

Derivatives

Derivative Rules

Assume c and n are constant, real numbers.

1. Power Rule

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

2. Derivative of a Constant Multiple

$$\frac{d}{dx}[cf(x)] = cf'(x)$$

3. Derivative of a Constant

$$\frac{d}{dx}(c) = 0$$

4. Addition/Subtraction of Derivatives

$$\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$$

5. Derivative of exponential functions

$$\frac{d}{dx}(a^x) = (\ln a)a^x$$

6. Product Rule

$$\frac{d}{dx}(f(x) \cdot g(x)) = f'(x)g(x) + f(x)g'(x)$$

7. Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

8. Chain Rule

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$$

Examples using the derivative rules

$$1. \frac{d}{dx}(x^3) = 3x^2$$

$$2. \frac{d}{dx}(5x^2) = 5 \frac{d}{dx}(x^2) = 5 \cdot 2x = 10x$$

$$3. \frac{d}{dx}(5) = 0$$

$$4. \frac{d}{dx}(2x + 4x^2) = \frac{d}{dx}(2x) + \frac{d}{dx}(4x^2) = 2 + 8x$$

$$5. \frac{d}{dx}(4^x) = (\ln 4)4^x$$

$$6. \frac{d}{dx}(2e^x(\sqrt{x} + 5x)) = 2e^x(\sqrt{x} + 5x) + 2e^x\left(\frac{1}{2}x^{-1/2} + 5\right)$$

$$7. \frac{d}{dx}\left(\frac{5x}{2 \cdot 3^x}\right) = \frac{5(2 \cdot 3^x) - 5x(2(\ln 3)3^x)}{(2 \cdot 3^x)^2}$$

$$8. \frac{d}{dx}(x^2 + 3x)^7 = 7(x^2 + 3x)^6 \cdot (2x + 3)$$

Derivatives Continued

Derivatives Rules for Special Functions

Assume k is a constant.

1. $\frac{d}{dx}(\sin x) = \cos x$
2. $\frac{d}{dx}(\cos x) = -\sin x$
3. $\frac{d}{dx}(\tan x) = \sec^2 x$
4. $\frac{d}{dx}(\sec x) = \sec x \tan x$
5. $\frac{d}{dx}(\cot x) = -\csc^2 x$
6. $\frac{d}{dx}(\csc x) = -\csc x \cot x$
7. $\frac{d}{dx}(e^{kx}) = ke^{kx}$
8. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
9. $\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$
10. $\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$

Examples using the derivative rules.

1. $\frac{d}{dx}(\sin(2x)) = 2\cos(2x)$
2. $\frac{d}{dx}(\cos(3x)) = -3\sin(3x)$
3. $\frac{d}{dx}(2\tan(3x)) = 6\sec^2(3x)$
4. $\frac{d}{dx}(\sec(2x)) = 2\sec(2x)\tan(2x)$
5. $\frac{d}{dx}(\cot(4x)) = -4\csc^2(4x)$
6. $\frac{d}{dx}(\csc(3x)) = -3\csc(3x)\cot(3x)$
7. $\frac{d}{dx}(e^{-3x}) = -3e^{-3x}$
8. $\frac{d}{dx}(2\ln x) = \frac{2}{x}$
9. $\frac{d}{dx}(4\arctan x) = \frac{4}{1+x^2}$
10. $\frac{d}{dx}(2\arcsin x) = \frac{2}{\sqrt{1-x^2}}$

Practice Problems

Differentiate.

