In group theory, it's possible to express a group in a number of very different looking ways. So, especially in the event that a group is infinite, it can be extremely difficult to tell whether or not a given pair of subgroups are equal to each other, or more generally, when they're "equal" with respect to some natural equivalence relation, such as conjugacy (two subgroups H, K of G are conjugate when there exists a group element g so that \( gHg^{-1} = K \)).

We'll study this problem in the context of surface groups, which are groups that encode different types of loops that can be drawn on an orientable surface of a given genus. One can translate the algebraic problem of determining when a given pair of subgroups are conjugate into the topological one of determining when a given pair of covering spaces of the surface are equivalent. The main theorem is that given your favorite pair of covering spaces, one can find a finite collection of loops on the surface such that if one understands how those loops behave with respect to each of the covering spaces, one can tell the spaces apart (or, decide if they're the same). This yields a new algorithm for solving the subgroup conjugacy problem in this setting.

I'll go slowly and try to define all of the above words (or, at least give a feel for them) and also try to convey some of the ideas behind the proofs. This represents joint work with Marissa Loving (Georgia Tech), Maxie Lahn (Michigan), and Nick Miller (Berkeley).