Horsin' Around with Math:
Content-Related Feedback in Math Pedagogy

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Swarthmore College, Spring 2011

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Acknowledgements

Professors David Harrison (Linguistics) and Ann Renninger (Educational Studies) provided advising throughout the writing of this thesis. Research data from the Math Forum was provided by Ann Renninger of the Educational Studies Department.

Many Swarthmore College students contributed by reading draft material and providing helpful suggestions through discussion. I am grateful to Dennis Fan ’11, Haley Most ’13, Brenna DiCola ’11, Julian Thomas ’10, Aaron Stein ’11, Jack Nicoludis ’12, Melissa Emmerson ’11, Will Scheuing ‘12 and Cecilia Jou ’11.

1. Introduction

The Problem

I was interested in how one can give content-related and metacognitive feedback in order to promote learning. More specifically, learning in this context is defined as understanding how to provide effective mentoring that focuses not only on the processes of problem solving but also the content of the problem. Can pre-service teachers (PTs) be encouraged to provide effective feedback? Is it more effective to provide implicit or explicit feedback? What is the role of metaphors in providing feedback? In order to study this problem, I looked at two contexts: linguistics and educational psychology.

2. Review of Literature: Educational Psychology

How does Educational Psychology inform our understanding of content-focused mentoring?

How do the concepts of scaffolding and the zone of proximal development provide a model for understanding the role of mentoring in learning?

2.1. Scaffolding and the Zone of Proximal Development

Wood, Bruner and Ross (1976) describe scaffolding as "a process that enables a child or novice to solve a problem, carry out a task or achieve a goal which would be beyond his unassisted efforts" (Wood, Bruner, and Ross, 90). This scaffolding consists essentially of the adult 'controlling' those elements of the task that are initially beyond the learner's capacity, thus permitting him to concentrate upon and complete only those elements that are within his range of competence. "Well executed scaffolding beings by luring the child into actions that produce recognizable-for-him solutions" (Wood, Bruner, and Ross, 96). Scaffolding instructional interactions are interactive and developmental, and promote learner autonomy through the gradual reduction of expert guidance. Central to the discussions of scaffolding is the reciprocity and evolution of the expert-learner interaction. Furthermore, scaffolding interactions depend upon the learning context and task, as well as upon the learner's interests, strengths, and needs.

The term scaffolding, however, is frequently, and inappropriately, applied to any form of verbal interaction that is undertaken with learning as a goal. Scaffolding should be distinguished from feedback that is a one-time and discrete or a pre-determined script intended to enhance performance, because it constitutes an ongoing process of providing feedback with the goal of supporting learning.

Closely related to the discussion of scaffolding is Vygotsky's Zone of Proximal Development (ZPD). Vygotsky (1978) describes a learner's ZPD as the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers. The ZPD permits us to delineate the child's "immediate future" and his "dynamic developmental state", allowing not only for what has already been achieved developmentally but also for what is in the course of maturing (Vygotsky, 87). The process of providing scaffolding then can be described as the expert's adjustment of his or her interaction within the learner's ZPD so that the learner becomes increasingly able to work independently.

Stone (1998) connects the concepts of scaffolding to the ZPD, identifying four main features of the scaffolding metaphor. The first feature of scaffolding involves the adult recruiting a child's participation in a meaningful and culturally desirable activity beyond the child's current understanding or control. Second, the adult diagnoses the child's understanding and skill level, and carefully calibrates the support provided to help accomplish the goal. The third feature of scaffolded interaction is that the adult provides a range of supports, including nonverbal assistance and extensive dialogue. The fourth and final feature of scaffolding is that the support of the adult is temporary and will be gradually withdrawn in order to foster a transfer of responsibility from the adult to the child. The "most important connection [is] the [explicit] link to Vygotsky's notion of the zone of proximal development, with its clear implication that not only the isolated learning of new concepts and procedures but also genuine conceptual reorganization results from scaffolded interaction" (Stone, 349).

What are the different types of scaffolds that a mentor might use when providing feedback, and how can we categorize these differences?

2.2. Conceptual, Metacognitive, Procedural, and Strategic Scaffolding

Hannafin, Land, and Oliver (1999) primarily categorize feedback into four major categories: conceptual, metacognitive, procedural, and strategic. Scaffolding approaches vary according to the focus of the problem posed and the demands posed in the enabling context.

Conceptual scaffolding is provided "when the problem under study is defined; it guides learners regarding what to consider" (Hannafin, Land, and Oliver, 132). This may be accomplished by identifying key conceptual knowledge related to a problem or creating structures that make conceptual organization readily apparent. These structures can be made available through a variety of mechanisms, ranging from the graphical depiction of relationships among concepts, to outlines featuring relationships, to information and hints provided by experts.

Metacognitive scaffolding provides guidance in how to think during learning, supporting the underlying processes associated with individual learning management (Hannafin, Land, and Oliver, 132-133). This type of scaffolding might also remind learners to reflect on goals or prompt them to relate a given resource or tool manipulation outcome to the problem at hand. In particular, mentors can design feedback to emphasize specific ways to think about the problem under study or they can focus on the processes of creating models, including finding ways to link models with prior knowledge and experience, linking representational models to current understanding, and enabling learners to manipulate ideas through modeling tools.

Procedural scaffolding emphasizes how to utilize available resources and tools (Hannafin, Land, and Oliver, 133). It orients to system features and functions, providing support for any resource that may aid in task completion.

Finally, strategic scaffolding focuses on approaches for identifying and selecting needed information, evaluating available resources, and relating new knowledge to existing knowledge and experience (Hannafin, Land, and Oliver, 133). Mentors may offer a range of alternatives to approach the problem at hand. They may use questions to provide an explicit strategy clue for those needing a place to begin, or to elicit related strategies for those who are in the intermediate stages of a problem. Furthermore, strategic scaffolding may take the form of "response-sensitive guidance" at key decision points; for example, when the mentee has completed the problem, he or she might be encouraged to test their understanding through reflection.

2.3. Implicit and Explicit

Hadwin and Winne (2001) distinguish between implicit and explicit supports to suggest how scaffolding can support the development of self-regulation, the process by which students monitor learning and motivation. Tacit, or implicit, scaffolds "refer to tools that are intended to cue students to attend to aspects of their studying without explicitly directing or instructing those studying activities" (Hadwin and Winne, 322). Tacit cues may include features such as headings in a textbook, bolded terms and underlining, or learning objectives. On the other hand, explicit scaffolds provide direct instruction to students, often including specific tactics and strategies to approach a problem. In particular, a strategy may include support for working with the available tools and is often overtly cued.

What is the role of metacognition in a student's learning and how can understanding this process promote content-focused mentoring?

2.4. Metacognition

Schoenfeld (1987) describes three related but distinct categories of intellectual behavior. The first is your knowledge about your own thought processes. How accurately can you describe your own thinking? The second category deals with control, or self-regulation: how well do you keep track of what you're doing when, for example, you're solving problems, and how, if at all, do you use the input from those observations to guide your problem solving actions? The final category of intellectual behavior is beliefs and intuitions: What ideas about mathematics do you bring to your work, and how does that shape the way that you do mathematics? Work in this area is important because good problem solving calls for using efficiently what you know.

Schoenfeld (1992) found that roughly 60 percent of college and high school students working on unfamiliar problems will "read, make a decision quickly, and pursue that direction come hell or high water." This first, quick, wrong decision, if not reconsidered and reversed, guarantees failure on these highly difficult problems. In contrast, a mathematics faculty member spent more than half of his allotted time trying to make sense of the problem. He spent a significant amount of time analyzing and exploring rather than committing to any one particular direction. By monitoring his solution with care, pursuing interesting leads and abandoning paths that didn't seem to bear fruit, he managed to solve the problem. These skills that characterize an "expert's" approach to an unfamiliar problem can be coached.

The instructor in Schoenfeld's (1992) experiment primarily focuses on three questions: What (exactly) are you doing? (i.e. can you describe it precisely?) Why are you doing it? (How does it fit into the solution?) and How does it help you? (What will you do with the outcome when you obtain it?) By the end of the semester, students were able to give themselves the opportunity to solve the problem through self-regulation. The students' behavior became expert-like; there was an associated increase in problem solving success.

Lester, Garofalo, & Kroll (1989) completed a study in order to foster students' metacognitive development. Ways of achieving this goal were to have the teacher serve as an external monitor during problem solving, encourage discussion of behaviors considered important for the internalization of metacognitive skills, and model good executive behavior. This held particularly for non-routine, "process" problems, for which there is no standard algorithm for solving. Lester, et. al. found that metacognition instruction is most effective when provided in a systematically organized manner under the direction of the teacher.

3. Review of Literature: Linguistics

How does Linguistics inform our understanding of content-focused mentoring?

What is the role of metaphor in everyday speech, and how can metaphors be used in providing feedback?

3.1. Role of Metaphors

According to Lakoff and Johnson (1980), metaphor is pervasive in everyday life, not just in language but in thought and action. Our ordinary conceptual system, in terms of which we both think and act, is fundamentally metaphoric in nature. Furthermore, the concepts that govern our thoughts also govern our everyday functioning, including the most mundane details. Our concepts structure what we perceive, how we get around in the world, and how we relate to other people. Thus, if our conceptual system is largely metaphorical, then the way we think, what we experience, and what we do every day is very much a matter of metaphor.

We often think and act automatically along certain lines as we go about everyday activities. One way to find out what these lines are is by looking at language, because communication is based on the same conceptual system that we use in thinking and acting. Thus, language can give us important insight into what our conceptual system is like. “Primarily on the basis of linguistic evidence, we have found that most of our ordinary conceptual system is metaphorical in nature” (Lakoff and Johnson, 4).

The essence of metaphor is understanding and experiencing one kind of thing in terms of another. The metaphorical concept *argument is war* structures what we do and how we understand what we are doing when we argue. Arguments can be won or lost, and we view our arguing counterpart as an opponent. We defend our position and attack his or hers, and we can

gain or lose ground. Though there is no physical battle, there is a verbal battle, and the structure of an argument reflects this. This metaphor structures the actions we perform in arguing. Most importantly, this metaphor, which we are hardly ever conscious of, is not merely in the words we use – it is in our very concept of an argument. We talk about arguments in this way because this is how we conceive of them, and we act in accordance with this. Thus, metaphor is not just a matter of language, or mere words. Human thought processes are largely metaphorical. Metaphors as linguistic expressions are possible only because there are metaphors in a person's conceptual system.

Furthermore, the very systematicity that allows us to comprehend one aspect of a concept in terms of another will necessarily hide other aspects of that concept. In allowing us to focus on one aspect of a concept, a metaphorical concept can keep us from focusing on other aspects of the concept that are inconsistent with that metaphor (Lakoff and Johnson, 10). For example, when we are attacking an opponent's position in the midst of an argument, we often lose sight of the cooperative aspects of arguments.

