

NAME _____

Honors Exam in Statistics

May 2005

Please read carefully:

- This three hour exam consists of 6 questions of equal weight.
- Answer all 6 questions.
- This is a closed book/closed notes exam.
- You may use a calculator that does not do algebra or calculus.
- Use additional answer sheets as needed.
- Label each sheet with the question number and your name at the top.
- Do not write the answer to more than one question on a single page.
- Do not write on the back of any sheet.
- Normal, t , χ^2 and F tables should be supplied with this exam.

Question 1

A systolic blood pressure (SBP) greater than 150 mmHg indicates the presence of hypertension. The definitive method for diagnosing hypertension requires that three consecutive systolic blood pressure readings each be greater than 150 mmHg. A new diagnosis method diagnoses hypertension if the mean of the three consecutive measures is greater than 150 mmHg. SBP measurements are independent and normally distributed with mean equal to the true underlying value and a variance of 30 mmHg.

- (a) What is the probability that a subject with a true underlying SBP of 155 mmHg will be given the correct diagnosis under the definitive method?
- (b) What is the probability that a subject with a true underlying SBP of 155 mmHg will be given the correct diagnosis under the new method?
- (c) What is the probability that these two methods applied to the same three readings will agree for a subject with a true underlying SBP of 155 mmHg?

Question 2

The following data are measures of pulmonary vascular resistance (PVR) in seven lambs before and after infusion of the drug histamine

Lamb	PVR before histamine	PVR after histamine
1	.09	.17
2	.10	.14
3	.09	.19
4	.13	.16
5	.09	.11
6	.08	.08
7	.13	.10

- (a) Perform a statistical test using $\alpha = 0.05$ to determine if histamine increases PVR on average. Be sure to state the null and alternative hypotheses, state all your assumptions and state the conclusions that can be drawn from the test.
- (b) You are now asked to design a similar study for a similar type of lamb from another region. How many lambs would you use in the study to ensure the test will have 90% power at the $\alpha = 0.05$ significance level for a difference in means of .03? Show your calculations.

Question 3

A new medical device has four components, A, B, C, D. Every time it is used each of the components may fail, independently, with probability 0.1. Failure is permanent. The device will not function if more than one of the components has failed. Assume no replacement of failed components.

- (a) What is the probability that the device fails during the first usage?
- (b) What is the probability that the device still works after 2 uses?
- (c) What is the expected number of uses that we will get from the device before it fails?

Question 4

Bananas on a special banana tree come in bunches whose size is Poisson with mean λ . Trucks deliver bunches to stores. At the store, a fraction $0 < p < 1$ of bananas are found to be bad. Bananas go bad independently of each other. Bad bananas are removed from their bunch before the bunch is put on sale. Bunches of size zero are not delivered and are not put on sale.

- (a) What is the distribution of the size of banana bunches on sale?
- (b) What fraction of bunches from the tree are never put on sale?
- (c) By going to the store and buying bunches of bananas, what function(s) of p and λ can you estimate?
- (d) Discuss whether, in reality, a Poisson model is appropriate for banana bunches.

Question 5

In a repeated measures design with n subjects, we take J measurements Y_{ij} , $j = 1, \dots, J$ on subject i . A commonly used model for this data is

$$Y_{ij} = \mu + \beta_i + \epsilon_{ij}$$

where

$$\begin{aligned}\beta_i &\sim N(0, D) \\ \epsilon_{ij} &\sim N(0, \sigma^2)\end{aligned}$$

and all β_i and all ϵ_{ij} are independent. The goal is to estimate μ . The usual estimator of μ is

$$\bar{Y} = (nJ)^{-1} \sum_{i=1}^n \sum_{j=1}^J Y_{ij} = n^{-1} \sum_{i=1}^n \bar{Y}_i,$$

defining \bar{Y} and \bar{Y}_i .

The cost of recruiting each subject is C_1 dollars. The cost of each measurement is C_2 dollars, including the first measurement.

A cross-sectional design has $J = 1$. The model is the same, $Y_{ij} = \mu + \beta_i + \epsilon_{ij}$ but now j only takes on the value 1. Assume m subjects in the cross-sectional design.

A measure of the efficiency of a design is

$$(\text{efficiency})^{-1} = \text{cost} * \text{variance}$$

where “cost” is the total cost of recruiting subjects and taking all measurements; “variance” is the variance of \bar{Y} .

- (a) What is the variance of \bar{Y} in the repeated measures design?
- (b) What is the cost of a repeated measures study with n subjects and J repeated measures?
- (c) How many subjects m could you afford in the cross-sectional study for the same price as the repeated measures study with given J , n , C_1 and C_2 ?
- (d) Suppose $m = 2n$ and $J = 4$,
 1. When does the repeated measures design have smaller variance of \bar{Y} than the cross-sectional design?
 2. And when is the repeated measures design cheaper than the cross-sectional design?
 3. When is the repeated measures design more efficient?

