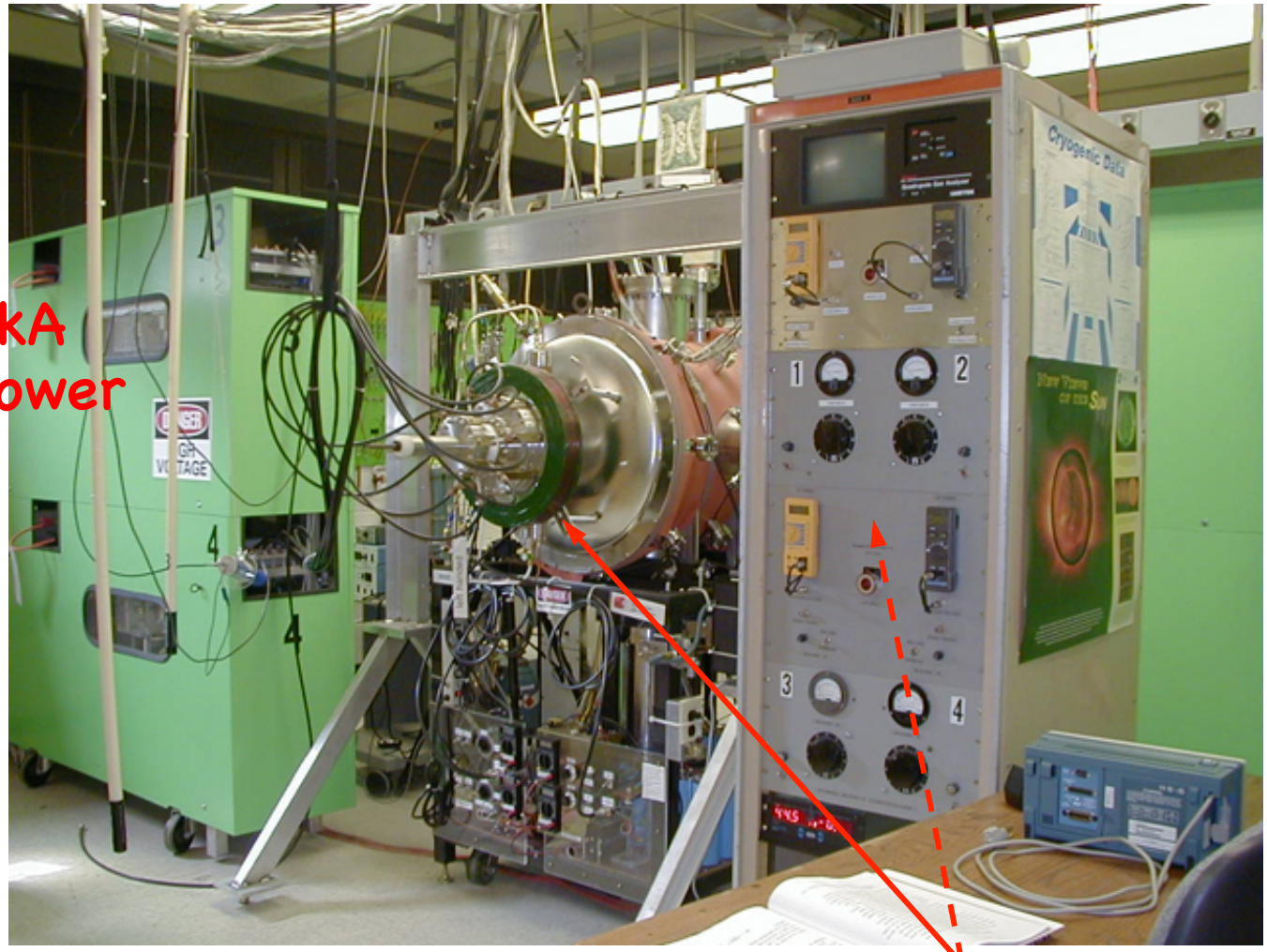
- The structure is usually formless, homogeneous, isotropic, stationary

- The spectrum of turbulence has a universal character ($k^{-5/3}$) in conventional fluids, also observed in magnetofluids

Brown, PSST review, 2014

The SSX Laboratory

10kV/100kA
Pulsed power



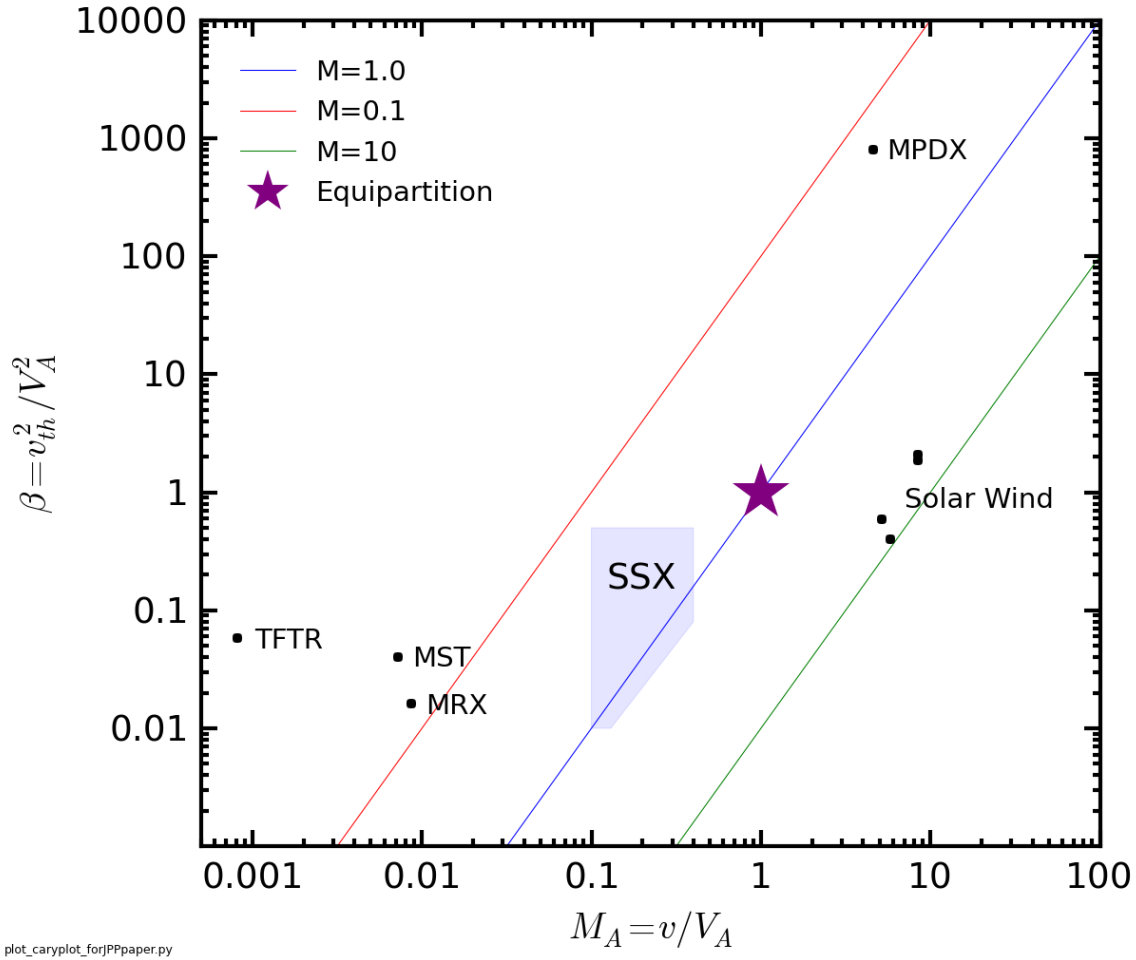
Cylindrical vacuum chamber
(D = 0.5 m, L = 1 m)

High voltage plasma
guns on each end

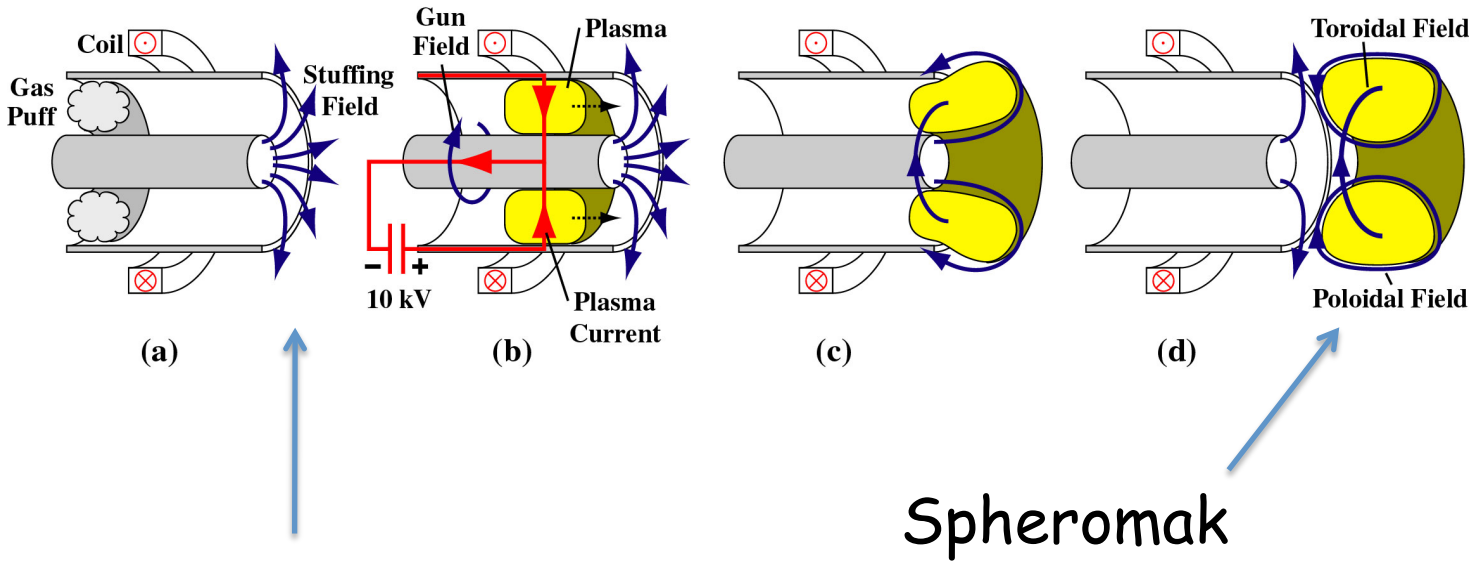
SSX parameters

Ion Density (protons)	10^{14} -10^{15} cm^{-3}
Temperature (T_e,T_i)	20 - 60 eV
Magnetic Field	> 0.1 Tesla
Ion gyroradius	< 0.5 cm
Alfvén speed	100 km/s
S (Lundquist number)	> 1000
Plasma β	0.1-1