Metaphorical structuring is partial, not total; if it were total, one concept would actually be the other, not merely be understood in terms of it. For example, time isn't really money: if you *spend your time* trying to do something and it doesn't work, you can't get your time back. Thus, a concept is only partially structured by a metaphor and it can be extended in some ways but not others.

All of the following examples of metaphors are provided by Lakoff and Johnson.

What are the metaphors that might be used in everyday life as well as in mentoring?

3.2. Metaphors in Everyday Life

Michael Reddy observes that our language about language is structured by the "conduit metaphor": *Ideas (or meanings) are objects; linguistic expressions are containers; communication is sending*. The speaker will put ideas (objects) into words (containers) and sends them (along a conduit) to a hearer who takes the idea/objects out of the word/containers. Here are some examples of the conduit metaphor: "It's hard to *get* that idea *across to* him"; "It's difficult to *put* my ideas *into* words"; "When you *have* a good idea, try to *capture* it immediately *in* words"; "Try to *pack* more thought *into* fewer words"; "His words *carry* little meaning"; "The idea is *buried in* terribly dense paragraphs" (Lakoff and Johnson, 11). *Linguistic expressions are containers for meanings* entails that words and sentences have meanings in themselves, independent of any context or speaker. The *conduit* metaphor does not fit cases where context is required to determine whether the sentence has any meaning at all and what meaning it has.

In structural metaphors, one concept is metaphorically structured in terms of another. For example, in contemporary English, a common metaphorical concept is time as money. One can "waste," "save," "invest," or "borrow" time, and time can "cost" an amount or be "spent."

3.3. Orientational Metaphor

Another kind of metaphorical concept is called an orientational metaphor, which organizes a whole system of concepts with respect to one another. These typically deal with spatial orientation, such as *up-down*, *in-out*, *front-back*, *on-off*, *deep-shallow*, and *central-peripheral*. Orientational metaphors give a concept a spatial orientation, such as *happy is up*. The fact that the concept *happy* is oriented as *up* gives rise to expressions such as "You're in *high* spirits"; "That *boosted* my spirits"; "I *fell* into a depression" or "I'm feeling *down."* These metaphorical orientations are not arbitrary; they have a basis in our physical and cultural experience. In particular, drooping posture is typically related to sadness and depression, whereas erect posture relates to a positive emotional state.

The fact that humans and most other mammals sleep lying down and stand up when they awaken gives rise to the metaphor *conscious is up; unconscious is down*: "Wake *up*. He *rises* early in the morning. He *fell* asleep. He *sank* into a coma. He's *under* hypnosis." Similarly, the physical basis for the metaphor *health and life are up*; *sickness and death are down* is that serious illness forces us to lie down physically and when one dies, he or she is physically down. This gives rise to "He's at the *peak* of health. He came *down* with the flu. His health is *declining*. He *dropped* dead." *Having control or force is up; being subject to control or force is down* occurs because physical size typically correlates with physical strength, and the victor in a fight is typically on top: "I have control *over* her. I am *on top* of the situation. He's in a *superior* position. He's at the *height* of his power. He's in the *upper* echelon. She's in the *high* command. He is *under* my control. He *fell* from power."

As you add more of a substance or of physical objects to a container or pile, the level goes up; thus, the metaphor *more is up; less is down*. "Her income *rose* last year. The number of errors he made is incredibly *low*. He is *under*age." *Foreseeable future events are up (and ahead)* because normally our eyes look in the direction in which we typically move, which is ahead or forward. "All *up*coming events are listed in the paper. I'm afraid of what's *ahead* of us. What's *up*?" Because status is correlated with social and physical power, *high status is up; low status is down*: "He has a *lofty* position. She'll *rise* to the *top*. He's at the *peak* up his career. He has little *upward* mobility. She *fell* in status."

Any physical basis for personal well-being, such as happiness, health, life, and control (i.e. the things that principally characterize what is good for a person) is up: *Good is up; bad is down*. "Things are looking *up*. We hit a *peak* last year, but it's been *downhill* ever since. Things are at an all-time *low*. He does *high*-quality work. Similarly, we have the metaphor *virtue is up; depravity is down* as a combination of two metaphors: *good is up* for a person (physical basis) and *society is a person*. To be virtuous is to act in accordance with the standards set by the society/person to maintain its well-being. *Virtue is up* because virtuous actions correlate with social well-being from the society/person's point of view. "He is *high*-minded. She has *high* standards. She is an *upstanding* citizen. Don't be *underhanded*. I wouldn't *stoop* to that. That would be *beneath* me." We also see *rational is up; emotional is down* because in our culture people view themselves as being in control over animals, plants, and their physical environment, and it is their unique ability to reason that places human beings above other animals. Thus, *control is up* provides a basis for *man is up* and therefore *rationale is up*: "We put

our *feelings* aside and had a *high-level intellectual* discussion of the matter. He couldn't *rise above* his *emotions*."

3.4. Ontological Metaphors

Our experience of physical objects and substances provides a further basis for understanding – one that goes beyond mere orientation. Understanding our experiences in terms of objects and substances allows us to pick out parts of our experience and treat them as discrete entities or substances of a uniform kind. Once we can identify our experiences as entities or substances, we can refer to them, categorize them, group them, and quantify them, and reason about them. When things are not clearly discrete or bounded, we still categorize them as such (i.e. mountains, street corners, hedges). Such ways of viewing physical phenomena are needed to satisfy certain purposes that we have: locating mountains, meeting at street corners, trimming hedges. Human purposes typically require us to impose artificial boundaries that make physical phenomena discrete just as we are: entities bounded by a surface.

Just as the basic experiences of human spatial orientations give rise to orientational metaphors, so our experiences with physical objects (especially our own bodies) provide the basis for an extraordinarily wide variety of ontological metaphors, ways of viewing events, activities, emotions, ideas, etc., as entities and substances. Ontological metaphors serve various purposes, and the various kinds of metaphors there are reflect the kinds of purposes served. For example, the experience of rising prices is often metaphorically viewed as an entity via the noun *inflation*. This gives us a way of referring to the experience. *Inflation is an entity*: "We

need to *combat inflation*. Buying land is the best way of *dealing with inflation*." Viewing inflation as an entity allows us to refer to it, quantify it, identify a particular aspect of it, see it as a cause, act with respect to it, and perhaps even believe that we understand it. Ontological metaphors are necessary for even attempting to deal rationally with our experiences.

These are some examples of the purposes and representative examples of ontological metaphors that serve them. *Referring*: "My *fear of insects* is driving my wife crazy. That was a *beautiful catch*. The *middle class* is a *powerful silent force* in *American politics*." *Quantifying*: "It will take *a lot of patience* to finish this book. There is *so much hatred* in the world. DuPont has *a lot of political power* in Delaware." *Identifying aspects*: "The *ugly side of his personality* comes out under pressure. The *brutality of war* dehumanizes us all. We never got to feel the *thrill of victory* in Vietnam." *Identifying causes*: "The *pressure of his responsibilities* caused his breakdown. He did it out of *anger*. *Internal dissension* cost them the pennant." *Setting goals and motivating actions*: "He went to New York to *seek fame and fortune*. The FBI will act quickly in the face of a *threat to national security*. She saw getting married as the *solution to her problems*."

3.5. Container Metaphors

We are physical beings, bounded and set off from the rest of the world by our skin, and we experience the rest of the world as outside us. Each of us is a container, with a bounding surface and an in-out orientation. We project our own in-out orientation onto other physical objects that are bounded by surfaces. Thus we also view them as containers with an inside and an outside. Rooms and houses are obvious containers. Moving from room to room is moving

from one container to another, moving *out of* one room and *into* another. We impose this orientation on our natural environment as well. A clearing in the woods is seen as having a bounding surface, and we can view ourselves as being *in* the clearing or *out of* the clearing, *in* or *out of* the woods. A clearing in the woods has something we can perceive as a natural boundary – the fuzzy area where the trees more or less stop and the clearing more or less begins. But even where there is no natural physical boundary that can be viewed as defining a container, we impose boundaries – marking off territory so that it has an inside and a bounding surface – whether a wall, a fence, or an abstract line or plane. Bounded objects have sizes: this allows them to be quantified in terms of the amount of substance they contain. Kansas, for example, is a bounded area – a *container*- which is why we say "there's a lot of land *in* Kansas."

3.6. The Visual Field

We conceptualize our visual field as a container and conceptualize what we see as being inside it. Even the term "visual *field*" suggests this. The metaphor is a natural one that emerges from the fact that, when you look at some territory, your field of vision defines a boundary of the territory, namely, the part that you can see. Given that a bounded physical space is a *container* and that our field of vision correlates with that bounded physical space, the metaphorical concept *visual fields are containers* emerges naturally. For example: "The ship is *coming into* view. I *have* him *in* sight. I can't see him – the tree is *in* the way. There's *nothing in* sight."

3.7. Ontological Metaphors: Examples

We use ontological metaphors to comprehend events, actions, activities, and states. Events and actions are conceptualized metaphorically as objects, activities as substances, states as containers. A race, for example, is an event, which is viewed as a discrete entity. The race exists in space and time, and it has well-defined boundaries. Hence we view it as a *container object*, having in it participants (which are objects), events like the start and finish (which are metaphorical objects), and the activity of running (which is a metaphorical substance). Thus, we can say of a race: "Are you *in* the race on Sunday? (race as *container object*). Did you *see* the race? (race as *object*). The *finish* of the race was really exciting (finish as *event object* within *container object*). There was *a lot of good running in* the race (running as a *substance* in a *container*). *Halfway into* the race, I ran out of energy (race as *container object*).

Activities in general are viewed metaphorically as *substances* and therefore as *containers*: "How did Jerry *get out of* washing the windows? How *much* window-washing did you do? How did you *get into* window-washing as a profession?" Activities are viewed as containers for the actions and other activities that make them up. They are also viewed as containers for the energy and materials required for them and for their by-products: "I *put a lot of energy into* washing the windows. I *get a lot of satisfaction out of* washing windows. *There is a lot of satisfaction in* washing windows." Various kinds of states may also be conceptualized as containers: "He's *in* love. We're *out of* trouble now. He's *coming out of* the coma. I'm *slowly getting into* shape. He *fell into* a depression."

3.8. Personification

Personification, where the physical object is further specified as being a person, allows us to comprehend a wide variety of experiences with nonhuman entities in terms of human motivations, characteristics, and activities: "*Life has cheated* me. The *experiment gave birth to* a new theory. *Cancer* finally *caught up with* him." The metaphor *inflation is a person* can be further specified as *inflation is an adversary*: "Our biggest *enemy* right now *is* inflation. Inflation *has pinned* us *to the wall*. The dollar *has been destroyed* by inflation."