Question 6

The following data are the remission times y_i in weeks for $n = 21$ independent leukemia patients:

1, 1, 2, 2, 3, 4, 4, 5, 8, 8, 8, 8, 11, 11, 12+, 12, 14, 15+, 17, 18+, 23+

The + indicates an observation whose exact value is unknown, but is at least as large as the given value. Let y denote the remission time of a generic patient.

- (a) Estimate the more than 5-week remission probability $P(y > 5)$. Do not assume any distribution for the y_i .
- (b) Give a 95% confidence interval for your estimate in (a).
- (c) Does not knowing the exact values of 4 of the observations matter for your calculations? Yes/No
- (d) Assume an exponential distribution $f(y_i|\lambda) = \exp(-y_i/\lambda)/\lambda$ for y_i with unknown mean λ . What is the probability, as a function of λ , that a $y_i > c$ for known c ? This is the likelihood contribution made by a '+' observation, where c is 12, 15, 18, or 23 as appropriate.
- (e) Write down the likelihood of λ . The total of the 17 known remission times is 119. The total of the 4 '+' times is 68.
- (f) Calculate the maximum likelihood estimate $\hat{\lambda}$ of λ .
- (g) Find the asymptotic standard deviation of $\hat{\lambda}$ as a function of λ . Calculate the asymptotic standard error of $\hat{\lambda}$ by substituting in the maximum likelihood estimate for the unknown λ .

Table entry for p and C is the critical value t^* with probability p lying to its right and probability C lying between $-t^*$ and t^* .

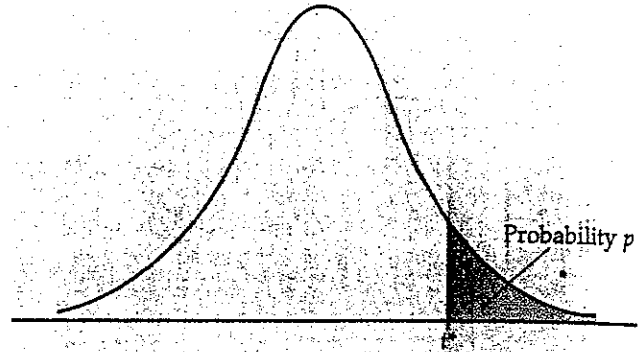


TABLE 3 t distribution critical values

df	Upper tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.884	1.100	1.384	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.374	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.684	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

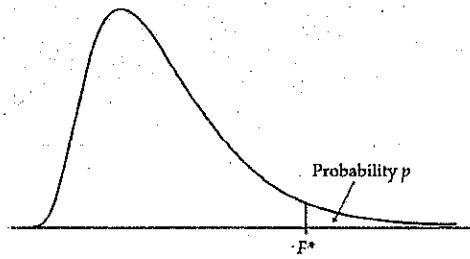


Table entry for p is the critical value F^* with probability p lying to its right.

TABLE E F critical values

Degrees of freedom in the denominator	p	Degrees of freedom in the numerator								
		1	2	3	4	5	6	7	8	9
1	.100	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86
	.050	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
	.025	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28
	.010	4052.2	4999.5	5403.4	5624.6	5763.6	5859.0	5928.4	5981.1	6022.5
	.001	405284	500000	540379	562500	576405	585937	592873	598144	602284
2	.100	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
	.050	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
	.025	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39
	.010	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
	.001	998.50	999.00	999.17	999.25	999.30	999.33	999.36	999.37	999.39
3	.100	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
	.050	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
	.025	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47
	.010	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35
	.001	167.03	148.50	141.11	137.10	134.58	132.85	131.58	130.62	129.86
4	.100	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
	.050	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
	.025	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90
	.010	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
	.001	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.47
5	.100	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
	.050	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
	.025	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68
	.010	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
	.001	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65	27.24
6	.100	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
	.050	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
	.025	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52
	.010	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98
	.001	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.69
7	.100	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
	.050	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
	.025	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82
	.010	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
	.001	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.33

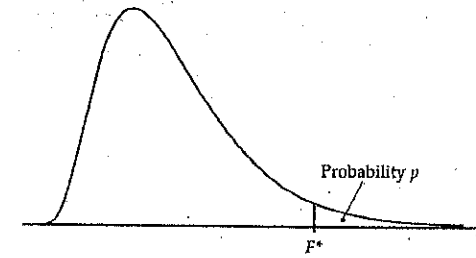


Table entry for p is the critical value F^* with probability p lying to its right.