Equipartition of flow, thermal, and magnetic energy



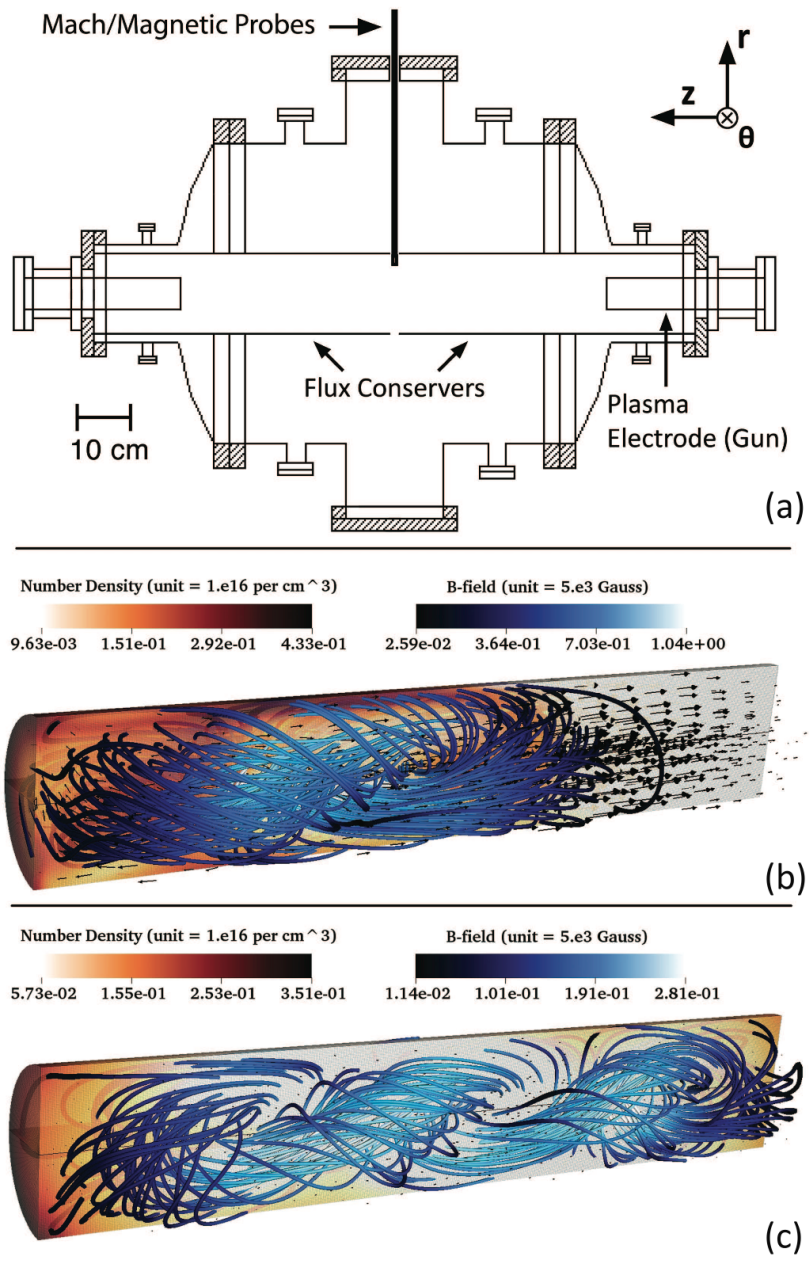
Spheromak formation



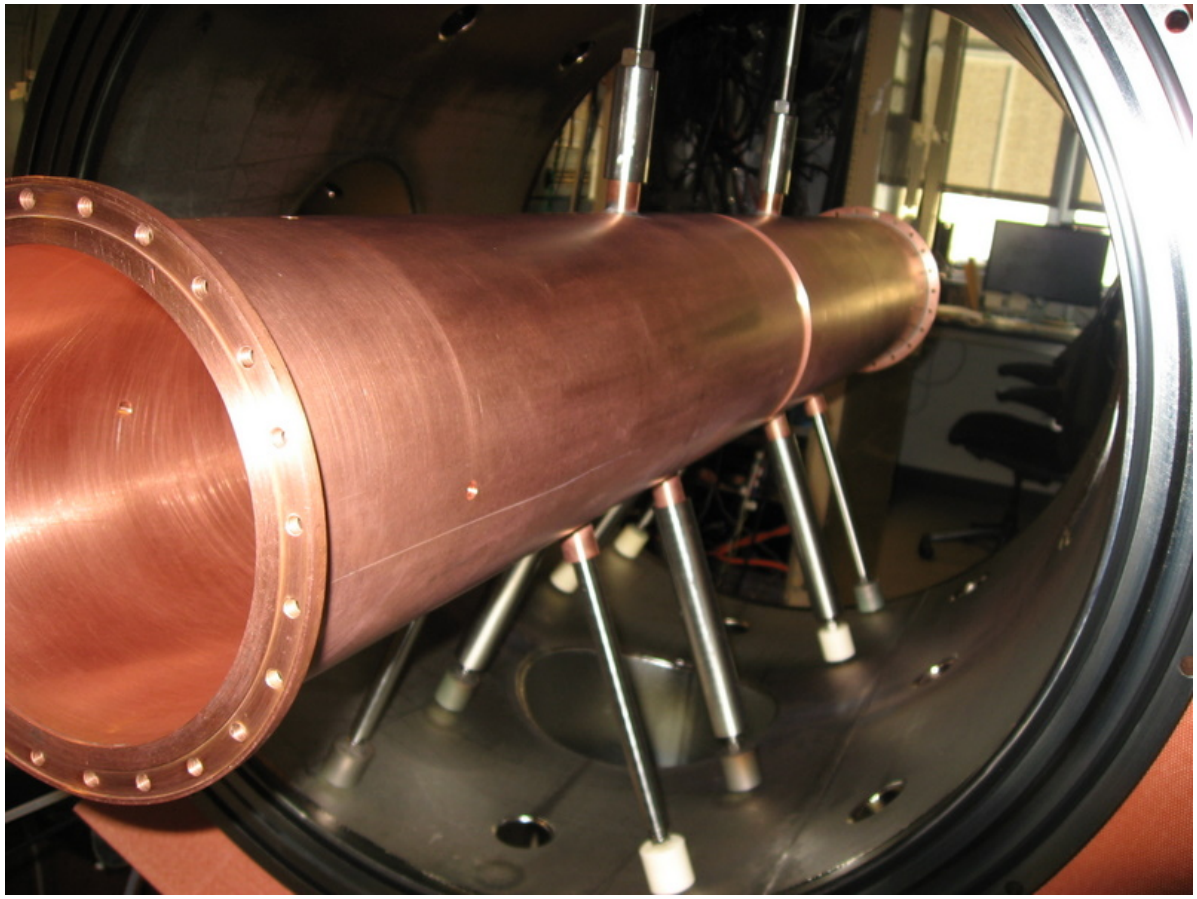
Stuffing flux acts like a nozzle

MHD wind tunnel

- 50 km/s flows, fully ionized and magnetized
- Kinetic, magnetic, thermal energies comparable
- Single plume (>5 kJ)
- Characterization of MHD turbulence
- Compare to solar wind

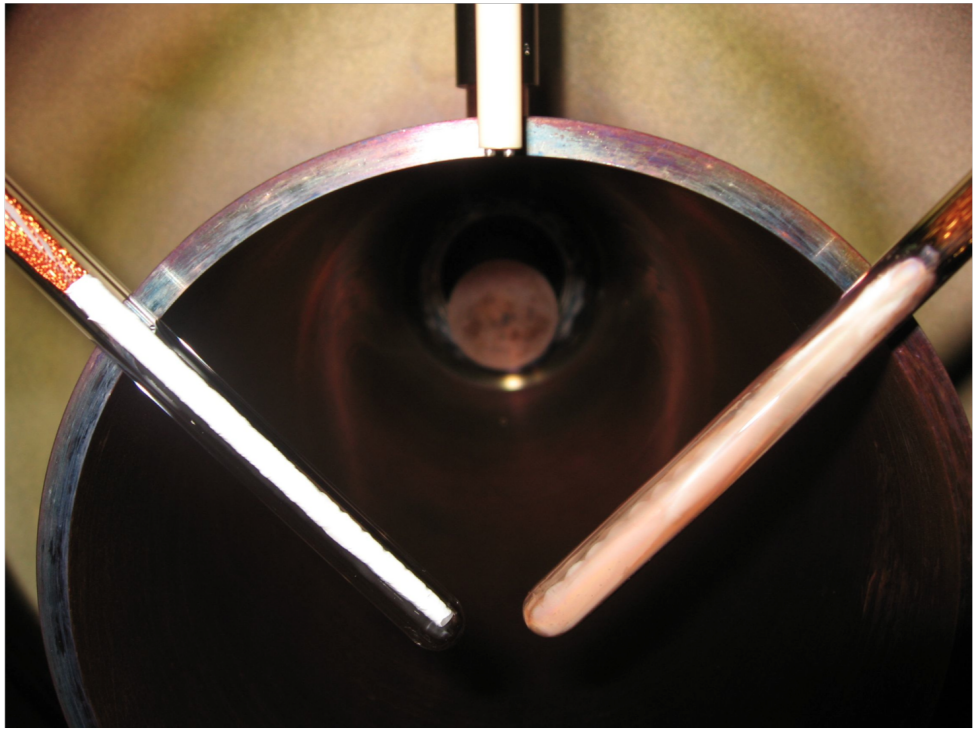

Mach/Magnetic Probes
r
z
θ
Flux Conservers
Plasma
Electrode (Gun)
10 cm
(a)
Number Density (unit = 1.e16 per cm ^ 3)
9.63e-03 1.51e-01 2.92e-01 4.33e-01
B-field (unit = 5.e3 Gauss)
2.59e-02 3.64e-01 7.03e-01 1.04e+00
(b)
Number Density (unit = 1.e16 per cm ^ 3)
5.73e-02 1.55e-01 2.53e-01 3.51e-01
B-field (unit = 5.e3 Gauss)
1.14e-02 1.01e-01 1.91e-01 2.81e-01
(c)

SSX MHD wind tunnel
50 km/s, magnetic and fluid turbulence

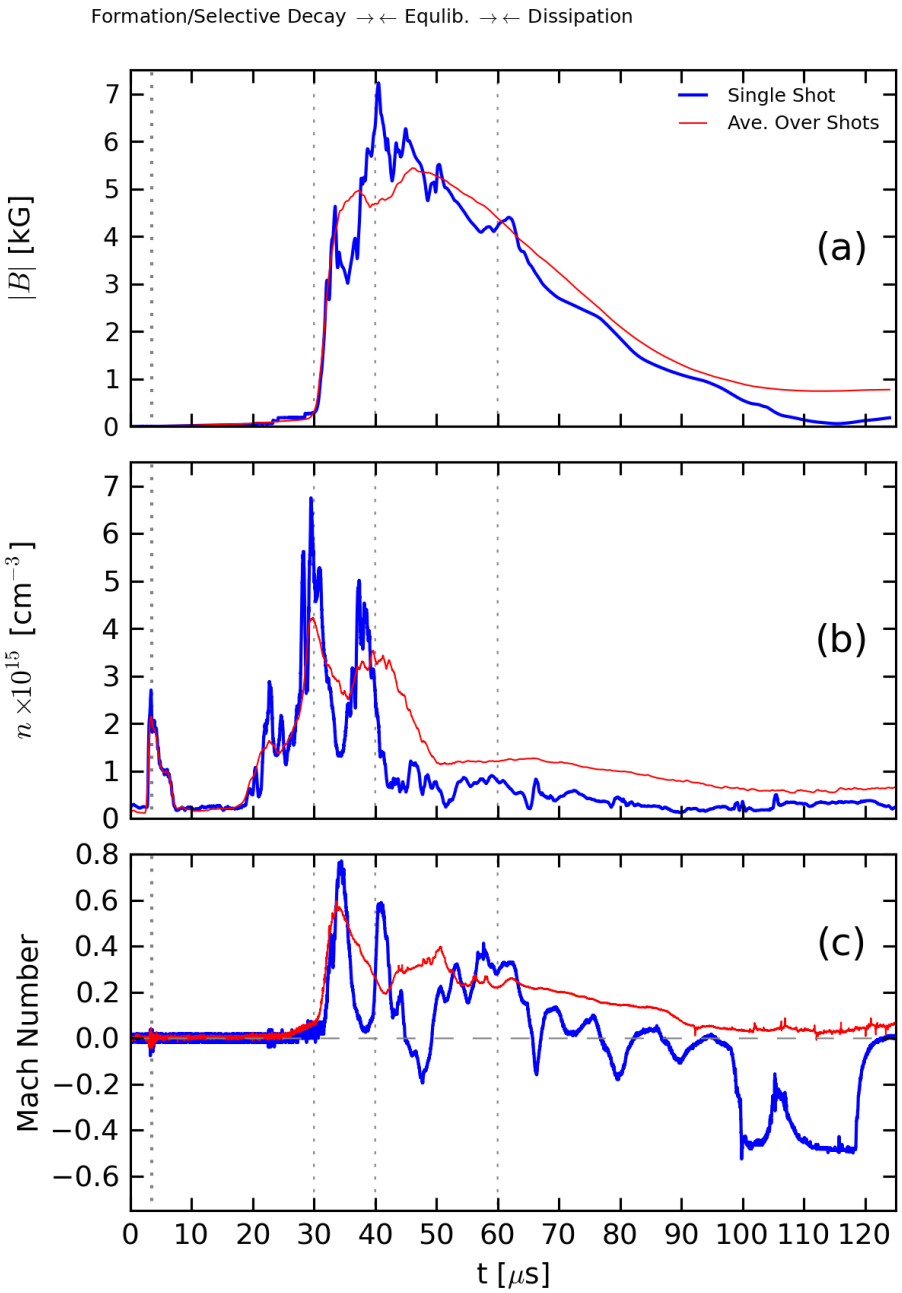


Diagnose with arrays of magnetic and velocity probes

Diagnostics at midplane (B and n_e)



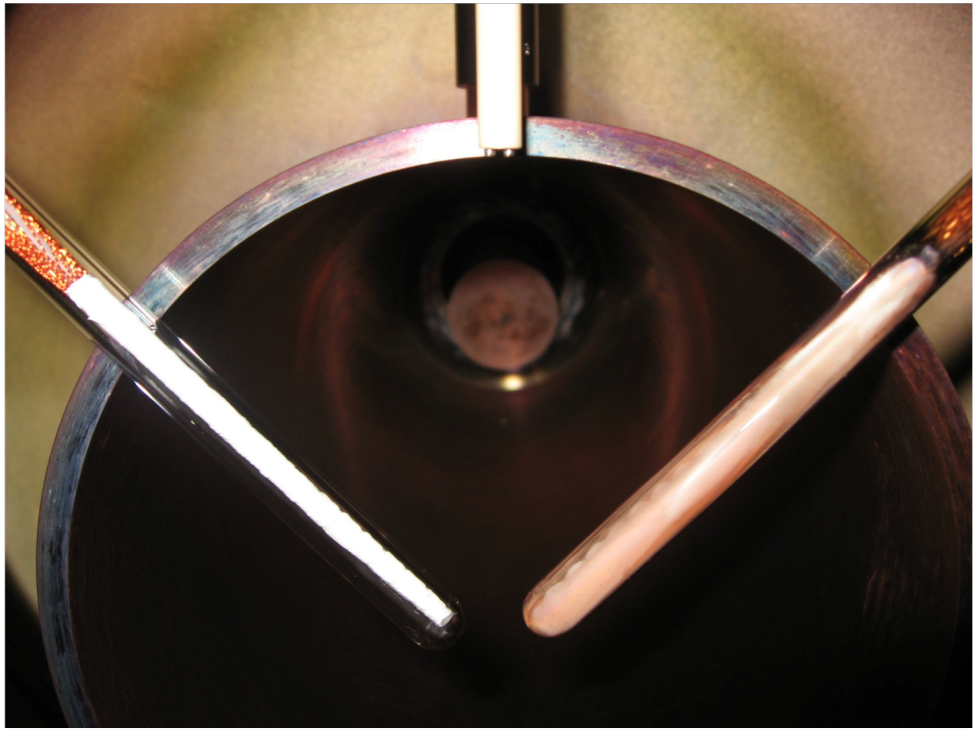
Line-averaged density with He-Ne, temperature from IDS