3.9. Metonymy

With metonymy, one entity is used to refer to another that is related to it. For example: "He's in *dance* (= the dancing profession). Mrs. Grundy frowns on *blue jeans* (= the wearing of blue jeans). He likes to read *J.K Rowling* (= the writings of J.K. Rowling)." A special case of metonymy is *synecdoche*, where the part stands for the whole: "We need a couple of *strong bodies* for our team (= strong people). I've got a new *set of wheels* (= car, motorcycle, etc)." When we say that we need some *good heads* on the project, we are using "good heads" to refer to "intelligent people. Get *your butt* over here!" The point is not just to use a part (head) to stand for a whole (person), but rather to pick out a particular characteristic of the person (intelligence) which is associated with the head. Other examples include: *the face for the person*: "She's just a *pretty face*. We need some *new faces* around here." *Producer for product*: "He bought a *Ford*. He's got a *Picasso* in his den." *Object for user*: "We need a better *glove* at third base." *Controller for controlled*: "*Napoleon* lost at Waterloo. A Mercedes rear-ended *me*."

Institution for people responsible: "I don't approve of the *government's* actions." *The place for the institution*: "*Hollywood* isn't what it used to be." *The place for the event*: "Remember *the Alamo*."

3.10. Additional Examples

Theories (and *arguments*) *are buildings*: "The theory needs more *support*. The argument is *shaky*. We need to *construct* a *strong* argument for that." *Ideas are food*: "What he said *left a bad taste in my mouth*. All this paper has in it are *raw facts* and *half-baked ideas*. Let me *stew* over that for a while. That's *food for thought*. He's a *voracious* reader. We don't need to *spoon-feed* our students." *Ideas are people*: "The theory of relativity *gave birth to* an enormous number of ideas in physics. He is the *father* of modern biology. His ideas will *live on* forever." *Ideas are plants*: "That's a *budding* theory. Mathematics has many *branches*. The *seeds* of his great ideas were *planted* in his youth." *Ideas are products*: "We've *generated* a lot of ideas this week. His *intellectual productivity* has decreased in recent years. It's a rough idea; it needs to be *refined*." *Ideas are commodities*: "It's important how you *package* your ideas. He won't *buy* that. That's a *worthless* idea." *Ideas are resources*: "He *ran out of* ideas. Let's *pool* our ideas. He's a *resourceful* man." *Ideas are money*: "Let me put in my *two cents' worth*. He has a *wealth* of ideas." *Ideas are cutting instruments*: "That *cuts right to the heart of* the matter. That was a *cutting* remark. He's *sharp*. He has a *keen mind*." *Ideas are fashions*: "That's an *outdated* idea. He keeps *up-to-date* by reading." *Understanding is seeing; ideas are light-sources; discourse is a light-medium*: "I *see* what you're saying. It *looks* different from my *point of view*. I've got the

whole picture. That's an *insightful* idea. That was a *brilliant* remark. The argument is *clear*. Could you elucidate your remarks? It's a *transparent* argument."

Love is a physical force (electromagnetic, gravitational, etc.): "I could feel the *electricity* between us. There were *sparks*. They *gravitated* to each other immediately. His whole life *revolves* around her. There is incredible *energy* in their relationship." *Love is a patient*: "They have a *strong, healthy* marriage. They've got a *listless* marriage. It's a *tired* affair." *Love is madness*: "I'm *crazy* about her. He constantly *raves* about her. I'm just *wild* about Harry." *Love is magic*: "She had me *hypnotized*. I'm *charmed* by her. She is *bewitching*." *Love is war*: "She *fought for* him, but his mistress *won out*. She *pursued* him *relentlessly*. He *won* her hand in marriage."

Wealth is a hidden object: "He's *seeking* his fortune. She's a *gold-digger*. He *lost* his fortune." *Significant is big*: "He's a *giant* among writers. That's the *biggest* idea to hit advertising in years. That was only a *little* white lie. That was one of the *greatest* moments in World Series history." *Seeing is touching; eyes are limbs*: "I can't *take* my eyes *off* her. He sits with his eyes *glued to* the TV. Their eyes *met*. He wanted everything *within reach of* his eyes." *Eyes are containers for the emotions*: "I could see the fear *in* his eyes. Her eyes *welled* with emotion." *Emotional effect is physical contact*: "His mother's death *hit* him *hard*. I was *struck* by his sincerity. That really *made an impression* on me. I was *touched* by his remark. That *blew me away*."

Physical and emotional states are entities within a person: "His depression *returned*. He could barely *contain* his joy. The smile *left* his face. *Wipe* that sneer *off* your face! His fears *keep coming back*." *Vitality is a substance*: "He's *devoid* of energy. I'm *drained*. That *took a lot out of*

me." *Life is a container*: "I've had a *full* life. There's *not much left* for him *in* life. *Get the most out of* life. Live your life *to the fullest*." *Life is a gambling game*: "I'll *take my chances*. The *odd are against me*. I've got an *ace up my sleeve*. It's a *toss-up*. If you *play your cards right*, you can do it. He *won big*. Where is he when the *chips are down*? He's *bluffing*. Let's *up the ante*. I think we should *stand pat*. That's *the luck of the draw*."

These metaphors, prevalent in everyday life, represent the way we think and what we experience. Thus, they must also play a pivotal role when a mentor provides feedback or when an instructor talks about the mathematics of a problem. I will identify which metaphors are most commonly found in the different types and content of feedback we have identified earlier. Because metaphors structure how we perceive the world, I suspect that different types of feedback may have characteristic metaphors associated with it.

4. Methods

4.1. Introduction

To understand the use of content-focused mentoring in mathematics, I examined data from four pre-service teachers' (PTs') mentoring of elementary students in an online environment. Students submitted solutions to the "Horsin' Around" Problem of the Week for the Math Forum. PTs responded with a set of suggestions, and were provided feedback regarding these suggestions by mentors working either with the Math Forum or their own instructors. The PTs then revised the initial feedback, and the revisions were analyzed to determine the effectiveness of the feedback and whether (and how) the suggestions were implemented (i.e. was the feedback simply copied or did the pre-service teacher understand the suggestion).

4.2. Participants

The four pre-service elementary school teachers were grouped based on interest level. The high interest group included Meghan and Avrila, while the low interest group included Melissa and Stephanie. Solutions were provided by elementary school students working with challenge problems in an online environment. Mentors providing feedback to the teachers are either the students' own instructor or a person trained to provide content-related scaffolding feedback.

4.3. Coding

For each of the four pre-service teachers, all discussion and feedback from the mentors was analyzed. First, the mentor's initial set of suggestions was coded; in particular, we focused on the content and style of each question or suggestion. Then, the teacher's revisions were coded and analyzed to determine which of the suggestions they had used and the effectiveness of the mentor's suggestions. Furthermore, we looked for hints that might provide insight into how well the teachers understood the suggestions. We coded the mentor's suggestions using two measures: the style and content of the feedback.

First, the style of the mentor's suggestions was coded as either implicit or explicit feedback. However, I noticed that a certain pattern began to emerge in the data: the mentors sometimes gave explicit feedback, but then also provided an explanation as well. Thus, I then made a further distinction when the mentor provided explicit feedback: whether or not a rationale for the suggestion was provided. A rationale for the feedback, however, could only be a characteristic of explicit feedback by definition. This distinction would not make sense for implicit feedback, as there is no command; in this case, the mentor is simply leading, or guiding the PTs.

Next, we coded the content of the feedback from the PTs, drawing primarily on the models of Hannafin, Land, and Oliver (1999) with one addition: math-related feedback. While Hannafin, Land, and Oliver provide a comprehensive theory of scaffolds and feedback, we felt that including a category for content-related feedback is essential for the purposes of this study. In this way, we hoped to apply the existing theoretical models, but modified to meet the

specific needs of our project. The first of three major categories representing the content of the feedback are is structural (including suggestions regarding formatting, typos, and paragraph structure). This category was largely ignored in the analysis of feedback; we primarily focused on the following two. The second type of feedback fell under the category of metacognitive/strategic. These categories include suggestions encouraging reflections and explanations, as well as providing various techniques and models. Here, the mentee is provided with specific support for the particular task. The suggestions typically are broader, though relating to the specific needs of the PT. Finally, the content of the feedback might be coded as math-related/contextual. In this case, the mentors are providing hints or prompts that are specifically related to the details in the problem. In particular, the suggestions relate specifically to the mathematics in the problem, not simply broad strategies. This type of feedback was the focal point of our investigation, not only studying it when it arose but more importantly, how we might be able to encourage the use of content-related feedback.

Suggestions can, and often are a mix of these various types of feedback and can vary in its content. For example, one particular comment may contain elements of both metacognitive/strategic feedback as well as math-related and contextual feedback. Also, the PTs were instructed to assign ratings (i.e. Apprentice, Novice, Expert) to the students for a particular aspect of their work. I ignored any suggestions relating to these ratings because the feedback tended to be non-informative: the mentor typically instructed the PTs to change the rating, which provided little information relating to this thesis.

4.4. Hypothesis

This paper focuses on two aspects of feedback and their effectiveness when dealing with content-related/contextual and metacognitive/strategic feedback. I wondered whether feedback that is explicit would be more effective, and whether this would be different for the low interest or high interest group. I hypothesized that explicit feedback may be more detrimental to the low interest PTs than implicit feedback because they may copy the suggestions without understanding them. Furthermore, I hypothesized that providing an explanation might be an effective and motivating tool for a deep understanding of the feedback. On the other hand, I suspected that feedback both explicit and implicit would be effective for the high interest teachers. Lastly, I hypothesized that metaphors, because of their prevalence in everyday life, would play a role in the effectiveness of the feedback, particularly in the mathematics of the problem.

5. Coded Data

In this section, I provide excerpts from both the feedback provided by the mentor and the revisions of the PT. I isolated what seemed to be each discrete instance of feedback and the response to this suggestion.