TABLE E F critical values (continued)

Degrees of freedom in the denominator	p	Degrees of freedom in the numerator										
		10	12	15	20	25	30	40	50	60	120	1000
1	.100	60.19	60.71	61.22	61.74	62.05	62.26	62.53	62.69	62.79	63.06	63.30
	.050	241.88	243.91	245.95	248.01	249.26	250.10	251.14	251.77	252.20	253.25	254.19
	.025	968.63	976.71	984.87	993.10	998.08	1001.4	1005.6	1008.1	1009.8	1014.0	1017.7
	.010	6055.8	6106.3	6157.3	6208.7	6239.8	6260.6	6286.8	6302.5	6313.0	6339.4	6362.7
	.001	605621	610668	615764	620908	624017	626099	628712	630285	631337	633972	636301
2	.100	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.47	9.48	9.49
	.050	19.40	19.41	19.43	19.45	19.46	19.46	19.47	19.48	19.48	19.49	19.49
	.025	39.40	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.48	39.49	39.50
	.010	99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.48	99.49	99.50
	.001	999.40	999.42	999.43	999.45	999.46	999.47	999.47	999.48	999.48	999.49	999.50
3	.100	5.23	5.22	5.20	5.18	5.17	5.17	5.16	5.15	5.15	5.14	5.13
	.050	8.79	8.74	8.70	8.66	8.63	8.62	8.59	8.58	8.57	8.55	8.53
	.025	14.42	14.34	14.25	14.17	14.12	14.08	14.04	14.01	13.99	13.95	13.91
	.010	27.23	27.05	26.87	26.69	26.58	26.50	26.41	26.35	26.32	26.22	26.14
	.001	129.25	128.32	127.37	126.42	125.84	125.45	124.96	124.66	124.47	123.97	123.53
4	.100	3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.79	3.79	3.78	3.76
	.050	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.70	5.69	5.66	5.63
	.025	8.84	8.75	8.66	8.56	8.50	8.46	8.41	8.38	8.36	8.31	8.26
	.010	14.55	14.37	14.20	14.02	13.91	13.84	13.75	13.69	13.65	13.56	13.47
	.001	48.05	47.41	46.76	46.10	45.70	45.43	45.09	44.88	44.75	44.40	44.09
5	.100	3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.15	3.14	3.12	3.11
	.050	4.74	4.68	4.62	4.56	4.52	4.50	4.46	4.44	4.43	4.40	4.37
	.025	6.62	6.52	6.43	6.33	6.27	6.23	6.18	6.14	6.12	6.07	6.02
	.010	10.05	9.89	9.72	9.55	9.45	9.38	9.29	9.24	9.20	9.11	9.03
	.001	26.92	26.42	25.91	25.39	25.08	24.87	24.60	24.44	24.33	24.06	23.82
6	.100	2.94	2.90	2.87	2.84	2.81	2.80	2.78	2.77	2.76	2.74	2.72
	.050	4.06	4.00	3.94	3.87	3.83	3.81	3.77	3.75	3.74	3.70	3.67
	.025	5.46	5.37	5.27	5.17	5.11	5.07	5.01	4.98	4.96	4.90	4.86
	.010	7.87	7.72	7.56	7.40	7.30	7.23	7.14	7.09	7.06	6.97	6.89
	.001	18.41	17.99	17.56	17.12	16.85	16.67	16.44	16.31	16.21	15.98	15.77
7	.100	2.70	2.67	2.63	2.59	2.57	2.56	2.54	2.52	2.51	2.49	2.47
	.050	3.64	3.57	3.51	3.44	3.40	3.38	3.34	3.32	3.30	3.27	3.23
	.025	4.76	4.67	4.57	4.47	4.40	4.36	4.31	4.28	4.25	4.20	4.15
	.010	6.62	6.47	6.31	6.16	6.06	5.99	5.91	5.86	5.82	5.74	5.66
	.001	14.08	13.71	13.32	12.93	12.69	12.53	12.33	12.20	12.12	11.91	11.72

TABLE E F critical values (continued)

		Degrees of freedom in the numerator									
		1	2	3	4	5	6	7	8	9	
Degrees of freedom in the denominator	8	.100	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
		.050	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
		.025	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36
		.010	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
		.001	25.41	18.49	15.83	14.39	13.48	12.86	12.40	12.05	11.77
		.100	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
		.050	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
		.025	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03
		.010	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
		.001	22.86	16.39	13.90	12.56	11.71	11.13	10.70	10.37	10.11
		.100	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
		.050	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
		.025	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78
		.010	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94
		.001	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.96
		.100	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27
		.050	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
		.025	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59
		.010	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63
		.001	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.35	8.12
	.100	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	
	.050	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	
	.025	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	
	.010	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	
	.001	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.48	
	.100	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	
	.050	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	
	.025	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	
	.010	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	
	.001	17.82	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.98	
	.100	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	
	.050	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	
	.025	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	
	.010	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	
	.001	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.58	
	.100	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	
	.050	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	
	.025	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	
	.010	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	
	.001	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.26	
	.100	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	
	.050	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	
	.025	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	
	.010	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	
	.001	16.12	10.97	9.01	7.94	7.27	6.80	6.46	6.19	5.98	
	.100	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	
	.050	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	
	.025	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	
	.010	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	
	.001	15.72	10.66	8.73	7.68	7.02	6.56	6.22	5.96	5.75	