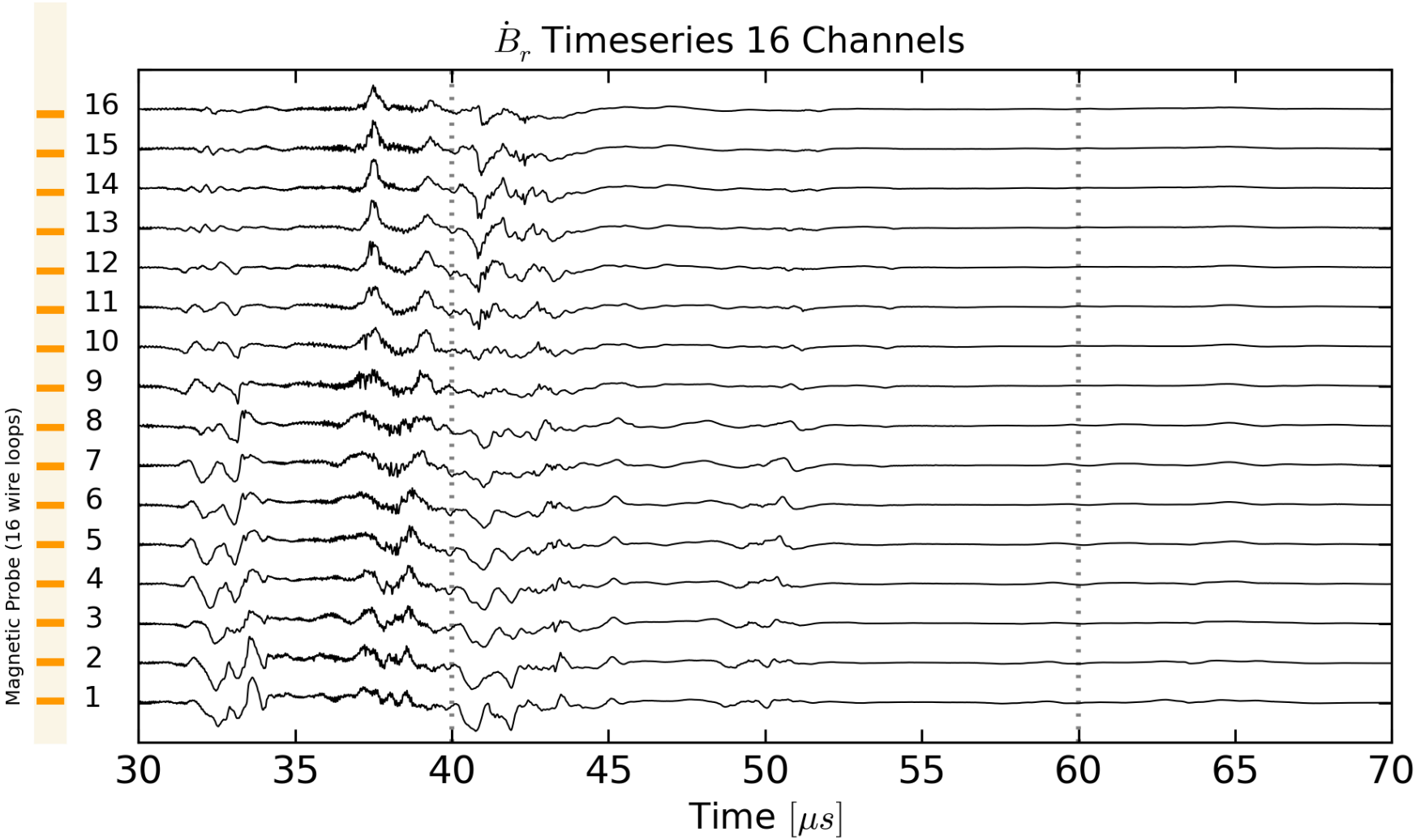

Formation/Selective Decay →← Equlib. →← Dissipation
Single Shot
Ave. Over Shots
(a)
(b)
(c)
$|B|$ [kG]
$n \times 10^{15}$ [cm^{-3}]
Mach Number
t [μs]

Four statistical metrics (SSX and solar wind)

- Frequency power spectrum $E_B(f)$
- PDF of temporal increments Δb
- Spatial correlation function $R(r)$
- Permutation entropy C-H
- many other metrics are possible

Diagnostics at midplane (B and n_e)