5.1. Data from Melissa (Low-interest PT)

Student: Jack

Feedback from instructor:

1. "You (Melissa) have some important information to tell the students but it is somewhat difficult to read. Try to break your mentoring into paragraphs. One paragraph for each suggestion, question or thought. Usually we leave one blank line between each paragraph" (Explicit with rationale, Structural)
2. "I would like you to ask Jack more questions rather than give him directions. For example, instead of telling Jack that he needs more explanation, you could give him a question about a part of his response that is not clear. This way you let Jack do the thinking on how to clarify his response by giving him a nudge in the right direction" (Explicit with rationale, Metacognitive)
3. "Your suggestion about always including units is good. You might also try to prompt Jack to display his calculations in equation form as well as words. Can you think of a question that will prompt Jack in this way?" (Implicit, Math-related)

Revisions by Melissa:

1. Formatting Changed; Broke mentoring into paragraphs, with a blank line between each paragraph (Made change, high understanding)
2. Adds a few reflection questions (seemed a little bit out of context): "A question I also have for you was this problem easy or difficult for you? Was your first approach at solving this problem correct? (Made changes, low understanding)
3. Adds a confusing and unrelated suggestion (doesn't deal with displaying calculations in equation form) and doesn't bother to come up with a question: "I see that you have a lot of good strategies and math concepts at solving this sort of problem, but it would make even better if you could enhance that by showing in your explanation in a way that others can follow as if they never saw the original problem" (No change, low understanding)

Student: Katy
Feedback from instructor:

1. "You are right about what Katy needs to improve but I think you used some words and concepts that could be above her understanding level. Try to remember how old the student is, Katy is 11, and work your response around that." (Explicit, Metacognitive)
2. "Katy definitely had some troubles understanding the problem but we want her to figure out why. What are some questions you could ask her to help her start thinking about where she went wrong in the problem? I really would like to see if she can figure out what went wrong and re-read the problem herself without you telling her the parts she needs. (Implicit, Metacognitive)

Revisions by Melissa:

1. Changed some of the vocabulary and explained concepts differently (It seems that the revisions might be more confusing): Changed "so I am going to help you out" to "I am going to give you some insight to help you understand the problem a little differently;" changed "The total journey is 50 miles that means for both the walking and horsebacking is 50 miles. It also states that they spent equal time for each event. So it is important to understand that you find out how many miles you go for each" to "The idea of the problem is they want to find equal time not equal distance; the total journey for the trip is 50 miles that means for both the walking and horsebacking is 50 miles total. A clue to note is that with the horseback riding and the walking they did this for the same amount of time. Remember that the final answer that you are wanting to find is how long (time) did it take for him to complete the journey. So it is important to understand that you find out how many miles you go for each." (Made change, low understanding)
2. Did not ask any questions (No change, low understanding)

Student: Maddy
Feedback from instructor:

1. "Try not to put the focus on her obtaining the correct answer but more on the process of understanding" (Explicit, Metacognitive)
2. "I think you also are right in that she needs to explain a lot more. Could you give her some questions that direct her where to start to clarify her response?" (Implicit, Metacognitive)
3. "It looks like she made a table, maybe you could encourage her to include it in her response?" (Explicit, Metacognitive)

Revisions by Melissa:

1. Left a comment in her response focusing on obtaining the correct answer: "Congratulations!!! At completing the problem correctly!!!"Did focus on the process, asking to explain thoughts and steps to problem-solving: "It would be nice if you could explain how you came up with the [table to figure out that problem], maybe write down your thoughts that brought you to this equation. (Made change, High Understanding)
2. Added reflection questions: "Was this problem difficult or easy for you? Do you think there is more than one way at solving this problem?" (Made changes, Low understanding)
3. Asked about a table, but did not ask to include it: "It seemed to me that you made some sort of a table to figure this out and that was good, but it would be nice if you could explain how you came up with this maybe write down your thoughts that brought you to this equation" (No change, Low understanding)

<u>Student: Amelia</u>

Feedback from instructor:

1. "We are more interested in the child's process than whether or not she is correct. You can leave comments that relate to correctness off and just concentrate on finding out more information about the student's thinking" (Explicit, Metacognitive)
2. "I was thinking of some specific questions to ask Amelia to get a better explanation from her. I was very interested to see what numbers she chose for her trial and error. I would also like to know what numbers and equations she used to work with the trial and error method. Perhaps you could ask her specific questions like these and any you think of to get her to explain herself more. (Explicit, Metacognitive /Math-related)

Revisions by Melissa:

1. Left the comment: "Congratulations at correctly answering the problem!!!" (No change, Low Understanding)
2. Simply added the questions provided in the feedback: "I was very interested to see what numbers you chose for the trial and error. I would also like to know what numbers and equations you used to work with the trial and error method" (Made changes, Low Understanding)

Student: Eric

Feedback from instructor:

1. "I would like to see you asking for more reflection from both of them. Some good questions to ask are: What was easy or hard for you about the problem? Why? Is there a way you can check your answer to make sure it is right? Does the answer you got make sense to you? Were you surprised by any of the results? Any other reflection questions you think of would be great to add as well" (Explicit, Math-Related/Metacognitive)

Revisions by Melissa:

1. Included the exact questions suggested in feedback (tacked onto the end of her response), even when redundant: "Here are some other questions I have for you: What was easy or hard for you about the problem? Why? Is there a way you can check your answer to make sure it is right? Does the answer you got make sense to you? Were you surprised by any of the results?" A questions she had originally included in her response (and included in her final response) were: "I think that in every instance there is always room for some sort of improvement at solving problems, so I am going to make a challenge for you try a new method called the verification method that means can you solve the problem another way or check your work?" (Made Changes, Low Understanding)

Student: VW

Feedback from instructor:

1. "I think it's a great idea to try to get VW to explain himself more. Are there any specific parts of his problem you would like to know more about? It might be more effective to ask VW specific questions to elicit explanation rather than asking the broader request for more explanation. You could ask him how he simplified his original equation and solved for x or why he divided 8 by 2 for the bonus. Think of some questions that will help VW explain himself" (Explicit, Metacognitive)
2. "You could also ask for labels in question format as well" (Explicit, Structural)

Revisions by Melissa:

1. Added a paragraph at the end with the exact question suggested in feedback: "I was wondering if you could explain a little more on how you were able to simplify your original equation and solved for x or why you divided 8 by 2 for the bonus" (Made change, Low Understanding)
2. Did not mention anything about labels, not in a statement or a question (No change, low understanding)

Student: Michaela

Feedback from instructor:

1. "I would like you to get rid of the section where you tell Michaela what to look for in the problem. Instead, what kinds of questions could you ask Michaela to get her to find her mistake. Instead of telling her the problem said equal time not distance, find a way to get her to look for that information herself" (Explicit, Metacognitive)

Revisions by Melissa:

1. Removed section reading "You may want to break up the data in maybe a chart form if that works best, or set it up algebra style and come up with variables. There are many ways at looking at this problem juts figure out what works best for you." Did not ask any questions to get Michaela to find her mistake; did not find a way to help Michaela look for the information that the problem said equal time, not distance. (No change, Low Understanding)

Student: Elizabeth

Feedback from instructor:

1. Gives same feedback as Eric: "[Elizabeth has] solved and explained the problem fully and only lack reflections. I would like to see you asking for more reflection from [her]. Some good questions to ask are: What was easy or hard for you about the problem? Why? Is there a way you can check your answer to make sure it is right? Does the answer you got make sense to you? Were you surprised by any of the results? Any other reflection questions you think of would be great to add as well" (Explicit, Math-Related/ Metacognitive)

Revisions by Melissa:

1. Removes a comment saying "Try next time to verify or solve the problem in a different way this is a good way to check your work" and tacks on a paragraph containing the exact questions provided by the instructor: "What was easy or hard for you about the problem? Why? Is there a way you can check your answer to make sure it is right? Does the answer you got make sense to you? Were you surprised by any of the results?" (Made changes, Low understanding)

5.2. Data from Stephanie (Low-interest PT)

Student: Briana
Feedback from instructor:

1. "I know this sounds all high and mighty, but don't tell [her] your failures. Just let [her] know that SHE misinterpreted the problem, not you" (Explicit, Structural)
2. "I am glad that you pointed out that .14 is not 14 minutes, but explain how you came up with .14 is 8 minutes and 24 seconds. (I know it ends up being irrelevant for this problem, but it is good knowledge to have anyway)" (Explicit, Metacognitive /Math-related)
3. "The second sentence of the paragraphs is a little awkward" (Explicit, Structural) – Feedback seems a little bit unclear; doesn't specify which sentence
4. "Your question 'Why did you subtract 9 hours from 60 hours?' could be stated better. Perhaps just ask her to explain how exactly she went from 9 hrs 84 min to 10 hrs 24 min"(Explicit, Structural)
5. Recheck calculations: "You also need to ask how they divided '25 miles by 9 miles (really per hour) and got 10 hrs 24 min.' That makes no sense" (Explicit, Math-Related)
6. "'Awesome job' is a little too high praise. Reserve that for truly excellent students." (Explicit, Structural)

Revisions by Stephanie:

1. Changed "When I first did this problem, I did the same thing that you did and I thought the problem said Zachary traveled an equal DISTANCE on foot and horseback rather than an equal TIME on food and horseback" to "It looks like you thought the problem said Zachary traveled an equal DISTANCE on foot and horseback rather than an equal TIME on foot and horseback. I look forward to seeing your revised answer when finding the equal time spent on foot and horseback" (Made change, high understanding)
2. Added in parenthesis: "With cross multiplication we find that .14 out of 1 = 8.4 minutes out of 60 minutes and .4 minutes out of 1 minute = 24 seconds out of 60 seconds" (Made change, high understanding)
3. No change, low understanding – perhaps Stephanie did not understand which sentence was awkward
4. Changed "Why did you subtract 9 hours from 60 hours?" to "Explain how you went from 9 hrs 84 min to 10 hrs 24 min" (Made change, high understanding)
5. Asked to recheck calculations: "How did you divide 25 miles by 9 miles and get 10 hrs and 24 min?" (Made change, low understanding)
6. Changed "Awesome job explaining your interpretation of the problem, I can't wait to see your revised answer!" to "Good effort in explaining your interpretation of the problem. I can't wait to see your revised answer!" (Made change, high understanding)

Student: D.J.

Feedback from instructor:

1. "I don't know what happened, but all the formatting is messed up on your replies. Could you please fix them so that there is not such big gaps between each line? It's weird. I wonder if you cannot see those on your computer. HHHMMMM?" (Explicit, Structural)
2. "Comment on the fact that their answer is wrong for # of hours, but they probably just made a minor math error. I know you hint at it, but tell them flat out" (Explicit, Metacognitive)
3. "I am not sure it is necessary to praise them exactly for their 14 miles answer, but praise them in general for doing nice work and being VERY VERY close to having the right answer" (Explicit, Structural)

Revisions by Stephanie:

1. Fixed formatting (Made change, high understanding)
2. Asked explicitly to recheck his answers: "Re-check your answer for the number of hours, you're really close to being right" (Made change, low understanding)
3. Did not change the praise, left "Awesome job explaining how you got 14 miles as the total miles for walking" (No change, low understanding)

Student: Heather

Feedback from instructor:

1. "I would just suggest to [her] that [she] shows all [her] calculations for wrong guesses too. (You asked them to show the wrong guesses, but they also need to show their multiplication and the other steps.)" (Explicit, Metacognitive)
2. "Also fix the formatting (if you see it)"(Explicit, Structural)

Revisions by Stephanie:

1. Adds: "Also show your calculations for each guess" (Made change, low understanding)
2. Formatting Fixed (Made change, high understanding)

Student: Caroline

Feedback from instructor:

1. "Caroline didn't really understand the 'equal time riding and walking' fact. I think she thought distance was time. That is, of course, not true. You should try to emphasize this to her. Therefore, when she tried 4 hours on horse and got 36 miles, she should have also tried 4 hours walking BECAUSE ZACHARY SPENT AN EQUAL TTTTIIIMMMEEEE WALKING AND RIDING. I put that in caps to emphasize what she did not understand" (Explicit, Math-related)