TABLE E F critical values (continued)

		Degrees of freedom in the numerator										
		10	12	15	20	25	30	40	50	60	120	1000
Degrees of freedom in the denominator	8	2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.35	2.34	2.32	2.30
		3.35	3.28	3.22	3.15	3.11	3.08	3.04	3.02	3.01	2.97	2.93
		4.30	4.20	4.10	4.00	3.94	3.89	3.84	3.81	3.78	3.73	3.68
		5.81	5.67	5.52	5.36	5.26	5.20	5.12	5.07	5.03	4.95	4.87
		11.54	11.19	10.84	10.48	10.26	10.11	9.92	9.80	9.73	9.53	9.36
		2.42	2.38	2.34	2.30	2.27	2.25	2.23	2.22	2.21	2.18	2.16
		3.14	3.07	3.01	2.94	2.89	2.86	2.83	2.80	2.79	2.75	2.71
		3.96	3.87	3.77	3.67	3.60	3.56	3.51	3.47	3.45	3.39	3.34
		5.26	5.11	4.96	4.81	4.71	4.65	4.57	4.52	4.48	4.40	4.32
		9.89	9.57	9.24	8.90	8.69	8.55	8.37	8.26	8.19	8.00	7.84
		2.32	2.28	2.24	2.20	2.17	2.16	2.13	2.12	2.11	2.08	2.06
		2.98	2.91	2.85	2.77	2.73	2.70	2.66	2.64	2.62	2.58	2.54
		3.72	3.62	3.52	3.42	3.35	3.31	3.26	3.22	3.20	3.14	3.09
		4.85	4.71	4.56	4.41	4.31	4.25	4.17	4.12	4.08	4.00	3.92
		8.75	8.45	8.13	7.80	7.60	7.47	7.30	7.19	7.12	6.94	6.78
		2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.04	2.03	2.00	1.98
		2.85	2.79	2.72	2.65	2.60	2.57	2.53	2.51	2.49	2.45	2.41
		3.53	3.43	3.33	3.23	3.16	3.12	3.06	3.03	3.00	2.94	2.89
		4.54	4.40	4.25	4.10	4.01	3.94	3.86	3.81	3.78	3.69	3.61
		7.92	7.63	7.32	7.01	6.81	6.68	6.52	6.42	6.35	6.18	6.02
	2.19	2.15	2.10	2.06	2.03	2.01	1.99	1.97	1.96	1.93	1.91	
	2.75	2.69	2.62	2.54	2.50	2.47	2.43	2.40	2.38	2.34	2.30	
	3.37	3.28	3.18	3.07	3.01	2.96	2.91	2.87	2.85	2.79	2.73	
	4.30	4.16	4.01	3.86	3.76	3.70	3.62	3.57	3.54	3.45	3.37	
	7.29	7.00	6.71	6.40	6.22	6.09	5.93	5.83	5.76	5.59	5.44	
	2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.92	1.90	1.88	1.85	
	2.67	2.60	2.53	2.46	2.41	2.38	2.34	2.31	2.30	2.25	2.21	
	3.25	3.15	3.05	2.95	2.88	2.84	2.78	2.74	2.72	2.66	2.60	
	4.10	3.96	3.82	3.66	3.57	3.51	3.43	3.38	3.34	3.25	3.18	
	6.80	6.52	6.23	5.93	5.75	5.63	5.47	5.37	5.30	5.14	4.99	
	2.10	2.05	2.01	1.96	1.93	1.91	1.89	1.87	1.86	1.83	1.80	
	2.60	2.53	2.46	2.39	2.34	2.31	2.27	2.24	2.22	2.18	2.14	
	3.15	3.05	2.95	2.84	2.78	2.73	2.67	2.64	2.61	2.55	2.50	
	3.94	3.80	3.66	3.51	3.41	3.35	3.27	3.22	3.18	3.09	3.02	
	6.40	6.13	5.85	5.56	5.38	5.25	5.10	5.00	4.94	4.77	4.62	
	2.06	2.02	1.97	1.92	1.89	1.87	1.85	1.83	1.82	1.79	1.76	
	2.54	2.48	2.40	2.33	2.28	2.25	2.20	2.18	2.16	2.11	2.07	
	3.06	2.96	2.86	2.76	2.69	2.64	2.59	2.55	2.52	2.46	2.40	
	3.80	3.67	3.52	3.37	3.28	3.21	3.13	3.08	3.05	2.96	2.88	
	6.08	5.81	5.54	5.25	5.07	4.95	4.80	4.70	4.64	4.47	4.33	
	2.03	1.99	1.94	1.89	1.86	1.84	1.81	1.79	1.78	1.75	1.72	
	2.49	2.42	2.35	2.28	2.23	2.19	2.15	2.12	2.11	2.06	2.02	
	2.99	2.89	2.79	2.68	2.61	2.57	2.51	2.47	2.45	2.38	2.32	
	3.69	3.55	3.41	3.26	3.16	3.10	3.02	2.97	2.93	2.84	2.76	
	5.81	5.55	5.27	4.99	4.82	4.70	4.54	4.45	4.39	4.23	4.08	
	2.00	1.96	1.91	1.86	1.83	1.81	1.78	1.76	1.75	1.72	1.69	
	2.45	2.38	2.31	2.23	2.18	2.15	2.10	2.08	2.06	2.01	1.97	
	2.92	2.82	2.72	2.62	2.55	2.50	2.44	2.41	2.38	2.32	2.26	
	3.59	3.46	3.31	3.16	3.07	3.00	2.92	2.87	2.83	2.75	2.66	
	5.58	5.32	5.05	4.78	4.60	4.48	4.33	4.24	4.18	4.02	3.87	