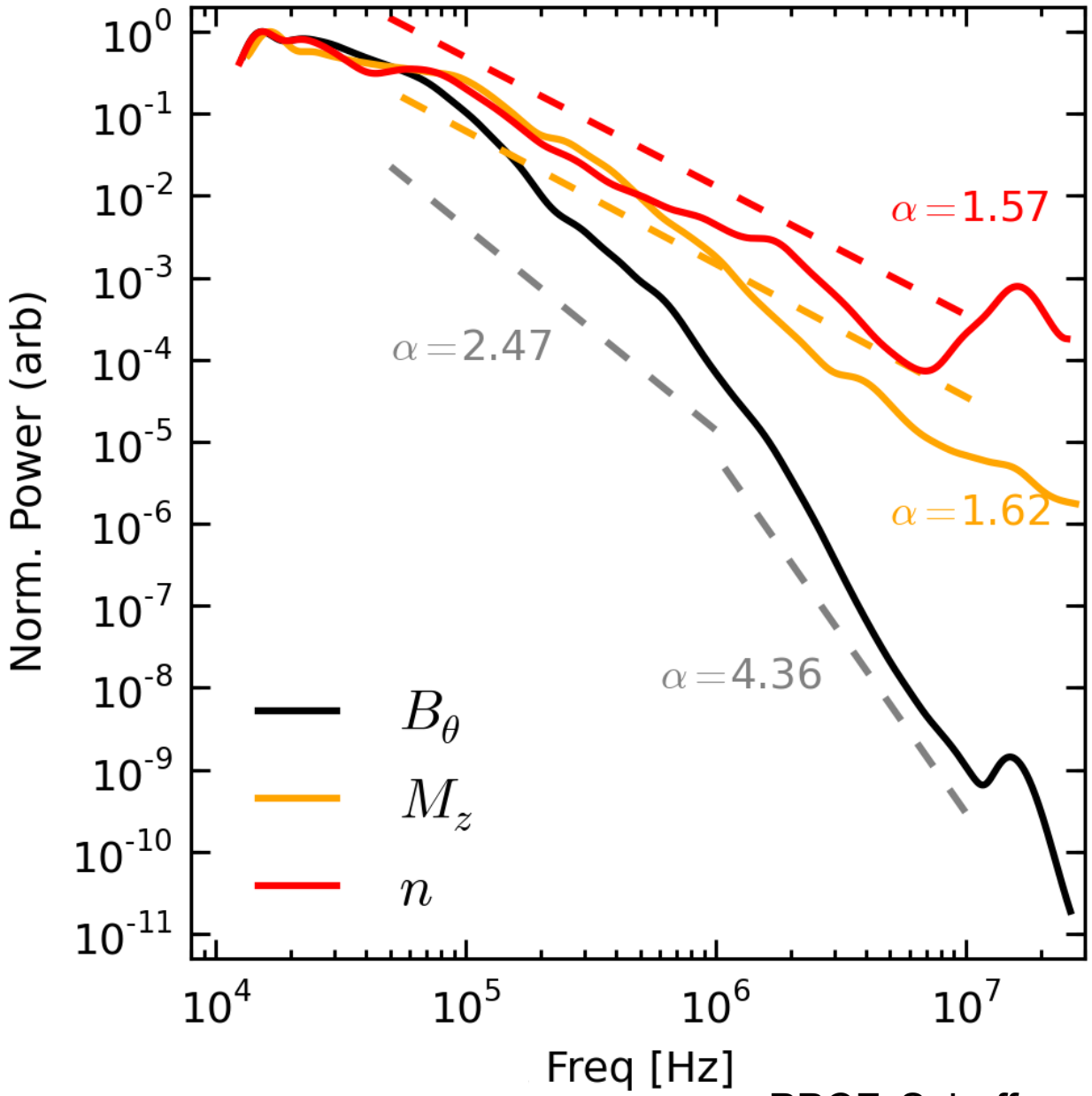


16 channels, 4 mm spacing, 65 MHz, 14 bit

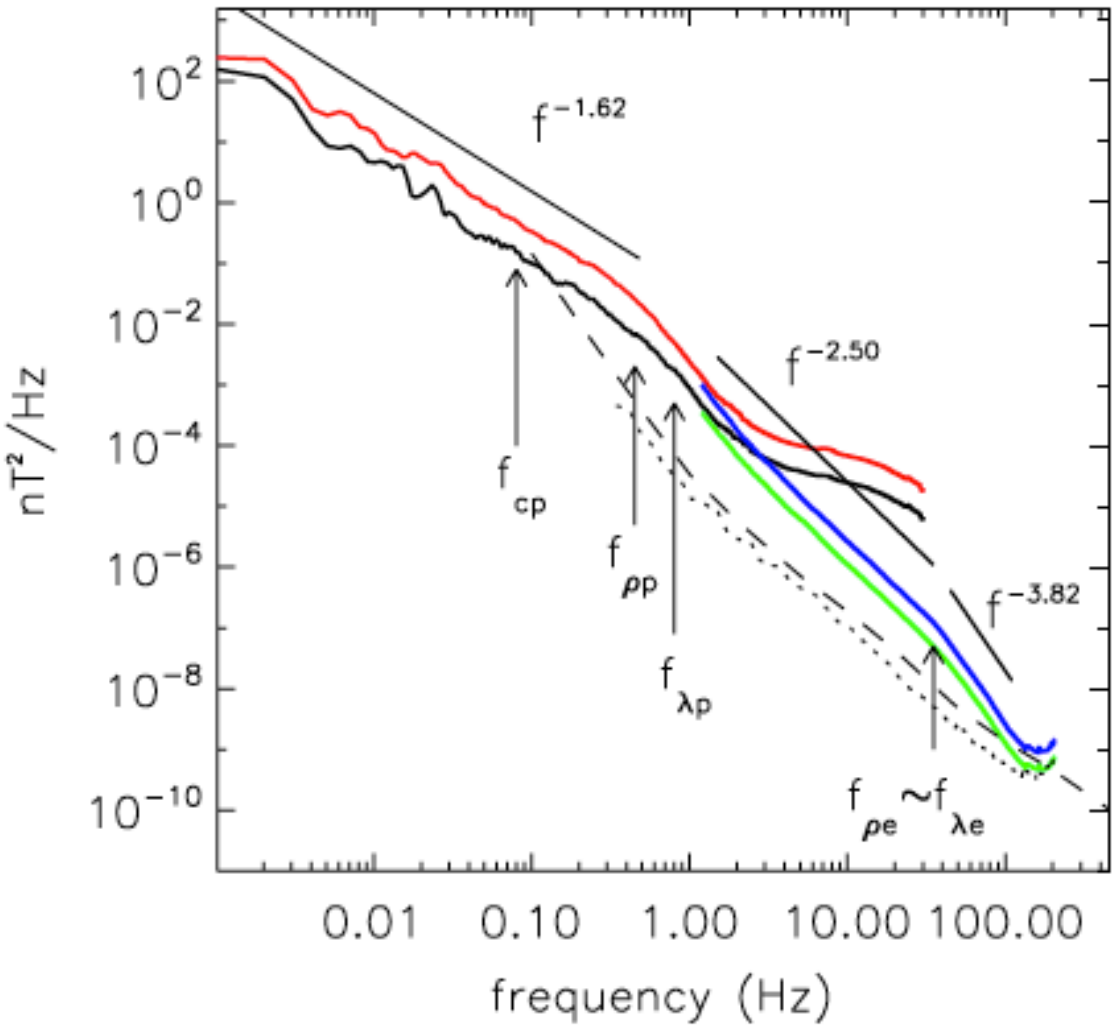
Frequency power spectrum $E_B(f)$

$$E(\omega) = \frac{1}{T}\left[\int B(t)e^{-i\omega t}dt\right]^2$$

- Use either FFT or Wavelets
- Convert spectrum of dB/dt to B

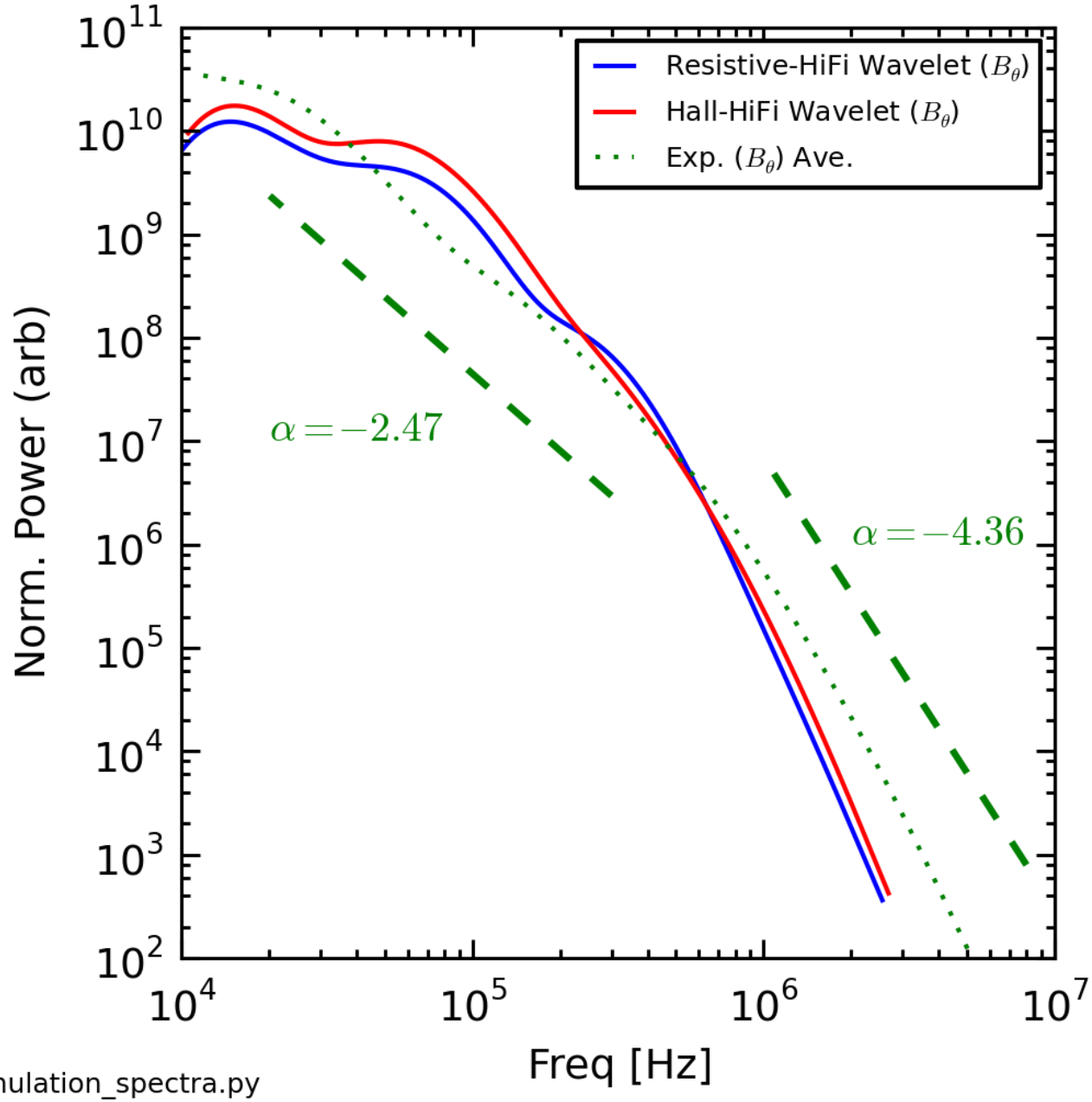


PPCF, Schaffner, et al (2014)