Revisions by Stephanie:

1. Added: "As soon as you decided that he rode for 4 hours, couldn't you have known that he walked for four hours? (Because you know ZACHARY SPENT EQUAL TIME RIDING AND WALKING)" (Made change, low understanding)

5.3. Data from Avrila (High-interest PT)

Student: Tim

Feedback from instructor:

1. "First and foremost, you accidentally erased their response. They need to know what they said. I have included their response at the end of this message. You can cut and paste it" (Explicit with rationale, Structural)

Revisions by Avrila:

1. Formatting Fixed (Made change, high understanding)

Student: Riley

Feedback from instructor:

1. "The first paragraph in this one needs to be revised. Starting out with 'One more thing,' sounds like you are being really nit picky and from the students' perspective he might get the impression he will never get it good enough and give up" (Explicit with rationale, Metacognitive)
2. "I had a hard time understanding the second sentence as well" (Implicit, Structural)
3. "Avoid the word 'you' if at all possible, it comes across too harsh" (Explicit with rationale, Metacognitive)

Revisions by Avrila:

1. Changed "There's one more thing I can see that you could improve on" to "I just want to point out one more possible area for improvement in case you'd like to try for a few more points. You're already doing great – this is for if you want a puzzle to figure out" (Made change, high understanding)
2. Changed "You don't seem to make the most use possible out of knowing from the problem that Zachary spends equal amounts of time riding and walking" to "It seems to me like there's a piece of information in the problem that doesn't get fully used in the solution: Zachary spends the same amount of time walking and riding" (Made change, low understanding)
3. Avoided using "you" harshly (Made change, high understanding)

Student: Francis

Feedback from instructor:

1. None

Revisions by Avrila:

1. N/A

5.4. Data from Meghan (High-interest PT)

Student: Super Geeks

Feedback from instructor:

1. "I think your tone could be even more encouraging if you could rephrase 'should' statements into questions." (Explicit with rationale, Structural)
2. "I really liked your comment about units cancelling out, because it shows the mathematical importance of labeling units. Is there a way you could ask a question about units based on that?" (Implicit, Math-Related/ Metacognitive)
3. "When you ask them to reflect, is there a way you could model for them what some more kinds of reflection are, rather than telling them to reflect?" (Implicit, Metacognitive)

Revisions by Meghan:

1. Rephrased "First you *should* double check your units" into "First you *could* double check your units" (Made change, high understanding)
2. Rephrased into a question, though with superficial changes: "You should label the hours in your table of answers so a mentor can see that the hours cancel out and you forgot to use units in your conclusion" to "Is there any way you could label the hours in your table of answers so a mentor can see that the hours cancel out or use units in your conclusion?" (Made change, low understanding)
3. Added an example of reflection, simply re-doing the students' work: "For example, 9 mph x 4 hrs = 36 mi, 3 ½ mph x 4 hrs = 14 mi, and so on" (Made change, low understanding)

Student: Adam

Feedback from instructor:

1. "Remember how in the first thread you reflected what the SuperGeeks understood back to them? I think that would be really good to do here, when you talk about their two strategies. If you named or described those strategies, Austin could see what he or she was doing, which helps him/her learn to reflect" (Explicit with rationale, Metacognitive)
2. "There was a lot of really interesting math in this problem: for example, Austin hypothesized that the mile of riding the horse should be around three times more than walking. Was that a valid hypothesis? Did it turn out to be true? Why? What is exactly the same about 9 and 3.5, and 14 and 36? In general, I think that asking for adding units is important, but challenging and pushing the math is even more important, especially when the student seems ready" (Explicit, Math-related/ Metacognitive)

Revisions by Meghan:

1. Explicitly named the two strategies: “And by two strategies I mean the table which is well labeled and basic math to check your answer. (Made change, low understanding)
2. Added exact questions (from the feedback) about the hypothesis: “I only have one question though, why did you hypothesize that the smaller number would be almost three times less than the larger one? Did it help you when filling out your table? Did your hypothesis true?” (Made change, low understanding)

Feedback from instructor:

1. “I think you may have left out a word or something when you said ‘Did your hypothesis true?” (Explicit, Structural)
2. “Now that there’s so much info here, it should help to break it into paragraphs, skipping lines in between each paragraph” (Explicit, Structural)

Revisions by Meghan:

1. Fixed, clarified her question: “Did your hypothesis about the smaller number being around three times smaller than the larger one come true?” (Made change, high understanding)
2. Fixed: broke into two paragraphs (Made change, high understanding)

<u>Student: Davy</u>

Feedback from instructor:

1. “Try to work with the strategy B Davis seems to be using, rather than going to a chart” (Explicit, Metacognitive)
2. “Help him see how to get the rest of the way. Why do you think he got 4 instead of 8 for his answer?” (Implicit, Math-related/ Metacognitive)
3. “I wouldn’t push him for a reflection just yet” (Explicit, Metacognitive)

Revisions by Meghan:

1. Removed suggestion to use a chart, as well as her sample chart: “Is there a way a table might help you find your answer? Can a table be helpful in solving the extra questions: about how many miles did he walk and how many miles were ridden at the end of his journey?” (Made change, low understanding)
2. Tells him outright that his answer needs to be doubled: “So if you’re finding the answer for one journey like walking or riding, the answer might need to be doubled” (Made change, low understanding)
3. Removed suggestion for a reflection: “Then as a reflection you can either find another way to solve the problem or back track your steps to see if you got the numbers you started with” (Made change, high understanding)

Student: Luke

Feedback from instructor:

1. "I noticed a typo in your response: Do '9 horsebacks' and '5 on foot' really equal '84'? I think it should be 5 horsebacks" (Explicit, Structural)
2. "I think it might help to focus a little more on the positive things Luke has done here, since his thinking is really great. Hearing specific things he did well will make him prouder and might help him do them again in the future" (Explicit with rationale, Metacognitive)

Revisions by Meghan:

1. Changed "Do '9 horsebacks' and '5 on foot' really equal…" to "Do '5 horsebacks' and '5 on foot' really equal…" (Made change, high understanding)
2. Added praise: "It really helped me to know you used the guess and check method by letting me know that you tried another number before you told me the right one" (Made change, high understanding)

Student: Alana

Feedback from instructor:

1. "I would stay away from telling her she misinterpreted the problem, though, since we know so little about what she did (maybe it was a calculation error of some sort)" (Explicit with rationale, Metacognitive)
2. "In this case it would be ok to be even more specific in your hints" (Explicit, Metacognitive)
3. "I like the idea of making a table, and I wonder if it would help to ask her specific questions about what goes in the table, or how she would set it up" (Implicit, Metacognitive)
4. "The people who wrote the problem did not use miles/hour on purpose, since they wanted kids to be able to think of it as 9 miles in 1 hour, not as a rate problem, so you may want to stay away from that language, especially if the kids don't use it" (Explicit with rationale, Math-related/Metacognitive)
5. "The hint about the answer being divisible by two may be a little misleading, especially if she does guess and check. Would you say a 5 hour trip is a reasonable guess?" (Implicit, Math-related)

Revisions by Meghan:

1. Deleted misinterpretation comment; changed "I would like to suggest that you re-read the problem because I believe you misinterpreted the question being asked" to "I would like to suggest that you re-read the problem at least twice because I believe it helps to really understand what the problem is asking you and gives you clues to solving it" (Made change, low understanding)
2. No change, low understanding
3. Set up a table, asked leading questions: "It might be easier if you try a table or chart to place your data in. It will help your process by placing at the top of your chart how many miles walked, how many miles riding, and how many miles were traveled total. Then for every hour write down how many miles were traveled for every mile, for example…" (Made change, high understanding)
4. Removed language of miles per hour appropriately: Changed "Remember he walks 3 ½ mph and rides 9 mph" to "Remember he walks 3 ½ mi. and rides 9 mi. for every 1 hour of travel" (Made change, high understanding)
5. Removed comment: "your answer should be divisible by 2" (Made change, low understanding)

Feedback from instructor:

6. "I liked how you started the chart, and then left some ?'s for Alana. However, I think the information in the chart isn't quite right or needs to be clarified. Please take another look at it" (Explicit, Math-related)
7. "I think you could probably leave all, or all but one of the spaces in the chart blank, since setting it up for her is a pretty big hint. The goal is to have her do as much thinking as possible, which is a pretty hard call. I'll leave it up to you, but please fix the chart if you leave some information in it" (Explicit with rationale, Metacognitive)

Revisions by Meghan:

6. Left chart blank (Made change, Low understanding)
7. Left chart blank (Made change, low understanding)

Feedback from instructor:

8. "Are you sure he goes 9 miles and 3.5 miles in one hour, or is it in one hour each of riding and walking (two hours total)?" (Implicit, Math-related)

Revisions by Meghan:

8. No change, Low understanding

Feedback from instructor:

9. Fix the same hint: "When you say 'he walks 3 ½ mi. plus rides 9 mi. for every 1 hour of travel,' that makes me think that in 1 hour he goes 12.5 miles (3.5 + 9). If he goes 12.5 miles in one hour, then he goes 50 miles in 4 hours, so the answer to the problem would be 4 hours. Is it possible you mean that 'in one hour EACH of riding and walking, he goes 3.5 plus 9 hours'?" (Explicit, Math-related)

Revisions by Meghan:

9. Fixed hint in her own words: Changes "Remember he walks 3 ½ mil. Plus rides 9 mi. for every 1 hour of travel" to "Remember he walks 3 ½ mi. in one hour and in another hour he rides 9 mi. So in two hours he walks 3 ½ mi. plus rides 9 mi." (Made change, high understanding)

<u>Student: Jonathan</u>
Feedback from instructor:

1. "I wish [Jonathan] had explained in more detail how he knew how far Zach went in two hours, then four hours. It seems like he left some equations or steps out – I understood what he did, but a classmate of his who was struggling with the problem might not" (Implicit, Math-related/ Metacognitive)

Revisions by Meghan:

1. No change, low understanding

<u>Student: Elisabeth</u>
Feedback from instructor:

1. "Could you clarify what you mean by 'Did you get '8 hours' in the equations or 'did you have to times a multiple by 2 or another number to get it?' I wasn't sure" (Implicit, Math-related)
2. "I think the suggestion that Elisabeth show her guesses is a good one, but could you also push her to be like Jonathan and do some logical reasoning about her guesses, not just say what they were?" (Implicit, Metacognitive)

Revisions by Meghan:

1. Clarified question somewhat: Changes "Did you get '8 hours' in the equations or did you have to times a multiple by 2 or another number to get it?" to "Did you actually get '8 hours' in the equation or did you have to multiply your answer by another number to get '8 hours'? (Made changes, Low understanding)
2. Asked for logic behind guesses explicitly as in the feedback instead of asking leading questions: Asked "Was there a reason that you started with one number and not get another or logic behind it? " (Made changes, low understanding)

Student: Gabriella
Feedback from instructor:

1. None

Revisions by Meghan:

1. N/A

6. Data Analysis

6.1. The Approach

In order to understand the effectiveness of different types of feedback and the four content categories for feedback, we first did a numerical analysis of the data provided through the Math Forum by creating a table. The PTs were divided into two groups, high or low, based on their interest level in mathematics. These two groups were analyzed distinctly to see whether different types of feedback or different content would be particularly helpful. Then, we coded the data, categorizing each suggestion or hint as one of three types (explicit, explicit with a rationale, or implicit) and describing the content of the data (Structural, Metacognitive, Math-Related, or Metacognitive and Math-Related). We determined whether the PTs responded to these suggestions by making a change or making no revisions. If they did in fact revise their response, we determined whether the PTs had a high or low understanding of the feedback provided. This was assessed through the revision (i.e. if the suggestions were copied and pasted into their response, we assumed the PTs did not have a thorough understanding of the hints). The results of this count are provided in the two tables below. Next, we recorded patterns that began to emerge from the data and looked specifically at certain characteristic exchanges between the instructor and the PTs. Because we are dealing with a small sample size, we hoped that analyzing these patterns and specific interactions might illuminate the effectiveness of certain types and content of feedback.