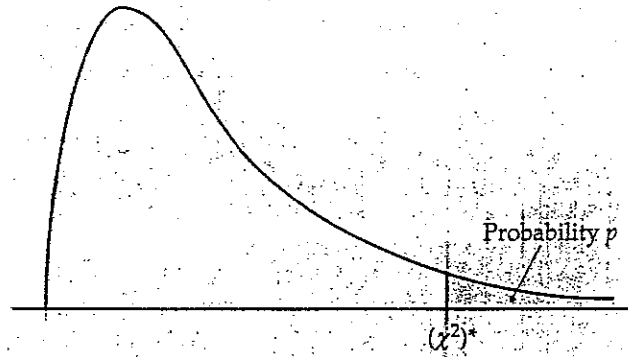


Table entry for p is the critical value $(\chi^2)^*$ with probability p lying to its right.

TABLE F χ^2 distribution critical values

df	Tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

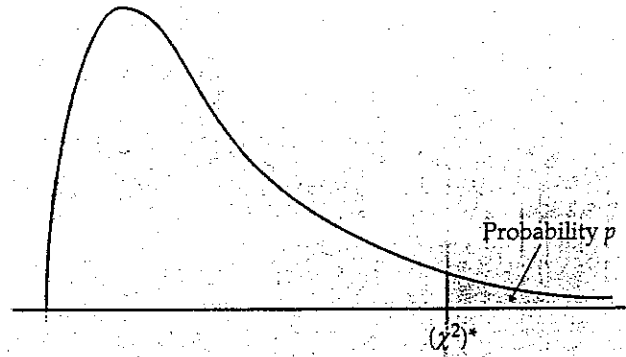


Table entry for p is the critical value $(\chi^2)^*$ with probability p lying to its right.

TABLE F χ^2 distribution critical values

df	Tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

Table entry for p and C is the critical value t^* with probability p lying to its right and probability C lying between $-t^*$ and t^* .

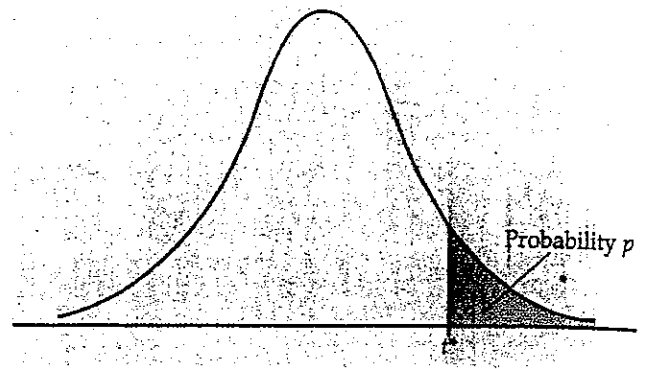


TABLE B t distribution critical values

df	Upper tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.888	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.882	1.100	1.384	1.833	2.262	2.408	2.821	3.250	3.690	4.297	4.781
10	0.700	0.877	1.093	1.374	1.812	2.226	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.684	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.684	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

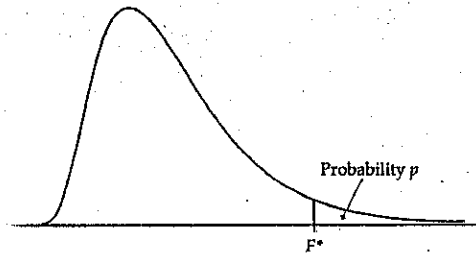


Table entry for p is the critical value F^* with probability p lying to its right.