Cluster FGM and STAFF-SC

Sahraoui, 2009



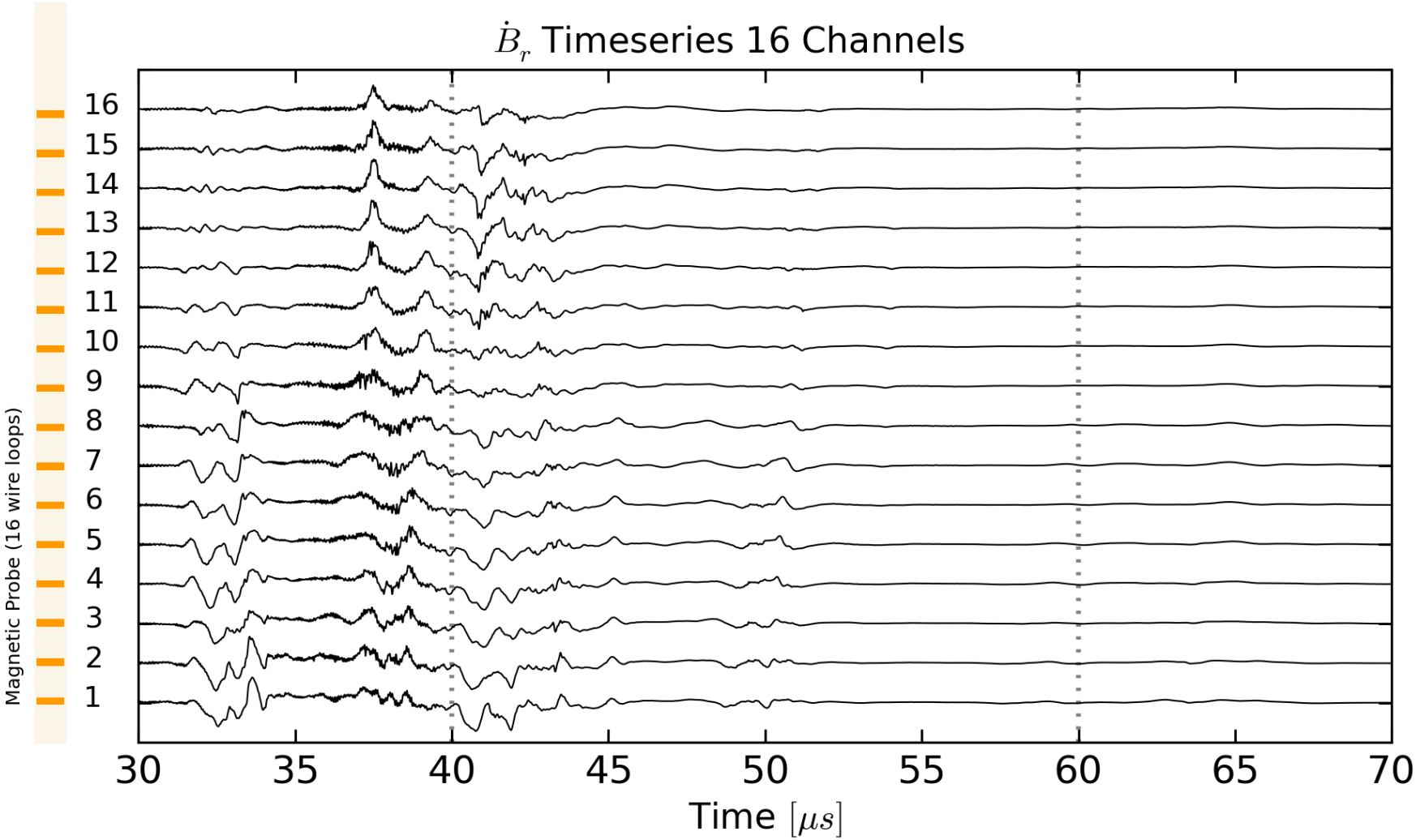
simulation_spectra.py

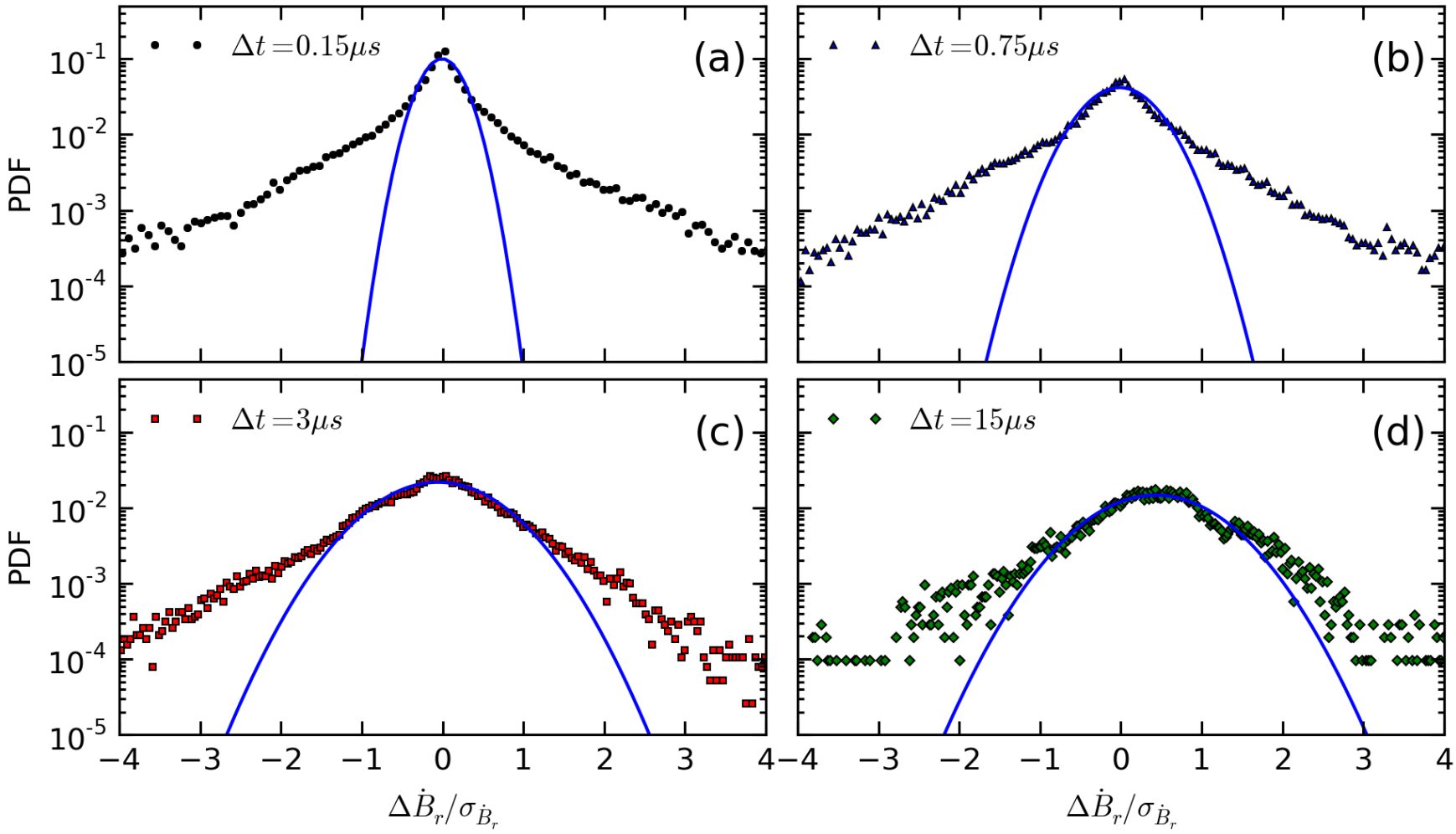
PDF of temporal increments

$$\Delta \mathbf{b}(t, \Delta t) = \mathbf{b}(t + \Delta t) - \mathbf{b}(t)$$

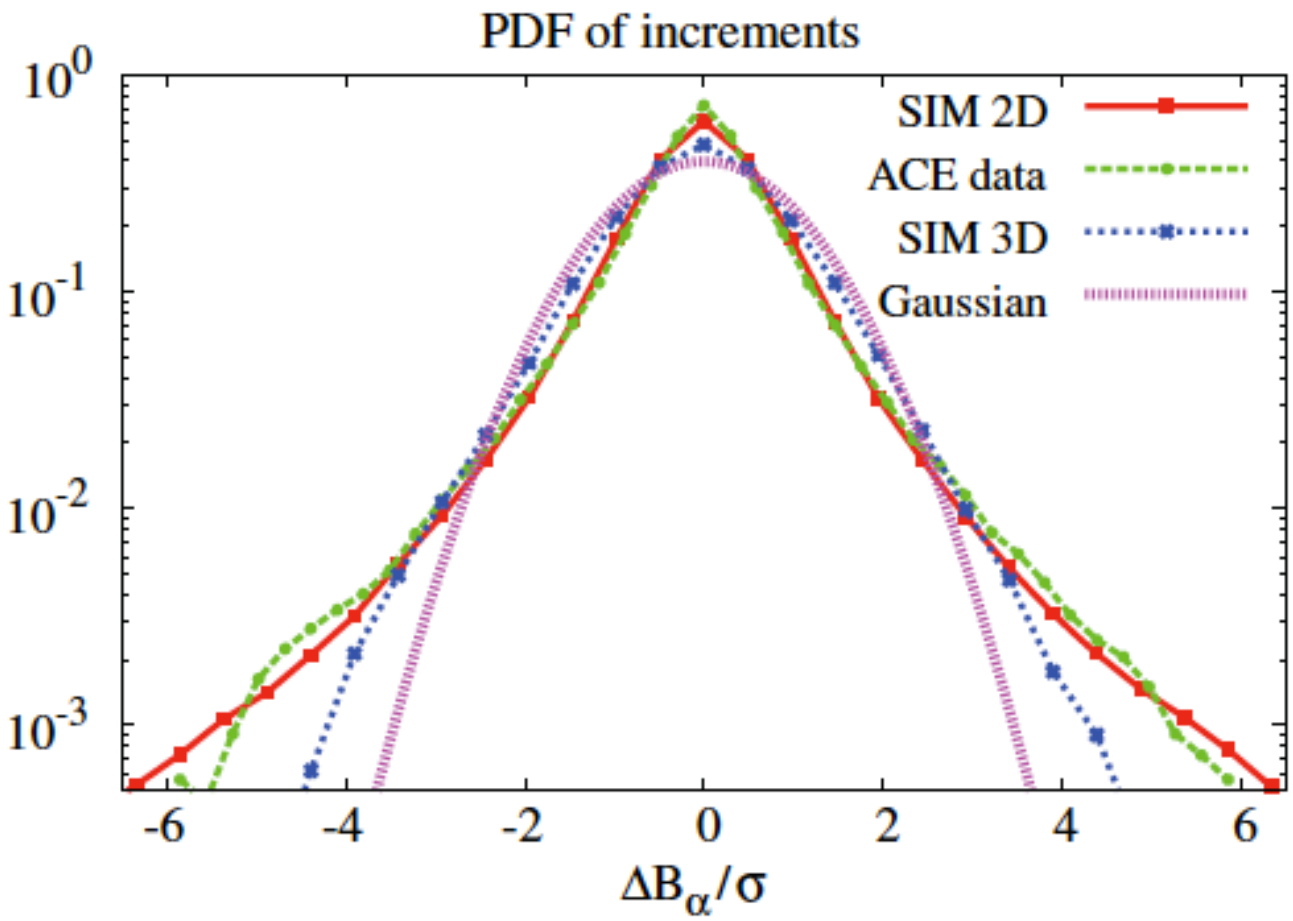
$$S^2(\Delta t) = \langle (\mathbf{b}(t + \Delta t) - \mathbf{b}(t))^2 \rangle$$

$$S^2(\Delta r) = \langle (\mathbf{b}(r + \Delta r) - \mathbf{b}(r))^2 \rangle$$

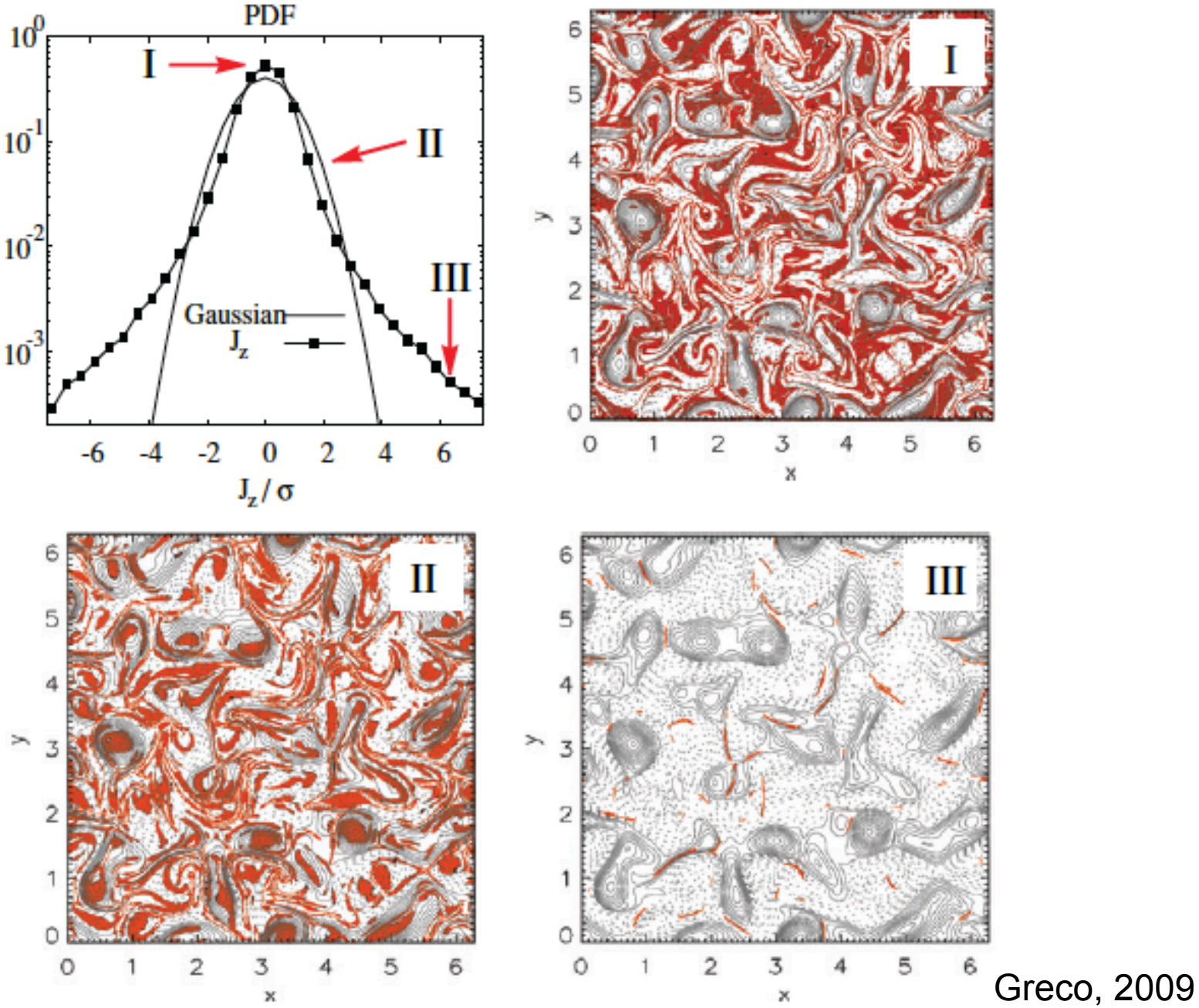

$\dot{B}_r$ Timeseries 16 Channels
Magnetic Probe (16 wire loops)
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
30
35
40
45
50
55
60
65
70
Time $[\mu s]$



PRL, Schaffner et al (2014)


PDF of increments
SIM 2D
ACE data
SIM 3D
Gaussian
10^0
10^{-1}
10^{-2}
10^{-3}
-6
-4
-2
0
2
4
6
$\Delta B_\alpha/\sigma$

Greco, 2009


PDF
I
II
III
Gaussian
J_z
J_z / σ
10^0
10^{-1}
10^{-2}
10^{-3}
-6 -4 -2 0 2 4 6
I
II
III
x
y
0 1 2 3 4 5 6

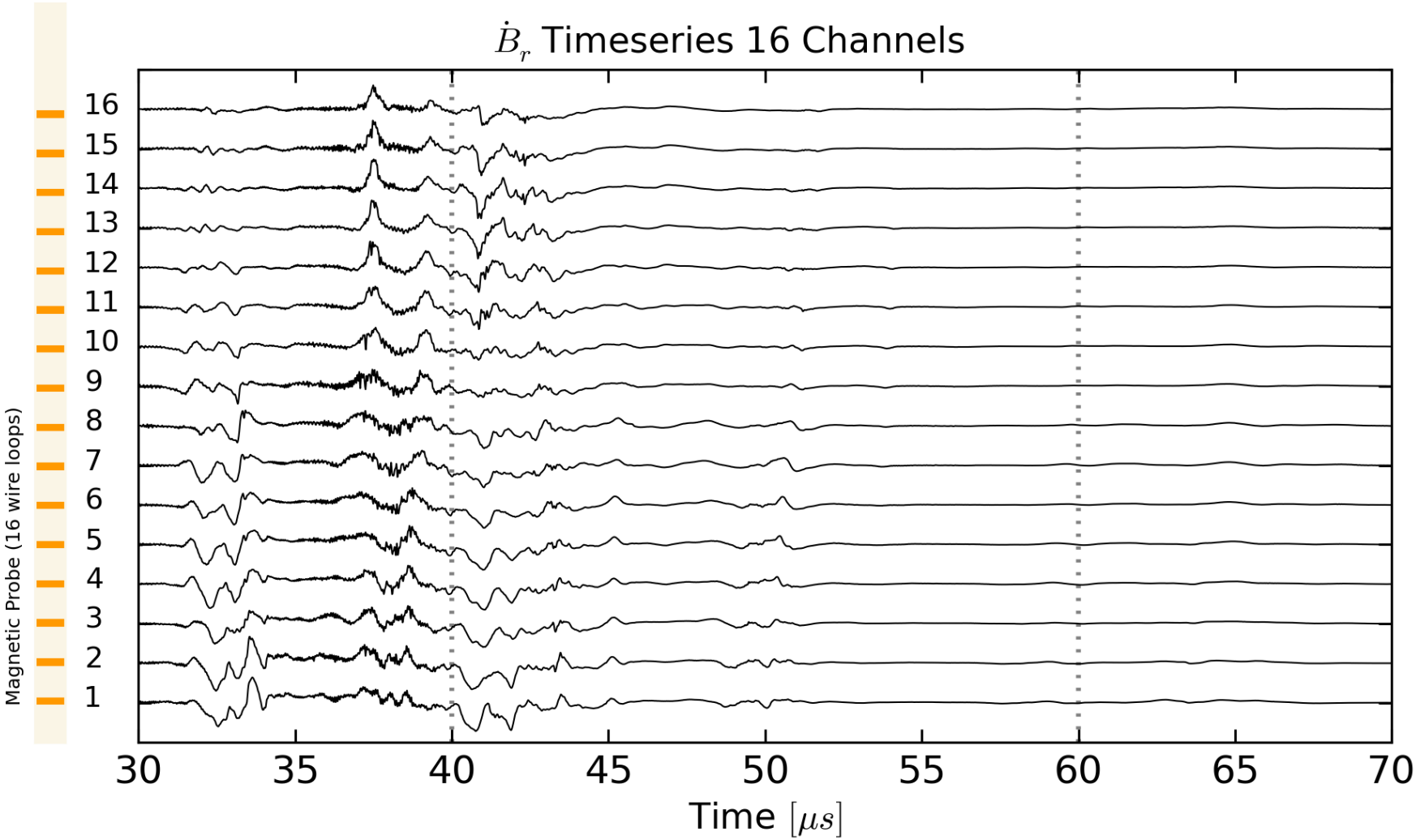
Greco, 2009

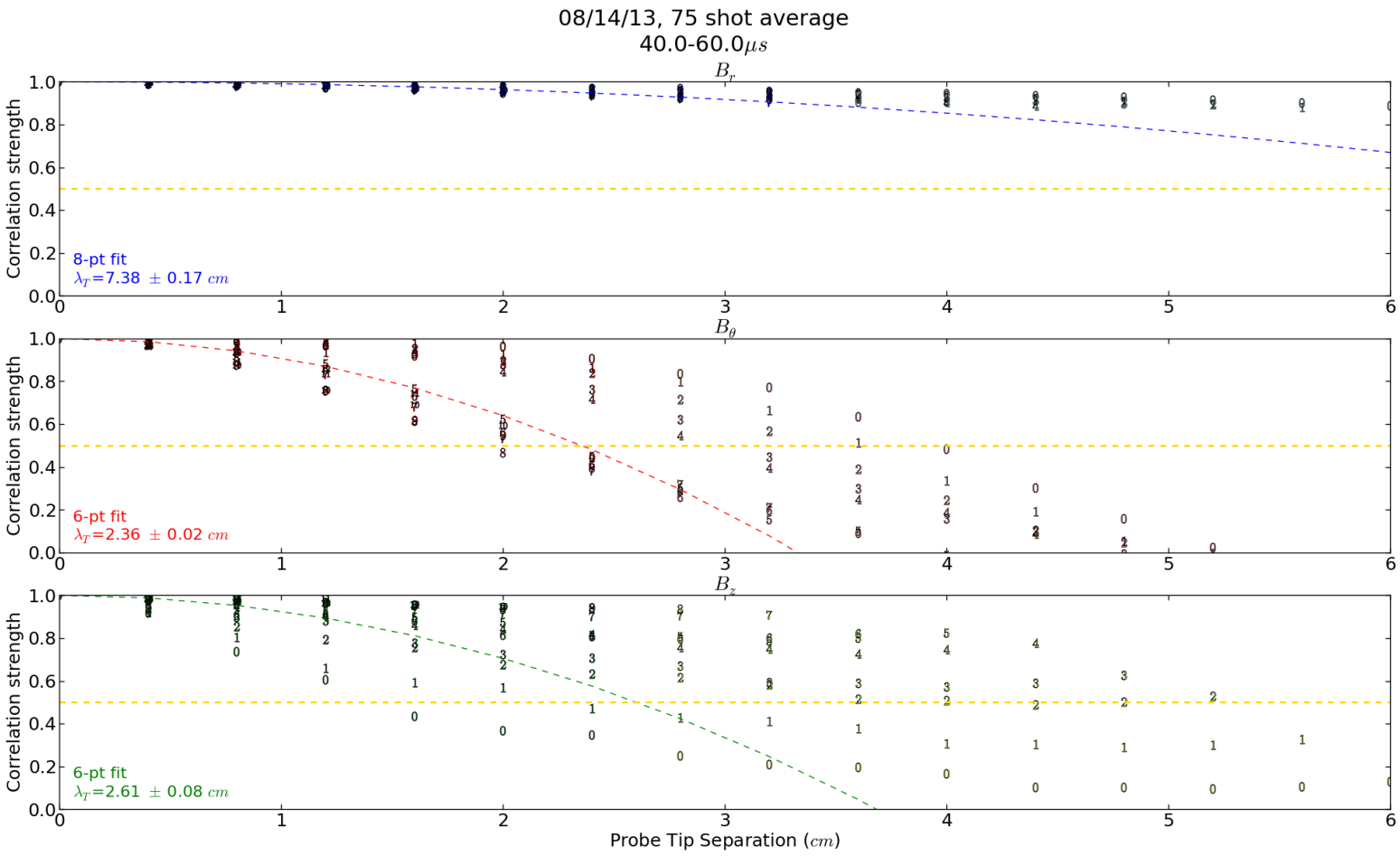
Spatial coherence function R(r)