1. $g(x) = x^2 + \frac{1}{x^2}$

2. $f(x) = (16x)^3$

3. $f(t) = \sqrt{t} - \frac{1}{\sqrt{t}}$

4. $f(t) = 6t^{-9}$

5. $f(x) = (x^2 + x + 1)(x^2 + 2)$

6. $y = 2e^{-x}$

7. $y = \frac{4t + 5}{2 - 3t}$

8. $y = \sin^2 x$

9. $y = \cos(3x^2 + 5)$

10. $f(x) = (1 - x^{-1})^{-1}$

11. $y = x\sqrt{x} + \frac{1}{x^2\sqrt{x}}$

12. $y = 3e^x(x^3 + 2x)$

13. $y = \tan(\sqrt{1-x})$

14. $y = \arctan(2x)$

15. $y = \ln(x^2 + 2)$

16. $f(x) = 3^x$

17. $y = \sin(\cos x)$

18. $y = \sec(x^2)$

19. $f(x) = \frac{5e^{2x}}{3x^4 + 7}$

20. $y = (\ln x)^{-2}$

21. $y = \arcsin(4x)$

Antiderivatives

Antiderivative Rules

Assume that k is a constant.

$$1. \int k \, dx = kx + C$$

$$2. \int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$3. \int \frac{1}{x} \, dx = \ln |x| + C$$

$$4. \int e^x \, dx = e^x + C$$

$$5. \int \cos x \, dx = \sin x + C$$

$$6. \int \sin x \, dx = -\cos x + C$$

$$7. \int \frac{1}{1+x^2} \, dx = \arctan x + C$$

$$8. \int \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin x + C$$

Examples using the antiderivative rules.

$$1. \int \pi^2 \, dx = \pi^2 x + C$$

$$2. \int x^5 \, dx = \frac{x^6}{6} + C$$

$$3. \int \frac{2}{x} \, dx = 2 \ln |x| + C$$

$$4. \int 7e^x \, dx = 7e^x + C$$

$$5. \int 3 \cos x \, dx = 3 \sin x + C$$

$$6. \int 4 \sin x \, dx = -4 \cos x + C$$

$$7. \int \frac{2}{1+x^2} \, dx = 2 \arctan x + C$$

$$8. \int \frac{4}{\sqrt{1-x^2}} \, dx = 4 \arcsin x + C$$

Properties of Antiderivatives: Sums and Constant Multiples

In indefinite integral notation,

$$1. \int [f(x) \pm g(x)] \, dx = \int f(x) \, dx \pm \int g(x) \, dx$$

An antiderivative of the sum (or difference) of two functions is the sum (or difference) of their antiderivatives.

$$2. \int cf(x) \, dx = c \int f(x) \, dx$$

An antiderivative of a constant times a function is the constant times an antiderivative of the function.

Practice Problems

Find an antiderivative

1. $f(x) = 7$

4. $g(x) = x^3 + x$

7. $h(t) = \frac{1}{t}$

2. $f(t) = 7t$

5. $h(t) = \cos t$

8. $g(x) = \frac{1}{x^2}$

3. $f(x) = x^4$

6. $f(x) = \sqrt{x}$

9. $f(t) = \frac{t^2 + 1}{t}$

Find the indefinite integrals.

10. $\int (3e^x + 2 \sin x) dx$

13. $\int \left(4t + \frac{4}{t}\right) dt$

16. $\int (3 \cos x - 7 \sin x) dx$

11. $\int 7e^x dx$

14. $\int \frac{x+1}{x} dx$

17. $\int \left(\frac{2}{x} + \pi \sin x\right) dx$

12. $\int (2 + \cos t) dt$

15. $\int \left(\sqrt{x^3} - \frac{2}{x}\right) dx$

18. $\int \left(\frac{x^2 + x + 1}{x}\right) dx$

Evaluate the definite integrals.

19. $\int_0^3 (x^2 + 4x + 3) dx$

22. $\int_0^1 \sin x dx$

25. $\int_{-3}^{-1} \frac{2}{t^3} dt$

20. $\int_0^{\pi/4} \cos x dx$

23. $\int_1^3 \frac{1}{t} dt$

26. $\int_0^3 x^2 dx$

21. $\int_0^2 4e^x dx$

24. $\int_0^2 \left(\frac{x^3}{3} + 2x\right) dx$

27. $\int_0^1 (x - x^3) dx$