TABLE E F critical values

Degrees of freedom in the denominator	p	Degrees of freedom in the numerator								
		1	2	3	4	5	6	7	8	9
1	.100	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86
	.050	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
	.025	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28
	.010	4052.2	4999.5	5403.4	5624.6	5763.6	5859.7	5928.4	5981.1	6022.5
	.001	405284	500000	540379	562500	576405	585937	592873	598144	602284
2	.100	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
	.050	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
	.025	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39
	.010	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
	.001	998.50	999.00	999.17	999.25	999.30	999.33	999.36	999.37	999.39
3	.100	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
	.050	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
	.025	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47
	.010	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35
	.001	167.03	148.50	141.11	137.10	134.58	132.85	131.58	130.62	129.86
4	.100	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
	.050	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
	.025	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90
	.010	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
	.001	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.47
5	.100	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
	.050	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
	.025	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68
	.010	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
	.001	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65	27.24
6	.100	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
	.050	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
	.025	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52
	.010	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98
	.001	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.69
7	.100	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
	.050	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
	.025	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82
	.010	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
	.001	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.33

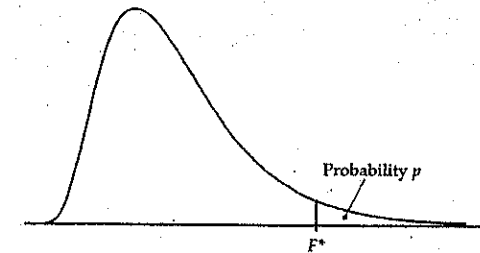


Table entry for p is the critical value F^* with probability p lying to its right.

TABLE E F critical values (continued)

Degrees of freedom in the denominator	p	Degrees of freedom in the numerator										
		10	12	15	20	25	30	40	50	60	120	1000
1	.100	60.19	60.71	61.22	61.74	62.05	62.26	62.53	62.69	62.79	63.06	63.30
	.050	241.88	243.91	245.95	248.01	249.26	250.10	251.14	251.77	252.20	253.25	254.19
	.025	968.63	976.71	984.87	993.10	998.08	1001.4	1005.6	1008.1	1009.8	1014.0	1017.7
	.010	6055.8	6106.3	6157.3	6208.7	6239.8	6260.6	6286.8	6302.5	6313.0	6339.4	6362.7
	.001	605621	610668	615764	620908	624017	626099	628712	630285	631337	633972	636301
2	.100	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.47	9.48	9.49
	.050	19.40	19.41	19.43	19.45	19.46	19.46	19.47	19.48	19.48	19.49	19.49
	.025	39.40	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.48	39.49	39.50
	.010	99.40	99.42	99.43	99.45	99.46	99.46	99.47	99.47	99.48	99.49	99.50
	.001	999.40	999.42	999.43	999.45	999.46	999.47	999.47	999.48	999.48	999.49	999.50
3	.100	5.23	5.22	5.20	5.18	5.17	5.17	5.16	5.15	5.15	5.14	5.13
	.050	8.79	8.74	8.70	8.66	8.63	8.62	8.59	8.58	8.57	8.55	8.53
	.025	14.42	14.34	14.25	14.17	14.12	14.08	14.04	14.01	13.99	13.95	13.91
	.010	27.23	27.05	26.87	26.69	26.58	26.50	26.41	26.35	26.32	26.22	26.14
	.001	129.25	128.32	127.37	126.42	125.84	125.45	124.96	124.66	124.47	123.97	123.53
4	.100	3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.79	3.78	3.76	3.76
	.050	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.70	5.69	5.66	5.63
	.025	8.84	8.75	8.66	8.56	8.50	8.46	8.41	8.38	8.36	8.31	8.26
	.010	14.55	14.37	14.20	14.02	13.91	13.84	13.75	13.69	13.65	13.56	13.47
	.001	48.05	47.41	46.76	46.10	45.70	45.43	45.09	44.88	44.75	44.40	44.09
5	.100	3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.15	3.14	3.12	3.11
	.050	4.74	4.68	4.62	4.56	4.52	4.50	4.46	4.44	4.43	4.40	4.37
	.025	6.62	6.52	6.43	6.33	6.27	6.23	6.18	6.14	6.12	6.07	6.02
	.010	10.05	9.89	9.72	9.55	9.45	9.38	9.29	9.24	9.20	9.11	9.03
	.001	26.92	26.42	25.91	25.39	25.08	24.87	24.60	24.44	24.33	24.06	23.82
6	.100	2.94	2.90	2.87	2.84	2.81	2.80	2.78	2.77	2.76	2.74	2.72
	.050	4.06	4.00	3.94	3.87	3.83	3.81	3.77	3.75	3.74	3.70	3.67
	.025	5.46	5.37	5.27	5.17	5.11	5.07	5.01	4.98	4.96	4.90	4.86
	.010	7.87	7.72	7.56	7.40	7.30	7.23	7.14	7.09	7.06	6.97	6.89
	.001	18.41	17.99	17.56	17.12	16.85	16.67	16.44	16.31	16.21	15.98	15.77
7	.100	2.70	2.67	2.63	2.59	2.57	2.56	2.54	2.52	2.51	2.49	2.47
	.050	3.64	3.57	3.51	3.44	3.40	3.38	3.34	3.32	3.30	3.27	3.23
	.025	4.76	4.67	4.57	4.47	4.40	4.36	4.31	4.28	4.25	4.20	4.15
	.010	6.62	6.47	6.31	6.16	6.06	5.99	5.91	5.86	5.82	5.74	5.66
	.001	14.08	13.71	13.32	12.93	12.69	12.53	12.33	12.20	12.12	11.91	11.72