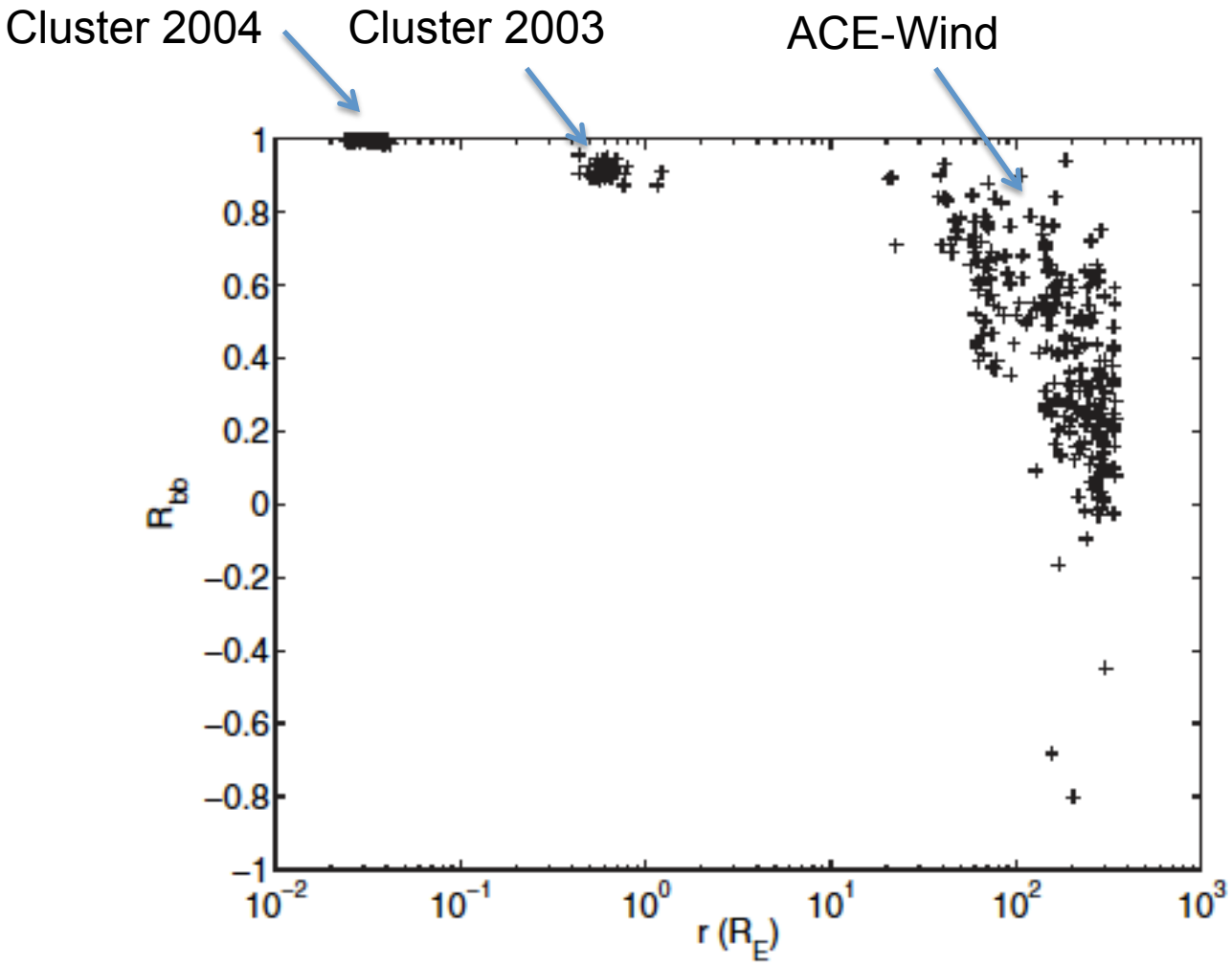
$$R_{ij}(\mathbf{r}) = \overline{b_i(\mathbf{x})b_j(\mathbf{x}+\mathbf{r})}$$

$$\frac{1}{2}\overline{b_i(\mathbf{x})b_i(\mathbf{x})} = \int_0^\infty E(k)dk$$

Work of Adrian Wan ‘15


$\dot{B}_r$ Timeseries 16 Channels
Magnetic Probe (16 wire loops)
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
30
35
40
45
50
55
60
65
70
Time $[\mu s]$

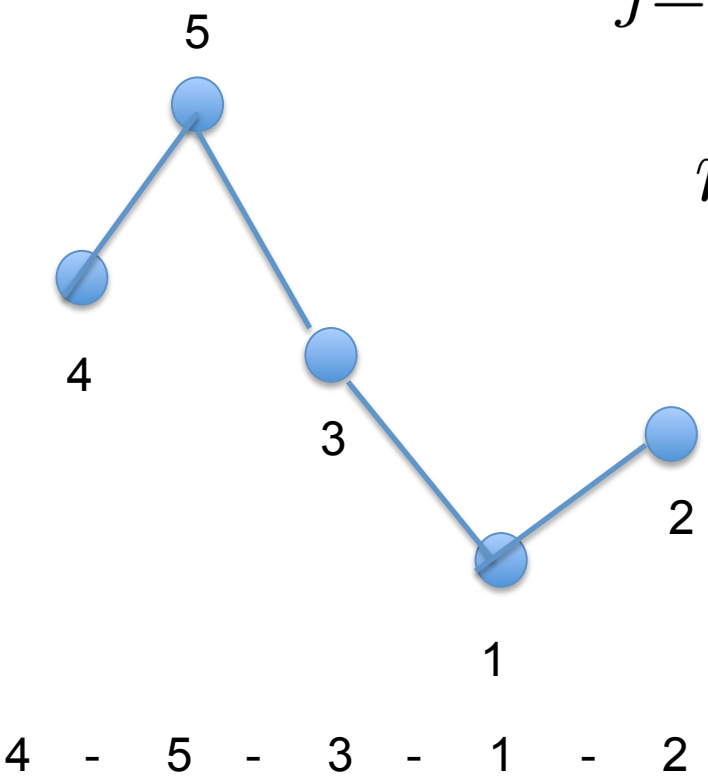

08/14/13, 75 shot average
40.0-60.0μs
B_r
8-pt fit
λ_T=7.38 ± 0.17 cm
B_θ
6-pt fit
λ_T=2.36 ± 0.02 cm
B_z
6-pt fit
λ_T=2.61 ± 0.08 cm
Correlation strength
Probe Tip Separation (cm)



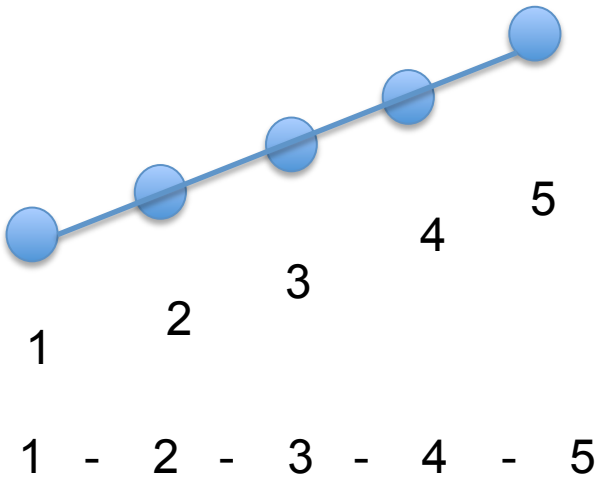
Matthaeus, 2005

Permutation Entropy

$$S[P] = -\sum_{j=1}^{N} p_j ln(p_j)$$



$$n = 5, N = n! = 120$$



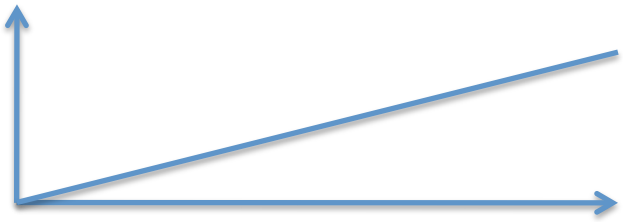
Permutation Entropy

$$S[P] = -\sum_{j=1}^{N} p_j ln(p_j)$$

Case 1: all N=120 permutations equally likely...
S = ln(N)… maximum

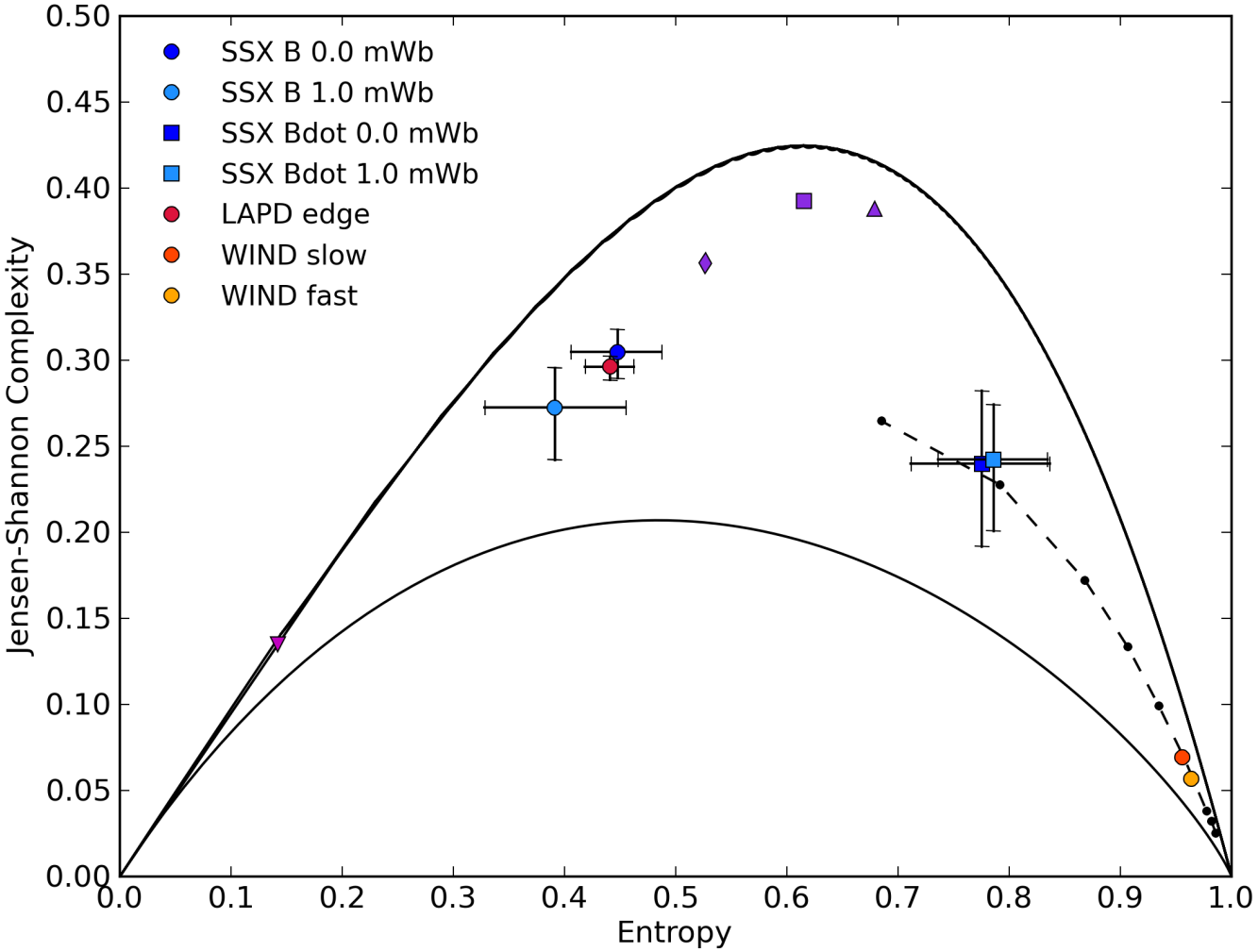
$$S = -\sum_{j=1}^{N} \frac{1}{N} ln(1/N) = N \frac{1}{N} ln(N) = ln(N)$$

