Derivatives

Derivative Rules

Assume *c* and *n* are constant, real numbers.

1. Power Rule

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$$

2. Derivative of a Constant Multiple

$$\frac{d}{dx}\left[cf(x)\right]=cf'(x)$$

3. Derivative of a Constant

$$\frac{d}{dx}(c)=0$$

4. Addition/Subtraction of Derivatives

$$\frac{d}{dx}\left(f(x)+g(x)\right)=f'(x)+g'(x)$$

5. Derivative of exponential functions

$$\frac{d}{dx}\left(a^x\right)=\left(\ln a\right)a^x$$

6. Product Rule

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right)=f'(x)g(x)+f(x)g'(x)$$

7. Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)=\frac{f'(x)g(x)-f(x)g'(x)}{\left[g(x)\right]^2}$$

8. Chain Rule

$$\frac{d}{dx}\left(f\left(g(x)\right)\right)=f'\left(g(x)\right)\cdot g'(x)$$

Examples using the derivative rules

1. $$\frac{d}{dx}\left(x^3\right)=3x^2$$

2. $$\frac{d}{dx}\left(5x^2\right)=5\frac{d}{dx}\left(x^2\right)=5\cdot 2x=10x$$

3. $$\frac{d}{dx}(5)=0$$

4. $$\frac{d}{dx}\left(2x+4x^2\right)=\frac{d}{dx}(2x)+\frac{d}{dx}\left(4x^2\right)=2+8x$$

5. $$\frac{d}{dx}\left(4^x\right)=\left(\ln 4\right)4^x$$

6. $$\frac{d}{dx}\left(2e^x\left(\sqrt{x}+5x\right)\right)=2e^x\left(\sqrt{x}+5x\right)+2e^x\left(\tfrac{1}{2}x^{-1/2}+5\right)$$

7. $$\frac{d}{dx}\left(\frac{5x}{2\cdot 3^x}\right)=\frac{5\left(2\cdot 3^x\right)-5x\left(2\left(\ln 3\right)3^x\right)}{\left(2\cdot 3^x\right)^2}$$

8. $$\frac{d}{dx}\left(x^2+3x\right)^7=7\left(x^2+3x\right)^6\cdot\left(2x+3\right)$$

Derivatives Continued

Derivatives Rules for Special Functions

Assume *k* is a constant.

1. $\frac{d}{dx}(\sin x) = \cos x$

2. $\frac{d}{dx}(\cos x) = -\sin x$

3. $\frac{d}{dx}(\tan x) = \sec^2 x$

4. $\frac{d}{dx}(\sec x) = \sec x \tan x$

5. $\frac{d}{dx}(\cot x) = -\csc^2 x$

6. $\frac{d}{dx}(\csc x) = -\csc x \cot x$

7. $\frac{d}{dx}(e^{kx}) = ke^{kx}$

8. $\frac{d}{dx}(\ln x) = \frac{1}{x}$

9. $\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$

10. $\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$

Examples using the derivative rules.

1. $\frac{d}{dx}(\sin(2x)) = 2\cos(2x)$

2. $\frac{d}{dx}(\cos(3x)) = -3\sin(3x)$

3. $\frac{d}{dx}\left(2\tan(3x)\right) = 6\sec^2(3x)$

4. $\frac{d}{dx}(\sec(2x)) = 2\sec(2x)\tan(2x)$

5. $\frac{d}{dx}(\cot(4x)) = -4\csc^2(4x)$

6. $\frac{d}{dx}(\csc(3x)) = -3\csc(3x)\cot(3x)$

7. $\frac{d}{dx}(e^{-3x}) = -3e^{-3x}$

8. $\frac{d}{dx}(2\ln x) = \frac{2}{x}$

9. $\frac{d}{dx}(4\arctan x) = \frac{4}{1+x^2}$

10. $\frac{d}{dx}(2\arcsin x) = \frac{2}{\sqrt{1-x^2}}$

Practice Problems

Differentiate.