6.2. Statistical Analysis

Provided below are two charts, divided between low and high interest PTs, which tally the responses of PTs (Made change with high or low understanding of the suggestions or Made no change to the mentor's feedback) and the various types (Explicit, Explicit with Rationale, and Implicit) and content (Structural, Metacognitive, Math-Related, Metacognitive and Math-Related) of the feedback.

While the most common responses of the PTs was to make a revision with a coded "low understanding" of the feedback, there are differences between the high and low interest group with the other responses. There are significantly more high interest PTs who had a high understanding of the suggestions (12 instances) than the low interest PTs (8 instances). Furthermore, the low interest PTs were more likely (more than twice as likely) to respond to the feedback with no revisions. In 8 instances, low interest teachers did not change their responses (compared to the 3 instances of high interest teachers). However, the most common response to the feedback was to make changes without fully understanding the suggestions, i.e. they have a low understanding of the suggestions (13 instances for the high interest group and 11 instances for the low interest group). This seems to suggest that both groups of PTs seem to be revising their responses without thinking about the feedback.

Within both groups, the content of the feedback provided was predominantly metacognitive. For the low interest groups, the vast majority of the feedback was explicit, while the feedback provided to the high interest group was evenly spread among explicit, explicit with a rationale, and implicit. At first glance, explicit feedback seems to be moderately effective while implicit feedback seems to be ineffective, and providing a rationale along with the explicit

feedback seems to be preferable. When the mentor provided a rationale, there were no instances in either the high or low interest groups in which there was a response of no change. In the high interest group, when provided with explicit feedback with a rationale, the most common response was for the PT to make a change with a high understanding. Furthermore, there were no instances in either group where implicit feedback elicited a response with a high understanding of the suggestion. It is possible that this is due to the small sample size of the exchanges. However, we also note that when the PT is provided with implicit feedback, the general response is not promising: in 7 cases the high interest group responds with a low understanding and 2 cases of no change while there is only one instance of a high understanding. In the low interest group, implicit feedback results in two instances of no change and one instance of low understanding. One thing that also must be noted is that explicit and structural feedback is typically correlated when the response is high understanding change. By the nature of this feedback, PTs tend to make the superficial structural changes, and earn the coding of "high understanding."

In both groups, the mentors most commonly provide metacognitive feedback and structural feedback. There are at least twice as many instances of metacognitive feedback then either math-related or metacognitive and math-related feedback. The content of the feedback seems to be insignificant when assessing the effectiveness of the mentor, except in the cases of explicit and structural feedback resulting in high understanding change (as discussed above).

High Interest Group

Style/Content of Feedback	**Made Change (High Understanding**	**Made Change (Low Understanding)**	**No Change**
Explicit with Rationale	6	3	0
Explicit	5	3	1
Implicit	1	7	2
Structural	5	1	0
Metacognitive	5	6	1
Math-Related	1	3	1
Metacognitive and Math-Related	1	3	1

Low Interest Group

Style/Content of Feedback	**Made Change (High Understanding**	**Made Change (Low Understanding)**	**No Change**
Explicit with Rationale	1	1	0
Explicit	7	9	6
Implicit	0	1	2
Structural	6	0	3
Metacognitive	1	6	4
Math-Related	0	2	1
Metacognitive and Math-Related	1	3	0

6.3. Analysis of Specific Exchanges

There are interesting aspects of an exchange that occurs between the mentor and PTs that are not expressed in this table. While the table tells us how a PT responds to different types and content of feedback, it does not convey how the PT is thinking about the feedback (i.e. the low or high level of understanding of the feedback). In the following exchange between the instructor and Melissa (a low interest PT), we see that Melissa prefers to simply copy and paste the provided feedback even though certain parts of her response are perfectly legitimate and do not need to be revised.

> The instructor tells Melissa: "[Elizabeth] has solved and explained the problem fully and only lack reflections. I would like to see you asking for more reflection from [her]. Some good questions to ask are: What was easy or hard for you about the problem? Why? Is there a way you can check your answer to make sure it is right? Does the answer you got make sense to you? Were you surprised by any of the results? Any other reflection questions you think of would be great to add as well."
>
> In her revision, Melissa adds the following: "What was easy or hard for you about the problem? Why? Is there a way you can check your answer to make sure it is right? Does the answer you got make sense to you? Were you surprised by any of the results?"
>
> (this is regarding the post by Elizabeth)

However, in her original response, Melissa had a redundant question but did not bother to post it. She had initially asked, “Try next time to verify or solve the problem in a different way; this is a good way to check your work.” However, she chose to include the mentor’s questions verbatim even though she had a legitimate question already. It seems as though Melissa either lacks confidence in her response or prefers not to think about the feedback and simply copy it into her response.

This type of exchange and revision by the PT is characteristic of responses coded as “low understanding.” In extreme cases such as this, PTs simply tack on the mentor’s suggestions verbatim (or with minimal restructuring) at the end of their original response, instead of integrating the suggestion into the response. In a similar case, the instructor provides feedback to Melissa, a low interest PT, regarding the student Amelia: “I was thinking of some specific questions to ask Amelia to get a better explanation from her. I was very interested to see what numbers she chose for her trial and error. I would also like to know what numbers and equations she used to work with the trial and error method. Perhaps you could ask her specific questions like these and any you think of to get her to explain herself more.” In her revision, Melissa simply adds the questions provided in the feedback, making minor changes to the suggestion: “I was very interested to see what numbers you chose for the trial and error. I would also like to know what numbers and equations you used to work with the trial and error method.” Melissa merely changes the “she” (regarding Amelia) from the instructor’s feedback to “you” in her revised response. The rest of the feedback is copied word for word, including the feelings of the instructor: “I was very interested… I would also like to know.” It is as if

Melissa is just the messenger of the feedback; we see no revisions regarding the thoughts and feelings of Melissa.

On the other hand, responses from the PTs characterized as “no change” is fairly self explanatory: this indicates that the PT did not make any changes regarding that particular suggestion or that the PT made changes inappropriate or unrelated to the feedback. When this happened, the mentor would either move on to the next exchange or ask the PT again to fix it. In this situation, it was much more common for the mentor the give the same feedback in a different way, by rewording the suggestion or utilizing a different method: this can be particularly illuminating when the same suggestion is provided in different styles (i.e. implicit at first, and then explicit). It may allow us to understand which feedback is effective and which isn’t; however, complications may arise when the feedback provided the second time is isolated.

PTs are given a rating of “high understanding” when they both make relevant changes regarding the feedback and somehow exhibit a full understanding of the mentor’s suggestions. If the PT makes these “high understanding” revisions, we consider the feedback to be effective. Because our goal is to understand how to encourage these revisions, I carefully examined the characteristics and the coding of the related feedback. An example of high understanding revision comes from an exchange with Stephanie, a low interest PT: the mentor advises Stephanie (regarding her response to Briana) “I am glad that you pointed out that .14 is not 14 minutes, but explain how you came up with .14 is 8 minutes and 24 seconds. (I know it ends up being irrelevant for this problem, but it is good knowledge to have anyway).” In her revision, Stephanie exhibits a full understanding of this suggestion, and adds in parenthesis to her

response: “With cross multiplication we find that .14 out of 1 = 8.4 minutes out of 60 minutes and .4 minutes out of 1 minute = 24 seconds out of 60 seconds.” She clearly understood the mathematics of the mentor’s suggestion, and was able to clearly communicate this to the student. This earned her a coding of “high understanding.” Similarly, Meghan, a high-interest PT, is provided feedback regarding her response to Alana: “When you say ‘he walks 3 ½ mi. plus rides 9 mi. for every 1 hour of travel,’ that makes me think that in 1 hour he goes 12.5 miles (3.5 + 9). If he goes 12.5 miles in one hour, then he goes 50 miles in 4 hours, so the answer to the problem would be 4 hours. Is it possible you mean that ‘in one hour EACH of riding and walking, he goes 3.5 plus 9 hours’?” In her revision, Meghan fixes the hint in her own words: from “Remember he walks 3 ½ mil. Plus rides 9 mi. for every 1 hour of travel” to “Remember he walks 3 ½ mi. in one hour and in another hour he rides 9 mi. So in two hours he walks 3 ½ mi. plus rides 9 mi.” Here, Meghan shows that she understands the feedback by making changes in her own words without copying the mentor’s suggestions.

One note regarding the coding of the responses as high or low understanding is that simple fixes were typically categorized as “high understanding.” In particular, a lot of the structural feedback that was provided related to formatting errors: for example, the mentor instructs Melissa, a low-interest PT, regarding her response to the student Jack, “You (Melissa) have some important information to tell the students but it is somewhat difficult to read. Try to break your mentoring into paragraphs. One paragraph for each suggestion, question or thought. Usually we leave one blank line between each paragraph.” Melissa revises her response, breaking her mentoring into paragraphs for each suggestion, question or thought, and leaves a blank line between each paragraph. Because Melissa addresses all aspects of the

feedback, her revision is categorized as “high understanding” even though the feedback was a bit superficial. This exchange is typical of structural feedback (and also occurs with other types of feedback), and because of the prevalence of this type of feedback, we understood that this approach of coding affects the analysis of the effectiveness of the feedback.

7. Discussion: Educational Psychology

7.1. Scaffolding and the Zone of Proximal Development

As we analyze exchanges between the instructors and the PTs, we notice that certain patterns in the feedback begin to emerge. In particular, there are many attempts by the mentors to encourage PTs to ask questions rather than give explicit directions. This is consistent with the theory behind the zone of proximal development (ZPD) and scaffolding. Students will not be able to progress, or expand their ZPD, when the answers are outright provided to them. Instead, they must be encouraged to grow and learn to do tasks independently. Thus, the instructors try to encourage the use of implicit, instead of explicit feedback.

