1. A college professor teaches Stat100 each fall to a large class of first-year students. For tests, she uses standardized exams that she knows from past experience produce bell-shaped grade distributions with a mean (μ) of 70 and a standard deviation (σ) of 12. Her philosophy of grading is to impose grading standards that will yield, in the long run, 14% A's, 20% B's, 32% C's, 20% D's, and 14% F's.
   a) Where should the cutoff be between the A's and the B's?
   b) Where should the cutoff be between the B's and the C's?

2. Suppose that plants of a particular species are randomly dispersed over an area, so that the number of plants in a given area follows a Poisson distribution with a mean of λ plants per unit area. In particular, this means that the number of plants in a given circle of radius *r* is Poisson with parameter $\lambda\pi r^2$. If a plant is randomly selected in this area, find the probability density function of the distance to the nearest neighboring plant.

   Hint: Let *R* denote the distance to the nearest neighboring plant. Calculate *P(R>r).*

3. Let $X_1, X_2, ...X_n$ be a random sample from a uniform pdf over the interval (0, θ) ($f(x) = \frac{1}{\theta}, \quad 0 < x < \theta$). Find the method of moments estimator for θ.

4. Suppose that $X_1, X_2, ...X_m$ is a random sample measuring yields per acres for corn variety A. Also, $Y_1, Y_2, ...Y_n$ is a separate random sample measuring yields per acre for corn variety B. Suppose that the X's are drawn from a population that has a normal distribution with mean μ_X and variance σ^2. Suppose also that the Y's are drawn from a population that has a normal distribution with mean μ_Y and variance σ^2 (notice that the two populations have the same variance). Assume that the X's and Y's are all independent. Also assume that μ_X and μ_Y are known. Find the maximum likelihood estimator for σ^2.

   Hint: You may find it easier to write σ^2 as θ (and σ as $\sqrt{\theta}$) and then find the m.l.e. for θ.

5. Suppose we observe n data points (x_i, y_i), $i = 1,...,n$. Assume that the data follow the statistical model:

$$y_i = \beta x_i + \varepsilon_i$$

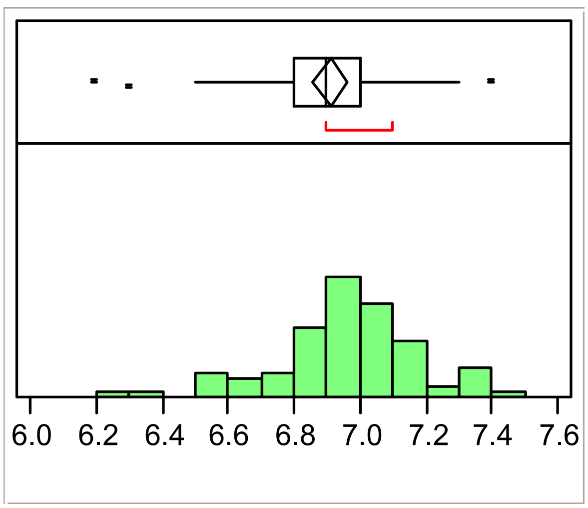
where $\varepsilon_i \sim N(0,\sigma^2)$ are independent and identically distributed random errors and the x_i are assumed to be fixed constants.

a) Find the least squares estimate ($\hat{\beta}$) of β.
b) Show that $\hat{\beta}$ is unbiased.
c) Compute the variance of $\hat{\beta}$.
d) What is the distribution of $\hat{\beta}$?
e) Suppose σ^2 is known. Derive a 100(1-α)% confidence interval for β. (You will need to use the distribution of $\hat{\beta}$. If you are unsure of this distribution, assume some distribution and go from there.) If necessary, make this problem easier by specifically calculating a 95% confidence interval (rather than a generic 100(1-α)% interval).

6. The diet pill combination "fen-phen" has been discovered to have potentially life-threatening side effects. However, a study published by the Mayo clinic (Dec 1, 1999) suggests that the heart damage associated with fen-phen goes away after use of the drug is discontinued. In their small study, they looked at 19 people who took fen-phen. Of these 19 people, 5 developed heart-valve abnormalities. Six months after the patients went off the drugs, 3 of these 5 people no longer appeared to have the drug-related valve problems.

a) This study suggests that 60% of all fen-phen heart-valve abnormality sufferers can expect their abnormalities to go away after discontinuation of the use of the drug. Obviously however, this is a very small study. Consider this a pilot study. Based on this information, how many patients with heart abnormalities caused by fen-phen would you need to study to obtain a 90% confidence interval for the proportion of fen-phen heart-valve abnormality sufferers that can expect their abnormalities to go away after discontinuation of the use of fen-phen? Assume that you want this confidence interval to have a margin of error of 5 percentage points. In other words, the confidence interval should have a width of 10 percentage points. [The "margin of error" is half the width of the confidence interval.]
b) A complication with this kind of study is that not all fen-phen users develop the heart-valve abnormality. Based on the information in this study, estimate the number of fen-phen users (past or present users) that you would need to find to get the required number of people with fen-phen heart abnormalities for your confidence interval in (a).

7. The routes of postal deliverers are carefully planned so that each deliverer works an average of 7 hours per shift. The planned routes assume an average walking speed of 2 miles per hour and no shortcuts across lawns. In an experiment to examine the amount of time deliverers actually spend completing their shifts, a random sample of 75 postal deliverers was secretly timed. The data are summarized below.



The sample mean is 6.909 hours
and the sample standard deviation is 0.226 hours.
Number of observations=75.

a) Is there enough evidence, at the 5% level of significance, to conclude that postal workers are spending less than 7 hours per day doing their jobs on average? (State your hypotheses, test statistic, relevant distribution, conclusion, interpretation, etc.)
b) From the above histogram, does the data look normal? Justify your answer. Is it important to have normally distributed data to do the test in (a)? Why or why not.
c) Construct a 95% confidence interval for the average shift time for postal deliverers.
d) Combining your answers from (a) and (c), write a one sentence summary about the results of your analysis. (Pretend you are summarizing for your boss who has some, but not much, statistical knowledge and who wants you to get to the point).