Case 2: linear ramp so only one permutation appears… S = 0… minimum

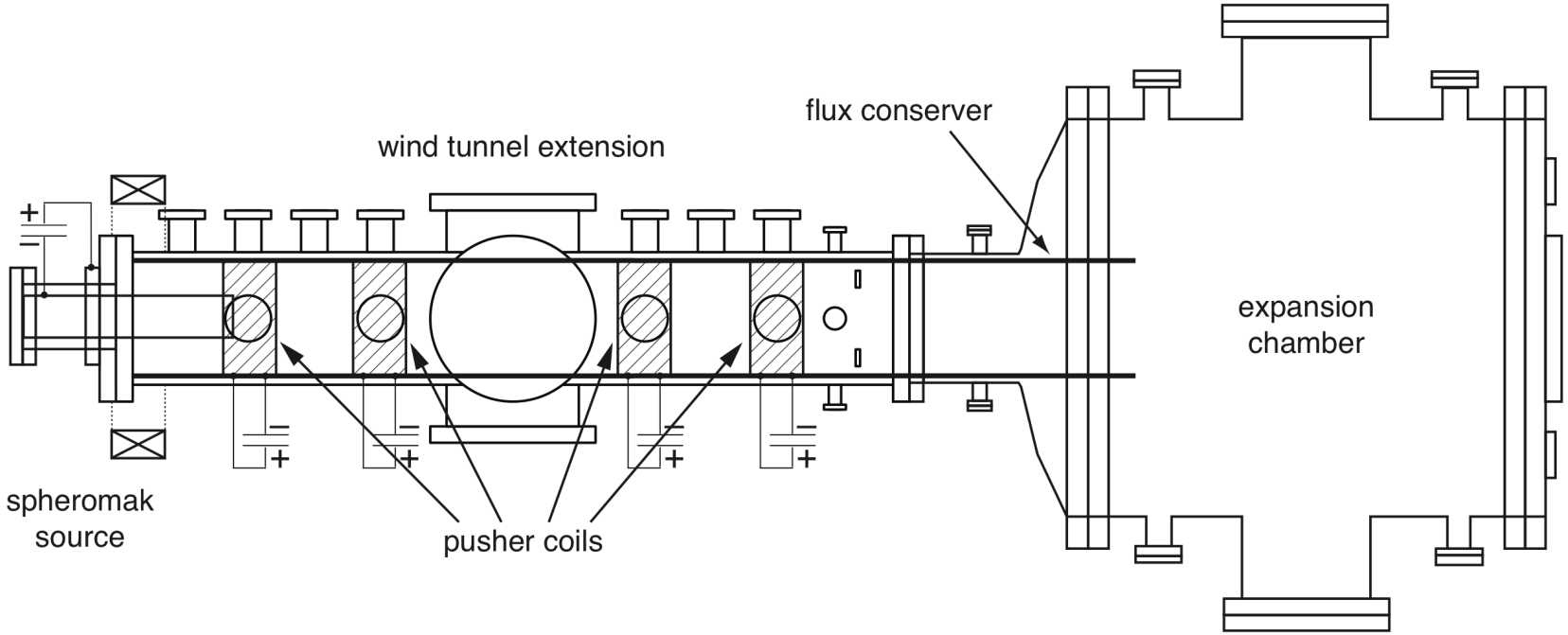


Complexity-Entropy map (SSX, solar wind, deterministic chaos)

PRE, Weck, et al (submitted 2014)

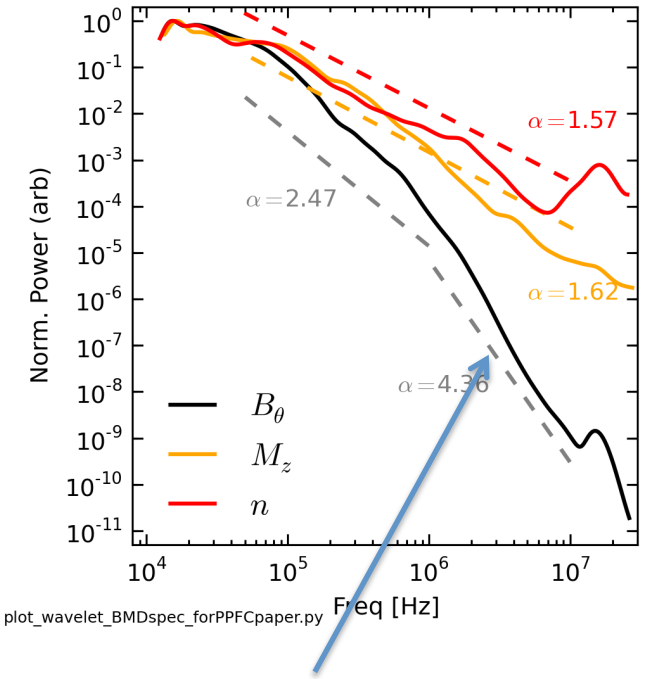


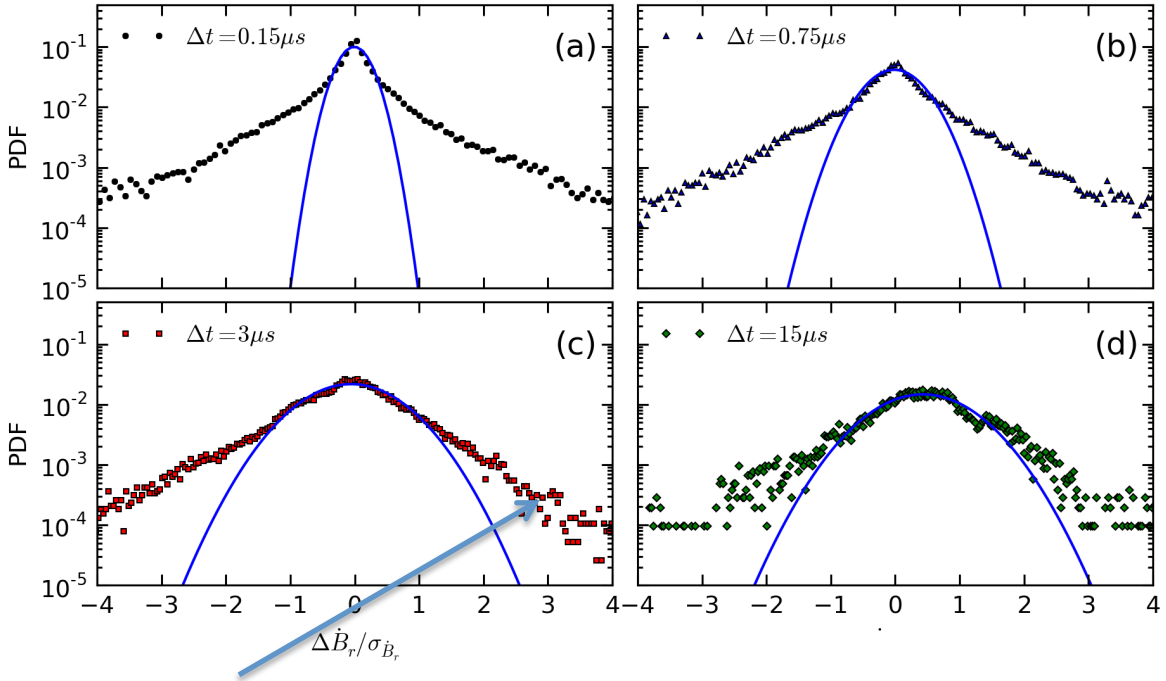
Extension and accelerator, 2015



Accelerate to v > 100 km/s, M > 2… fast camera, probes in chamber

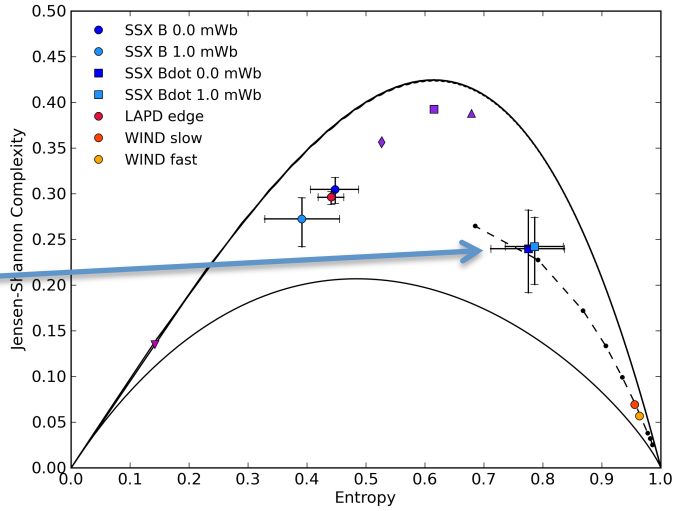
Summary





Cascade from large to small scales

Intermittency in the PDF

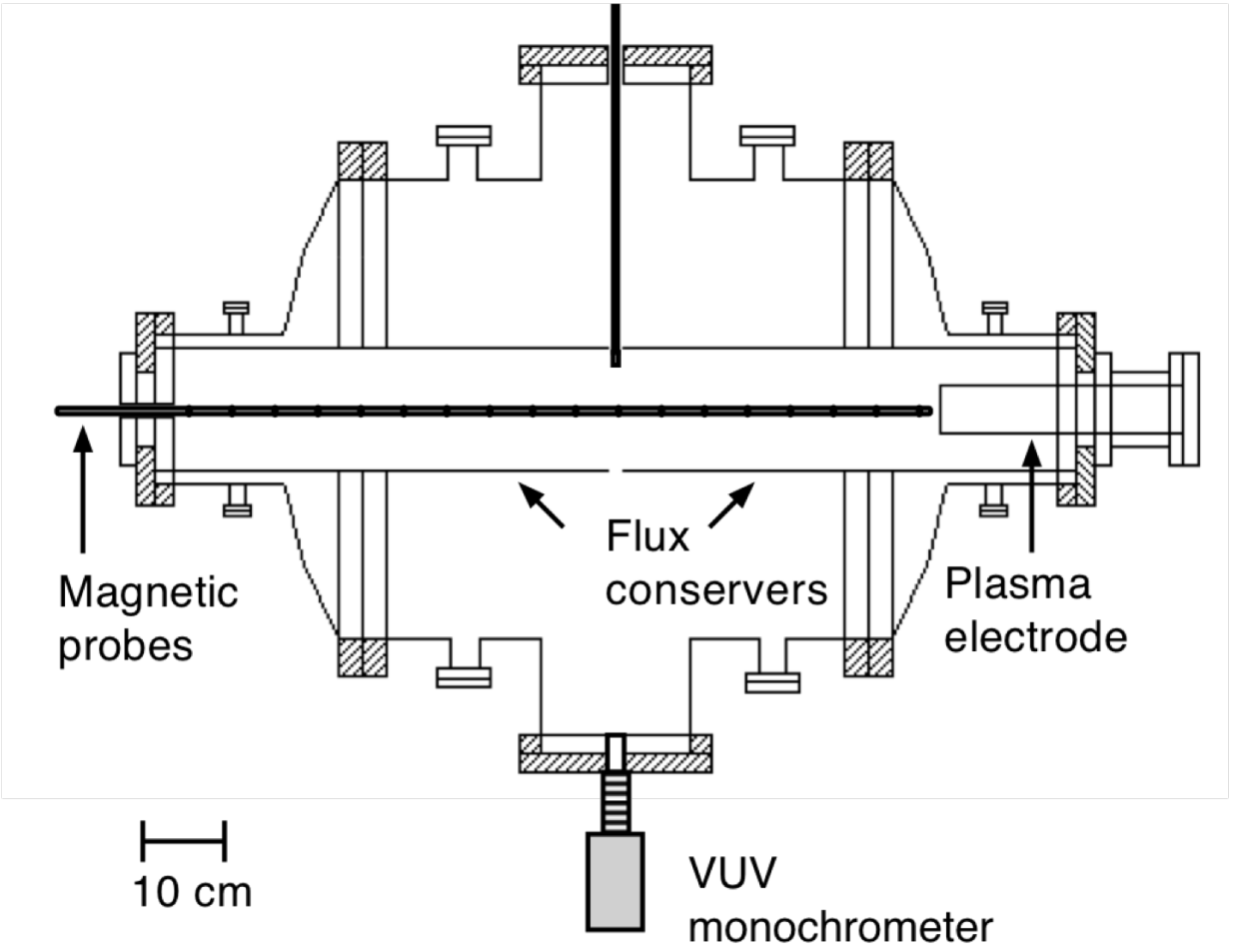


Entropy in the waveform

Thank you!