TABLE E F critical values (continued)

		Degrees of freedom in the numerator									
		1	2	3	4	5	6	7	8	9	
Degrees of freedom in the denominator	8	.100	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
		.050	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
		.025	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36
		.010	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
		.001	25.41	18.49	15.83	14.39	13.48	12.86	12.40	12.05	11.77
		.100	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
		.050	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
		.025	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03
		.010	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
		.001	22.86	16.39	13.90	12.56	11.71	11.13	10.70	10.37	10.11
		.100	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
		.050	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
		.025	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78
		.010	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94
		.001	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.96
		.100	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27
		.050	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
		.025	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59
		.010	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63
		.001	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.35	8.12
	.100	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	
	.050	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	
	.025	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	
	.010	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	
	.001	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.48	
	.100	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	
	.050	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	
	.025	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	
	.010	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	
	.001	17.82	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.98	
	.100	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	
	.050	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	
	.025	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	
	.010	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	
	.001	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.58	
	.100	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	
	.050	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	
	.025	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	
	.010	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	
	.001	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.26	
	.100	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	
	.050	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	
	.025	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	
	.010	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	
	.001	16.12	10.97	9.01	7.94	7.27	6.80	6.46	6.19	5.98	
	.100	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	
	.050	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	
	.025	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	
	.010	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	
	.001	15.72	10.66	8.73	7.68	7.02	6.56	6.22	5.96	5.75	

TABLE E F critical values (continued)