1. $g(x) = x^2 + \frac{1}{x^2}$

2. $f(x) = (16x)^3$

3. $f(t) = \sqrt{t} - \frac{1}{\sqrt{t}}$

4. $f(t) = 6t^{-9}$

5. $f(x) = \left(x^2 + x + 1\right)\left(x^2 + 2\right)$

6. $y = 2e^{-x}$

7. $y = \frac{4t + 5}{2 - 3t}$

8. $y = \sin^2 x$

9. $y = \cos\left(3x^2 + 5\right)$

10. $f(x) = \left(1 - x^{-1}\right)^{-1}$

11. $y = x\sqrt{x} + \frac{1}{x^2\sqrt{x}}$

12. $y = 3e^x\left(x^3 + 2x\right)$

13. $y = \tan\left(\sqrt{1 - x}\right)$

14. $y = \arctan(2x)$

15. $y = \ln\left(x^2 + 2\right)$

16. $f(x) = 3^x$

17. $y = \sin(\cos x)$

18. $y = \sec\left(x^2\right)$

19. $f(x) = \frac{5e^{2x}}{3x^4 + 7}$

20. $y = (\ln x)^{-2}$

21. $y = \arcsin(4x)$

Antiderivatives

Antiderivative Rules

Assume that *k* is a constant.

1. $\int k \; dx = kx + C$

2. $\int x^n dx = \frac{x^{n+1}}{n+1} + C,\; n \neq -1$

3. $\int \frac{1}{x} \; dx = \ln|x| + C$

4. $\int e^x dx = e^x + C$

5. $\int \cos x \; dx = \sin x + C$

6. $\int \sin x \; dx = -\cos x + C$

7. $\int \frac{1}{1+x^2} dx = \arctan x + C$

8. $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$

Examples using the antiderivative rules.

1. $\int \pi^2 dx = \pi^2 x + C$

2. $\int x^5 dx = \frac{x^6}{6} + C$

3. $\int \frac{2}{x} \; dx = 2\ln|x| + C$

4. $\int 7e^x dx = 7e^x + C$

5. $\int 3\cos x \; dx = 3\sin x + C$

6. $\int 4\sin x \; dx = -4\cos x + C$

7. $\int \frac{2}{1+x^2} dx = 2\arctan x + C$

8. $\int \frac{4}{\sqrt{1-x^2}} dx = \arcsin x + C$

Properties of Antiderivatives: Sums and Constant Multiples

In indefinite integral notation,

1. $\int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx$

An antiderivative of the sum (or difference) of two functions is the sum (or difference) of their antiderivatives.

2. $\int cf(x) dx = c\int f(x) dx$

An antiderivative of a constant times a function is the constant time an antiderivative of the function.

Practice Problems

Find an antiderivative

1. $f(x) = 7$
2. $f(t) = 7t$
3. $f(x) = x^4$
4. $g(x) = x^3 + x$
5. $h(t) = \cos t$
6. $f(x) = \sqrt{x}$
7. $h(t) = \dfrac{1}{t}$
8. $g(x) = \dfrac{1}{x^2}$
9. $f(t) = \dfrac{t^2 + 1}{t}$

Find the indefinite integrals.

10. $\int \left(3e^x + 2\sin x\right) dx$
11. $\int 7e^x dx$
12. $\int \left(2 + \cos t\right) dt$
13. $\int \left(4t + \dfrac{4}{t}\right) dt$
14. $\int \dfrac{x+1}{x} dx$
15. $\int \left(\sqrt{x^3} - \dfrac{2}{x}\right) dx$
16. $\int \left(3\cos x - 7\sin x\right) dx$
17. $\int \left(\dfrac{2}{x} + \pi \sin x\right) dx$
18. $\int \left(\dfrac{x^2 + x + 1}{x}\right) dx$

Evaluate the definite integrals.

19. $\int_0^3 \left(x^2 + 4x + 3\right) dx$
20. $\int_0^{\pi/4} \cos x \, dx$
21. $\int_0^2 4e^x dx$
22. $\int_0^1 \sin x \, dx$
23. $\int_1^3 \dfrac{1}{t} dt$
24. $\int_0^2 \left(\dfrac{x^3}{3} + 2x\right) dx$
25. $\int_{-3}^{-1} \dfrac{2}{t^3} \, dt$
26. $\int_0^3 x^2 dx$
27. $\int_0^1 \left(x - x^3\right) dx$