However, Wood, Brunner, and Ross (1976) describe one issue that we face when categorizing feedback as scaffolding: The term scaffolding is frequently, and inappropriately, applied to any form of verbal interaction that is undertaken with learning as a goal. Scaffolding should be distinguished from feedback that is a one-time, discrete directive or a hard-wired script intended to enhance performance, because it constitutes an ongoing process of providing feedback with the goal of supporting learning. Because the feedback here is occurring via the internet on the Math Forum, it is typically one-time and discrete. In only a few cases does true dialogue between the mentor and the mentee occur. However, scaffolding does provide a model with which we can better understand the feedback provided here, though it may not strictly fall under the category of scaffolding.

7.2. Content-Related and Metacognitive Feedback

Our primary goal was to understand how one might provide mentoring in order to promote content-related and metacognitive feedback. According to the numerical data presented in the chart above, low interest PTs rarely respond with high understanding changes to metacognitive and math-related feedback while high interest PTs respond slightly better. However, in the majority of cases of either group, it seems very difficult to encourage the effective use of the feedback. The following are typical shortcomings when PTs are provided with metacognitive and content-related feedback. Meghan, a high interest PT, is encouraged by the instructor to "help [Davy] see how to get the rest of the way. Why do you think he got 4 instead of 8 for his answer?" In response to this implicit, math-related and metacognitive feedback, Meghan simply tells Davy outright that his answer needs to be doubled: "So if you're finding the answer for one journey like walking or riding, the answer might need to be doubled." This is a common low understanding response, characterized by a lack of guidance and mentoring and consisting of the PT outright telling the student where the mistake is. In another example, the mentor attempts to convey where the student's misunderstanding of the math occurred: "Caroline didn't really understand the 'equal time riding and walking' fact. I think she thought distance was time. That is, of course, not true. You should try to emphasize this to her. Therefore, when she tried 4 hours on horse and got 36 miles, she should have also tried 4 hours walking BECAUSE ZACHARY SPENT AN EQUAL TTTTIIIMMMEEEE WALKING AND RIDING. I put that in caps to emphasize what she did not understand." In this content-related feedback, the mentor seems to desperately be trying to express the mathematical errors.

However, Stephanie responds with low understanding changes: she adds “As soon as you decided that he rode for 4 hours, couldn’t you have known that he walked for four hours? (Because you know ZACHARY SPENT EQUAL TIME RIDING AND WALKING).” Despite the instructor’s attempt, the PT simply copies the mentor’s feedback into her response, in particular the part in all capital letters. These two examples exhibit some of the common low understanding responses from the PTs to metacognitive and/or content-related feedback.

One way that the mentors often attempted to push the PTs into giving effective feedback was to urge them to ask questions instead of simply telling the students the answers. In this case we viewed “learning” as properly being able to ask leading questions. This is particularly important because in order to do this, the PT must first understand the math in the problem as well as what the student is doing wrong, which is what the feedback is related to. Then, the PT must apply the feedback and ask a question, which if properly done reflects learning and effective feedback.

7.3. Asking Questions

The PTs are often given the general suggestion to “ask questions” rather than simply giving the students “explicit directions,” which, similar to implicit feedback, seems to confuse the PTs or paralyze them because of the broadness of the suggestion. Relating to the feedback that Melissa (a low interest PT) provides to Michaela, the mentor instructors Melissa to “get rid of the section where you tell Michaela what to look for in the problem. Instead, what kinds of questions could you ask Michaela to get her to find her mistake? Instead of telling her the problem said equal time not distance, find a way to get her to look for that information

herself.” In response to this general statement to ask questions, Melissa removes the section reading “You may want to break up the data in maybe a chart form if that works best, or set it up algebra style and come up with variables. There are many ways at looking at this problem juts figure out what works best for you.” She does not ask any questions to direct Michaela to find her mistake; she does not find a way to help Michaela look for the information that the problem said equal time, not distance. In particular, we note that Melissa responds to the specific feedback regarding the deletion of a section. However, Melissa seems unable to implement the general feedback instructing her to ask leading questions.

On the other hand, when Melissa (a low interest PT) is provided with a set of sample questions, she simply copies the provided questions into her revisions. The mentor instructs Melissa to “ask for more reflection from [Eric]. Some good questions to ask are: What was easy or hard for you about the problem? Why? Is there a way you can check your answer to make sure it is right? Does the answer you got make sense to you? Were you surprised by any of the results? Any other reflection questions you think of would be great to add as well.” In her revision, Melissa simply includes these exact questions suggested in the feedback, tacked onto the end of her response: “Here are some other questions I have for you: What was easy or hard for you about the problem? Why? Is there a way you can check your answer to make sure it is right? Does the answer you got make sense to you? Were you surprised by any of the results?” Interestingly, Melissa had included a similar question in her original response: “I think that in every instance there is always room for some sort of improvement at solving problems, so I am going to make a challenge for you try a new method called the verification method that means can you solve the problem another way or check your work?” Even though some of the

questions provided by the mentor were redundant with this question, Melissa chose to delete her suggestion and include the mentor's questions verbatim. This seems to suggest that perhaps Melissa lacks confidence in her work or she is not bothering to spend the time to understand the feedback she is given. Thus, as a low interest PT, we see that it is both ineffective to broadly suggest that he or she ask questions and ineffective to provide a set of specific questions. Interestingly, this seems to be the two predominant forms of feedback relating to questions. It would be interesting to see if a hybrid of these two approaches would be effective in encouraging PTs to ask questions instead of giving explicit direction. This would be an area for future research.

7.4. Implicit Feedback

I found that there were very few instances in which implicit feedback seemed effective for either the high interest or low interest PTs. It seems that this type of feedback is too vague and that Melissa, a low interest PT, in particular did not respond well. The mentor provides Melissa with the following feedback: "Katy definitely had some troubles understanding the problem but we want her to figure out why. What are some questions you could ask her to help her start thinking about where she went wrong in the problem? I really would like to see if she can figure out what went wrong and re-read the problem herself without you telling her the parts she needs." In particular, I note that the mentor does not provide the PT with any sample questions: instead, she simply gives the broad suggestion to help Katy figure out where she went wrong. Perhaps it is not surprising that Melissa does not include any questions (regarding this specific feedback) in her revisions.

Similarly, regarding Melissa's response to Maddy, the mentor provides the following feedback: "I think you also are right in that she needs to explain a lot more. Could you give her some questions that direct in her where to start to clarify her response?" Again the instructor simply directs Melissa to asking questions, but does not provide any specific instruction. In her revision, Melissa adds a couple unrelated reflection questions that do not relate in any way to Maddy's specific problem solving or the math in the problem: "Was this problem difficult or easy for you? Do you think there is more than one way at solving this problem?"

It seems that implicit feedback is similarly ineffective for high interest PTs, as well as the low interest PTs. There seem to be a comparable response to the low interest PTs when Meghan (high interest PT) revises her feedback to the student Davy. She is instructed to: "Help him see how to get the rest of the way. Why do you think he got 4 instead of 8 for his answer?" Again, the mentor provides little explicit direction in this comment and instead cues the PT to potential areas of improvement. In her revision, Meghan simply tells Davy that his answer must be doubled: "So if you're finding the answer for one journey like walking or riding, the answer might need to be doubled." Her response to "helping [Davy] see how to get the rest of the way" is similar to the revisions of the low interest PTs. It seems as if Meghan does not think about why he gets 4 instead of 8 for his answer, as hinted.

Similarly, when Meghan is instructed: "When you ask them to reflect, is there a way you could model for them what some more kinds of reflection are, rather than telling them to reflect?" (relating to the submission of the SuperGeeks), she simply adds an example of reflection and re-does the students' work: "For example, 9 mph x 4 hrs = 36 mi, 3 ½ mph x 4 hrs = 14 mi, and so on." In the same thread, the mentor tells Meghan: "I really liked your comment

about units cancelling out, because it shows the mathematical importance of labeling units. Is there a way you could ask a question about units based on that?" In her revision, Meghan simply rephrases her original response into a question with superficial changes: "You should label the hours in your table of answers so a mentor can see that the hours cancel out and you forgot to use units in your conclusion" to "Is there any way you could label the hours in your table of answers so a mentor can see that the hours cancel out or use units in your conclusion?"

Thus, we see that in response to implicit feedback, both low interest and high interest PTs seem to be unable to provide effective revisions. It appears that implicit feedback is more difficult to grasp with a high understanding. Because the nature of implicit feedback, as outlined by Hadwin and Winne, include cues, hints, and tips without explicitly directing or instructing students, the PTs seem to be unable to utilize the feedback because it is so broad. This is contrary to my hypothesis, as I believed that implicit feedback would be effective for the high interest PTs because they have a deeper understanding of the mathematics of the problem. Furthermore, implicit feedback seems to be as detrimental, if not more, as explicit feedback for the low interest PTs. In particular, we see that PTs often directly copy the questions provided by the mentor; however, when Melissa is simply instructed to include questions without any sample ones, she is unable, or chooses not to, include any at all. Interestingly, this lack of response to broad feedback seems to mirror the revisions of the PTs when encouraged to ask questions rather than give explicit direction.

Furthermore, in the exchange between Meghan, a high interest PT, and an instructor, we see that Meghan revises her response only when given explicit directions. The mentor initially provides Meghan with implicit feedback, asking: "Are you sure he goes 9 miles and 3.5

miles in one hour, or is it in one hour each of riding and walking (two hours total)?" Despite the hint, Meghan does not do any revision. The instructor then explicitly tells Meghan: "When you say 'he walks 3.5 miles plus rides 9 miles for every 1 hour of travel,' that makes me think that in 1 hour he goes 12.5 miles (3.5 + 9). If he goes 12.5 miles in one hour, then he goes 50 miles in 4 hours, so the answer to the problem would be 4 hours. Is it possible you mean that 'in one hour EACH of riding and walking, he goes 3.5 plus 9 hours'?" This explicit feedback then pushes Meghan to revise her response to Alana (the student). This is particularly interesting because the same hint is provided in two different styles, and the PT responds only to one of them.

7.5. Explicit Feedback with Rationale

I hypothesized that if a PT is provided with a rationale for the mentor's feedback, the feedback would be more effective because the PT would have a deeper understanding of the mentor's thought process. Instead of simply being told what to do, they are pushed to understand the reasoning behind the feedback, and if it is solid reasoning, then the feedback should be more effective. After coding the data and examining the ten instances where a rationale was provided along with the feedback, it seems that the explanation is in fact more effective. In every exchange where a rationale was provided, the PT responded with some sort of revision (i.e. there was never a response of "no change" to feedback that is explicit with a rationale). Furthermore, in the majority of cases (6 out of 10), the revisions made by the PTs were coded as "high understanding." Although it is a small sample size, these facts suggest that something about providing a rationale along with explicit feedback is effective.