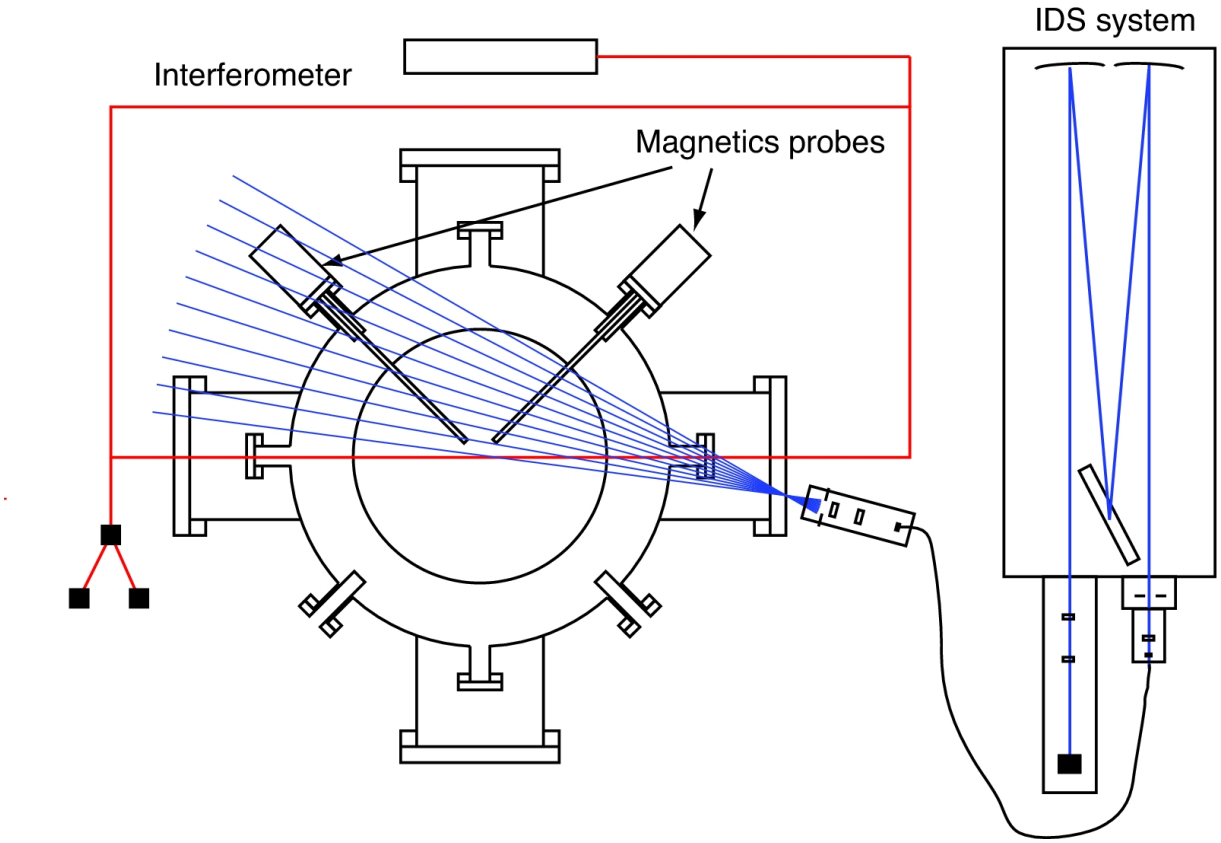
Questions?

SSX MHD wind tunnel



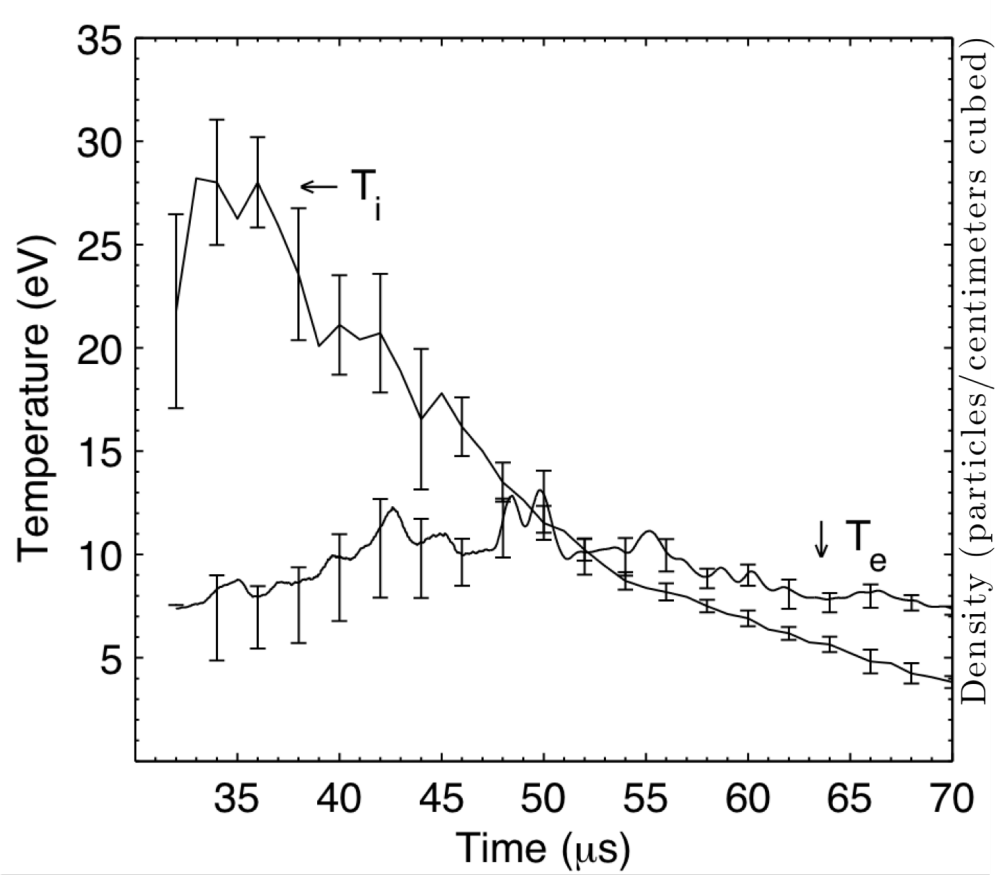
Chamber is baked and glow discharge cleaned, P = 10^{-8} torr

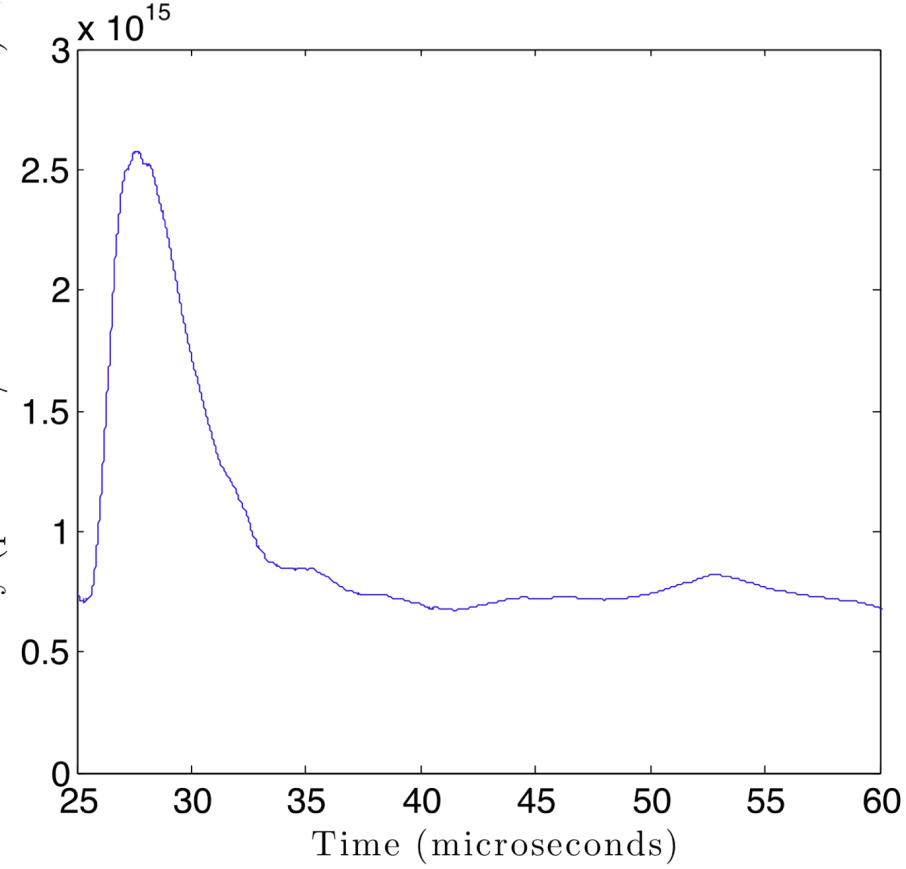
Ion Doppler spectrometer on SSX



Interferometer chord and two magnetic probes also shown

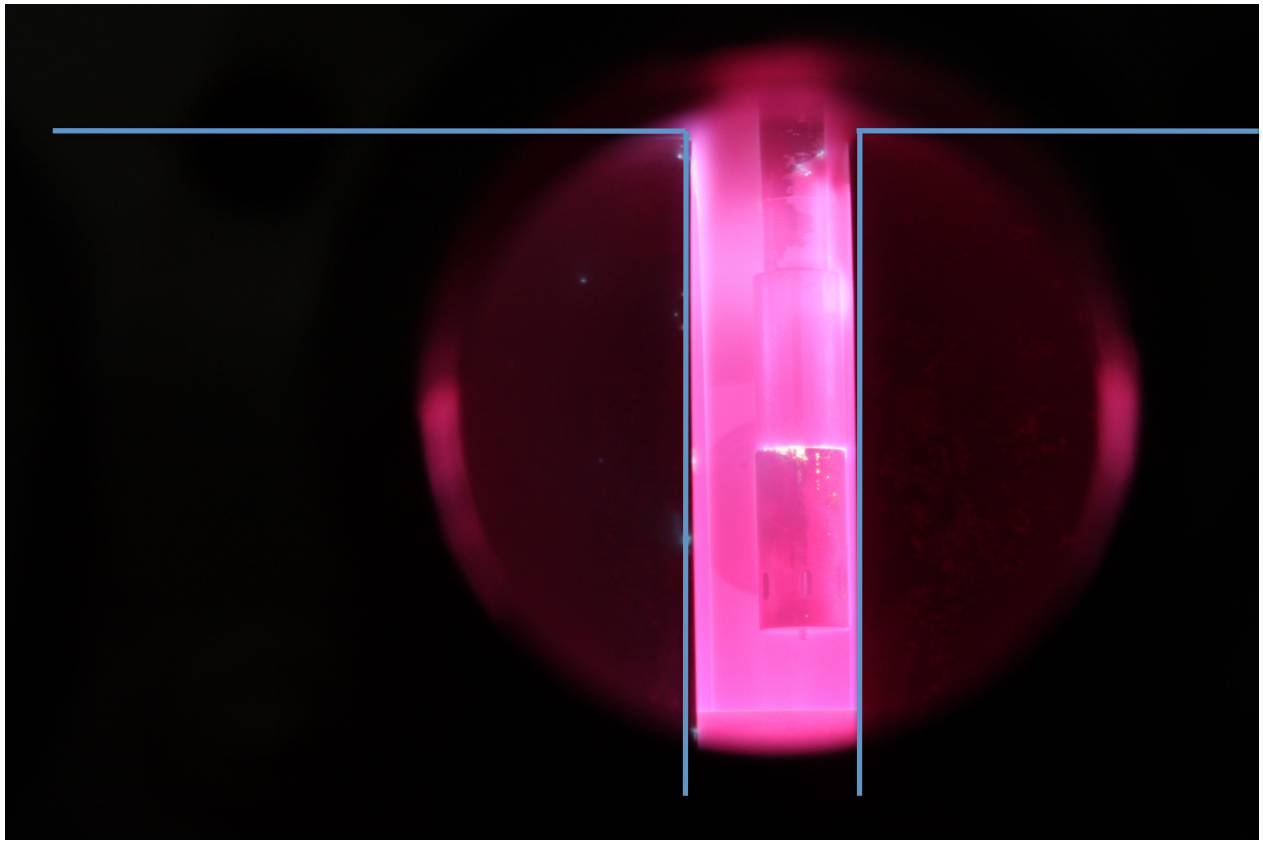
Temperature and density measurements





Wind tunnel plasma is well-characterized at the midplane (line-averaged)

Mach probe measures local flow



X. Zhang