		Degrees of freedom in the numerator										
		10	12	15	20	25	30	40	50	60	120	1000
Degrees of freedom in the denominator	8	2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.35	2.34	2.32	2.30
		3.35	3.28	3.22	3.15	3.11	3.08	3.04	3.02	3.01	2.97	2.93
		4.30	4.20	4.10	4.00	3.94	3.89	3.84	3.81	3.78	3.73	3.68
		5.81	5.67	5.52	5.36	5.26	5.20	5.12	5.07	5.03	4.95	4.87
		11.54	11.19	10.84	10.48	10.26	10.11	9.92	9.80	9.73	9.53	9.36
		2.42	2.38	2.34	2.30	2.27	2.25	2.23	2.22	2.21	2.18	2.16
		3.14	3.07	3.01	2.94	2.89	2.86	2.83	2.80	2.79	2.75	2.71
		3.96	3.87	3.77	3.67	3.60	3.56	3.51	3.47	3.45	3.39	3.34
		5.26	5.11	4.96	4.81	4.71	4.65	4.57	4.52	4.48	4.40	4.32
		9.89	9.57	9.24	8.90	8.69	8.55	8.37	8.26	8.19	8.00	7.84
		2.32	2.28	2.24	2.20	2.17	2.16	2.13	2.12	2.11	2.08	2.06
		2.98	2.91	2.85	2.77	2.73	2.70	2.66	2.64	2.62	2.58	2.54
		3.72	3.62	3.52	3.42	3.35	3.31	3.26	3.22	3.20	3.14	3.09
		4.85	4.71	4.56	4.41	4.31	4.25	4.17	4.12	4.08	4.00	3.92
		8.75	8.45	8.13	7.80	7.60	7.47	7.30	7.19	7.12	6.94	6.78
		2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.04	2.03	2.00	1.98
		2.85	2.79	2.72	2.65	2.60	2.57	2.53	2.51	2.49	2.45	2.41
		3.53	3.43	3.33	3.23	3.16	3.12	3.06	3.03	3.00	2.94	2.89
		4.54	4.40	4.25	4.10	4.01	3.94	3.86	3.81	3.78	3.69	3.61
		7.92	7.63	7.32	7.01	6.81	6.68	6.52	6.42	6.35	6.18	6.02
	2.19	2.15	2.10	2.06	2.03	2.01	1.99	1.97	1.96	1.93	1.91	
	2.75	2.69	2.62	2.54	2.50	2.47	2.43	2.40	2.38	2.34	2.30	
	3.37	3.28	3.18	3.07	3.01	2.96	2.91	2.87	2.85	2.79	2.73	
	4.30	4.16	4.01	3.86	3.76	3.70	3.62	3.57	3.54	3.45	3.37	
	7.29	7.00	6.71	6.40	6.22	6.09	5.93	5.83	5.76	5.59	5.44	
	2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.92	1.90	1.88	1.85	
	2.87	2.80	2.73	2.66	2.61	2.58	2.54	2.51	2.49	2.45	2.41	
	3.25	3.15	3.05	2.95	2.88	2.84	2.78	2.74	2.72	2.66	2.60	
	4.10	3.96	3.82	3.66	3.57	3.51	3.43	3.38	3.34	3.25	3.18	
	6.80	6.52	6.23	5.93	5.75	5.63	5.47	5.37	5.30	5.14	4.99	
	2.10	2.05	2.01	1.96	1.93	1.91	1.89	1.87	1.86	1.83	1.80	
	2.60	2.53	2.46	2.39	2.34	2.31	2.27	2.24	2.22	2.18	2.14	
	3.15	3.05	2.95	2.84	2.78	2.73	2.67	2.64	2.61	2.55	2.50	
	3.94	3.80	3.66	3.51	3.41	3.35	3.27	3.22	3.18	3.09	3.02	
	6.40	6.13	5.85	5.56	5.38	5.25	5.10	5.00	4.94	4.77	4.62	
	2.06	2.02	1.97	1.92	1.89	1.87	1.85	1.83	1.82	1.79	1.76	
	2.54	2.48	2.40	2.33	2.28	2.25	2.20	2.18	2.16	2.11	2.07	
	3.06	2.96	2.86	2.76	2.69	2.64	2.59	2.55	2.52	2.46	2.40	
	3.80	3.67	3.52	3.37	3.28	3.21	3.13	3.08	3.05	2.96	2.88	
	6.08	5.81	5.54	5.25	5.07	4.95	4.80	4.70	4.64	4.47	4.33	
	2.03	1.99	1.94	1.89	1.86	1.84	1.81	1.79	1.78	1.75	1.72	
	2.49	2.42	2.35	2.28	2.23	2.19	2.15	2.12	2.11	2.06	2.02	
	2.99	2.89	2.79	2.68	2.61	2.57	2.51	2.47	2.45	2.38	2.32	
	3.69	3.55	3.41	3.26	3.16	3.10	3.02	2.97	2.93	2.84	2.76	
	5.81	5.55	5.27	4.99	4.82	4.70	4.54	4.45	4.39	4.23	4.08	
	2.00	1.96	1.91	1.86	1.83	1.81	1.78	1.76	1.75	1.72	1.69	
	2.45	2.38	2.31	2.23	2.18	2.15	2.10	2.08	2.06	2.01	1.97	
	2.92	2.82	2.72	2.62	2.55	2.50	2.44	2.41	2.38	2.32	2.26	
	3.59	3.46	3.31	3.16	3.07	3.00	2.92	2.87	2.83	2.75	2.66	
	5.58	5.32	5.05	4.78	4.60	4.48	4.33	4.24	4.18	4.02	3.87	



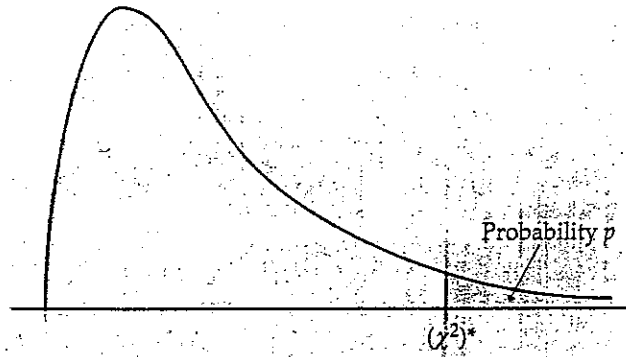


Table entry for p is the critical value $(\chi^2)^*$ with probability p lying to its right.

TABLE F χ^2 distribution critical values

df	Tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2