First, we note that in both instances where a rationale is provided with structural feedback, the PT's response was characterized as "high understanding." Melissa, a low-interest PT, is suggested regarding her response to the student Jack: "You (Melissa) have some important information to tell the students but it is somewhat difficult to read. Try to break your mentoring into paragraphs. One paragraph for each suggestion, question or thought. Usually we leave one blank line between each paragraph." Instead of simply instructing Melissa to break the mentoring into paragraphs, the mentor explains that the reasoning behind this request is because the mentoring is difficult to read. In response, Melissa changes the formatting, breaking the mentoring into paragraphs and inserting a blank line between each paragraph. She addresses every issue that the mentor brings up. In the second case, Avrila, a high-interest PT is instructed regarding her response to Tim: "First and foremost, you accidentally erased their response. They need to know what they said. I have included their response at the end of this message. You can cut and paste it." In her revision, Avrila addresses these issues and fixes the formatting. However, it is important to realize that the effectiveness of a rationale cannot be judged simply on these two cases. In particular, many of the responses to structural feedback are coded as high understanding, simply by the nature of structural feedback (as discussed earlier). Thus, we focus on the other instances in which mentors provide a rationale along with explicit feedback.

Furthermore, the other specific exchanges regarding explicit feedback with a rationale also support the numerical analysis that seems to suggest that providing an explanation increases the effectiveness of the feedback. In the following exchange with Avrila, a high-interest PT, the instructor suggests (regarding the student Riley): "The first paragraph in this

one needs to be revised. Starting out with 'One more thing,' sounds like you are being really nitpicky and from the students' perspective he might get the impression he will never get it good enough and give up." In her revision, Avrila changes "there's one more thing I can see that you could improve on" to "I just want to point out one more possible area for improvement in case you'd like to try for a few more points. You're already doing great – this is for if you want a puzzle to figure out." Similarly, the instructor provides Meghan, a high-interest PT with the following feedback regarding the student Luke: "I think it might help to focus a little more on the positive things Luke has done here, since his thinking is really great. Hearing specific things he did well will make him prouder and might help him do them again in the future." In her revision, Meghan adds the following relevant praise: "It really helped me to know you used the guess and check method by letting me know that you tried another number before you told me the right one." In both cases, the PT exhibits a high understanding of the feedback, not only making the necessary adjustments but also effectively applying the mentor's suggestions to the response.

However, there is one situation in which providing a rationale still seems to be ineffective: in the case of the low-interest PTs, we see similar patterns as exhibited with other types of feedback. The mentor instructs Melissa, a low interest PT, regarding the student Jack's response: "I would like you to ask Jack more questions rather than give him directions. For example, instead of telling Jack that he needs more explanation, you could give him a question about a part of his response that is not clear. This way you let Jack do the thinking on how to clarify his response by giving him a nudge in the right direction." In her revision, Melissa adds a few reflection questions that seem to be out of context and unrelated to the feedback: "A

question I also have for you was this problem easy or difficult for you? Was your first approach at solving this problem correct?" This is similar to the responses to both implicit feedback and explicit feedback without a rationale: we saw earlier that PTs, in response to feedback urging them to ask questions rather than give explicit direction, often either copied and pasted the mentor's questions or added irrelevant reflection questions. This seems to be a similar case with Melissa, a low-interest PT. Somehow, the effectiveness of a rationale does not seem to transfer in this case.

8. Discussion: Metaphors

When analyzing the feedback provided by mentors, we were able to identify three basic types of metaphors that occurred consistently: *Ideas are objects*; *knowledge as a pathway*; and *seeing as understanding* (i.e. *to see is to know*). In the metaphor *knowledge as a pathway*, the PT is seen as someone who guides the student along a pathway, clarifying (here is an example of *seeing/clarity as understanding*) the issue at hand. The PTs are encouraged to "nudge" the student along or "push" them in the right direction. For example, Melissa, a low interest PT, is provided with the following feedback regarding the student Davy: "My suggestion here is that you try to work with the strategy Davy seems to be using, rather than going to a chart. I think his strategy could be effective, if you could help him *see how* to get the *rest of the way*." In this example, the instructor uses both the metaphors *knowledge as a pathway* when he or she says "to get *the rest of the way*" and utilizes *seeing as understanding* when he or she encourages Melissa to "help him *see.*" Similarly, Melissa is encouraged (in her response to the student Michaela) "I would like you to get rid of the section where you tell Michaela *what to look for* in the problem. Instead, what kinds of questions could you ask Michaela to get her to *find her mistake*. Instead of telling her the problem said equal time not distance, find a way to get her *to look for* that information herself." Here we see another example of *seeing as understanding* when Melissa is encouraged to get Michaela *to look for* information. There is also an example of the metaphor *ideas are objects*: The mentor advises Melissa to help Michaela *find her mistake*. Interestingly, we found that some metaphors seem to be more common in certain types of feedback.

8.1. Metaphors in Content-Related Feedback

Because this problem has to do with time (the time *spent* on horseback and the time *spent* on foot), we commonly see the metaphor *time is money/ time is a limited resource/ time is a valuable commodity* in math-related feedback. By the very nature of this problem, mentors often use this metaphor in order to relay the information that the PTs are missing in the math. For example, regarding the response Stephanie, a low-interest PT, provided to the student Caroline, the instructor provides the following feedback: "Caroline didn't really understand the 'equal time riding and walking' fact. I think she thought distance was time. That is, of course, not true. You should try to emphasize this to her. Therefore, when she tried 4 hours on horse and got 36 miles, she should have also tried 4 hours walking BECAUSE ZACHARY *SPENT AN EQUAL TTTTIIIMMMEEEE* WALKING AND RIDING. I put that in caps to emphasize what she did not understand." The mentor emphasizes the *time spent* and with this utilizes the metaphor *time is money/ time is a limited resource/ time is a valuable commodity*. This is a characteristic response by mentors when the PT or the student has a difficult time grasping the "equal time" fact.

8.2. Metaphors in Implicit Feedback

Interestingly, we saw a prevalence of the metaphor *knowledge as a pathway* in instances where the mentor provides implicit feedback. In fact, there seems to be almost an exclusive use of this metaphor; this is interesting because we have just examined the effectiveness, or lack thereof, of implicit feedback. In the feedback provided to Melissa, a low

interest PT, regarding the student Katy, the mentor utilizes this metaphor: “Katy definitely had some troubles understanding the problem but we want her to figure out why. What are some questions you could ask her to help her start thinking about *where she went wrong* in the problem? I really would like to see if she can figure out what went wrong and re-read the problem herself without you telling her the parts she needs.” This is a typical exchange including the metaphor *knowledge as a pathway*. There is a beginning to this pathway, as illustrated by the mentor’s suggestions to Melissa, regarding the student Maddy: “I think you also are right in that she needs to explain a lot more. Could you give her some questions that *direct* her *where to start* to clarify her response?” There is also an end to this path (mentor’s suggestions to Meghan, a high interest PT, regarding Davy): “Help him see how to *get the rest of the way*. Why do you think he got 4 instead of 8 for his answer?” Finally, students can be *pushed* along the path (Meghan, high interest PT; Elisabeth, student) and we can find out how the student Briana (Stephanie, low interest PT) “*went from* 9 hrs 84 min *to* 10 hrs 24 min.”

However, we do see one instance in which the metaphor *clarity is understanding* is used in implicit feedback. In his or her suggestions to Meghan, a high interest PT, regarding the student Elisabeth, the mentor gives the following feedback: ““Could you *clarify* what you mean by ‘Did you get ‘8 hours’ in the equations or did you have to times a multiple by 2 or another number to get it? I wasn’t sure.” However, this use of *clarity is understanding* seems to be an anomaly; it does indeed seem to be the case that implicit feedback is dominated by the metaphor *knowledge as a pathway*.

8.3. Metaphors in Explicit Feedback with Rationale

While implicit feedback seems to be dominated by the metaphor *knowledge as a pathway*, explicit feedback with a rationale seems to contain predominately *clarity is understanding*. Again, we see a few instances of *knowledge as a pathway*, which seem to be the exception, not the rule. In the feedback provided to Melissa, a low interest PT, regarding her student Jack, we see an example of both metaphors: "I would like you to ask Jack more questions rather than *give him directions*. For example, instead of telling Jack that he needs more explanation, you could give him a question about a part of his response that is not *clear*. This way you let Jack do the thinking on how to *clarify* his response by giving him *a nudge in the right direction*." Melissa is encouraged to *give directions* and *nudge Jack in the right direction*, utilizing the metaphor of a pathway. In addition, Melissa is told that her response is not *clear* and that she could *clarify* Jack's response. This is a characteristic use of *clarity is understanding* in explicit feedback with a rationale. In another example, Meghan, a high interest PT, is provided with the following feedback regarding her response to the student Adam: "Remember how in the first thread you reflected what the SuperGeeks understood back to them? I think that would be really good to do here, when you talk about their two strategies. If you named or described those strategies, Austin could *see* what he or she was doing, which helps him/her learn to reflect."

Interestingly, the pattern observed in explicit feedback with a rationale (a dominant use of *clarity is understanding* with minimal use of *knowledge as a pathway*) is the exact opposite of the pattern observed in implicit feedback (a near exclusive use of *knowledge as a pathway* and a minimal use of *clarity is understanding*). Because we speculated earlier that explicit

feedback with a rationale might be more effective than implicit feedback, it is tempting to conclude that the use of metaphors may play a role in this difference. I wondered whether there is any causation or if this is simply a correlation, or perhaps if there is a minimal effect of these metaphors. This could definitely be included in a future study, isolating the use of metaphors to understand whether there truly is an effect.

9. Areas for future research

One of the primary issues of this study is its speculative nature. In a future study, I would like to analyze a larger sample size, which would allow more certainty with the patterns and numerical analysis. Furthermore, I was unable to find any causation between variables; it might be useful to isolate variables and have the instructors specifically utilize certain types or content of feedback in order to establish causality. Furthermore, all of the PTs that we have been analyzing have been female; it might be interesting or useful to note whether there is a difference between genders.

The data suggests that explicit feedback with a rationale is the most effective form of feedback and is characterized by the use of the metaphor *clarity is understanding*, while implicit feedback is the least effective and its use is accompanied by the metaphor *knowledge as a pathway*. The research suggests that there may be something inherent about both implicit feedback and explicit feedback with a rationale that pushes us to conceptualize them with a particular metaphor. In particular, the concepts that govern our thoughts also govern what we do in everyday life, including providing feedback. Thus, I believe it will be useful to conduct future research to help us understand the association between the types of feedback and its accompanying metaphor in greater detail; this will help us understand the basis of this association and will also have practical implications in the schools.

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