# A few thoughts on the meaning of few and a few

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#### 1 Introduction

While there has been a great deal of work done on English determiners in the past two decades, there is a curious lack of scholarship that focuses the determiners *few* and *a few*. Some of this can be attributed to more comprehensive approaches taken by, for example, Barwise and Cooper (1981) and Keenan and Stavi (1986), that seek to make more general observations about determiners in natural languages. That work provided a superb framework in which future work on determiners could be done. Unfortunately, there have been no in-depth analyses on the meaning *few* and *a few*. What follows is an attempt to characterize both the semantics and pragmatics of the determiners *a few* and *few*.

Section 2 is an examination of the truth conditions of *a few* and *few*. The evidence presented there argues for two senses of *few*, a proportional sense and a cardinal sense. Other semantic properties, including monotonicity, determiner strength, and pronominal reference are explored in section 3. The pragmatics of *a few* and *few* are examined in section 4. One subsection therein discusses scalar implicature. The other looks at the effects of tense on the strength of implicature. Section 5 returns to the semantics of *few* to explore some difficult issues more fully.

# 1.1 An overview of generalized quantifiers

B&C provides the standard account of noun phrases as generalized quantifiers that will be assumed for the rest of the paper. A brief overview of the relevant aspects of that account follows.

The quantifiers of predicate logic are inadequate for capturing natural language quantifiers. If quantifiers did correspond to determiners, it would be possible to define 1) from 2).

<sup>&</sup>lt;sup>1</sup> Hereafter, B&C and K&S, respectively.

- (1) More than half of John's arrows...
- (2) More than half of all things...

As such a definition is impossible, quantifiers must correspond to complete NPs rather than just the determiner portion of the NP. In other words, the syntactic category NP is analogous to the semantic category of quantifiers. The following trees make this analogy clear:

(3) Most dogs like dog food

quantifier returns a truth value of 1 (true). For example,

is true just in case there are more dogs that like dog food than dogs that do not like dog food. Therefore, the denotation of 'most dogs' is the set X such that a simple majority of individuals that have the property of being a dog are contained in X.

What remains to be shown, then, is the role that determiners such as *most* and *some* play in contributing to the denotation of the quantifier as a whole. Since quantifiers are interpreted as families of sets and common nouns (the other component of NPs, as seen above) are interpreted as sets of entities, determiners can be seen as functions from sets of things to families of sets.

To sum up, those tokens which may superficially appear to be quantifiers (*every*, *most*, *some*, etc.) are actually functions from set expressions to quantifiers.

#### 2 Truth Conditions

The truth conditions of sentences involving the determiners *a few* and *few* will first be examined informally, then described more formally using the notation of Keenan (1996).

#### 2.1 A few

#### 2.1.1 Informal discussion

Suppose a group of students ask their professor how the class (of, say, 15 students) did on a recent exam. The professor responds with (4).

## (4) A few students passed.

When will this statement be considered true? Let's examine a few possibilities:

Situation 1: Zero students passed the exam.

In this case, the professor's statement is false.

Situation 2: One student passed the exam.

The professor's statement is false in this situation; surely the students would protest and accuse the professor of deception if it turned out only one of them had passed the exam.

Situation 3: Two students passed the exam.

In this case, the professor's statement is true.

Situation 4: Twelve students passed the exam.

At first blush, it might appear that the professor's statement is false; it seems that more than just a few students passed the exam.

(5) A few students passed. In fact, twelve did.

Since (5) is not a contradiction, *a few* does not establish an upper limit on the number of students that passed the exam. The intuition that the professor's statement is false is based on conversational implicature, a topic discussed below in section 4.1.

Situation 5: Fifteen students in the class passed the exam.

The professor's statement is once again true. Discussion of Situation 4 suggested that *a few* does not establish either an upper limit. Situation 5 confirms that *a few* establishes no upper limit, not even when all individuals that fulfill the restriction also fulfill the nuclear scope.

The preceding discussion reveals that for sentences with *a few* to be true, at least two entities must have the property denoted by the common noun following *a few* and fulfill the predicate of the sentence. A formal statement of this observation follows.

## 2.1.2 Formal definition

(6) Let A and B be subsets of U, the universe. Assuming standard set notation,

(A FEW) (A) (B) = T iff 
$$|A \cap B| \ge 2$$

In other words, if the intersection of the sets A and B has at least two members, sentences of the form "A few A B," (allowing for proper syntactic form)<sup>2</sup> will be true. An example follows.

 $\underline{\mathbf{M}^{1}}$ 

U:

 ${a, b, c, d, e, f, g, h, i, j, k}$ 

STUDENT:

 $\{a, c, e, g, i, k\}$ 

**VEGETARIAN:** 

 $\{a, c\}$ 

(STUDENT  $\cap$  VEGETARIAN) = {a, c}

 $|\{a, c\}| = 2$ 

Therefore, relative to M<sup>1</sup>, "A few students are vegetarians," is true.

 $<sup>^{2}</sup>$  See Appendix A for a listing of the possible forms of such sentences.

2.2 Informal discussion of few

Unfortunately, the truth conditions for *few* are not as easily determined as those for *a few*. Discussion begins with circumstances similar to those discussed above, with one modification: the professor now responds with (7).

(7) Few students passed.

The truth of this statement will be examined for each of the five situations described above.

In Situation 1, zero students passed the exam. It has been argued that *few* makes an existential claim (Foster 1985: 79). If this argument were correct, (7) would be false in Situation 1. It can be shown, however, that *few* does not make an existential claim. One piece of evidence against this claim is the fact that a discourse such as "Few students passed the exam. In fact, none of them did" is not a contradiction. Further evidence is seen in (8) and (9).

- (8) Few, if any, teams are playing at a higher level than USC.<sup>3</sup>
- (9) If any teams are playing at a higher level than USC, the number of teams playing at a higher level than USC is small.<sup>4</sup>

Suppose that no teams are playing better than USC. (9) is true under these circumstances, as whenever the antecedent of a conditional is false, the conditional as a whole is false. Since (9) is a paraphrase of (8) with identical truth conditions, (8) is also true under these circumstances. Removing the "if any" portion of (8) does not change the truth conditions of the sentence. The "if any" makes explicit the non-existential possibility of few.

The truth of (7) with respect to Situation 1 is exactly analogous to the football situation just described. Therefore, (7) is also true. Few-sentences do not require the existence of an individual that fulfills both the restriction and the nuclear scope to be true. In other words, few

<sup>4</sup> What constitutes "small" is a slippery question and is discussed below in section 5.1.

<sup>&</sup>lt;sup>3</sup> This example is drawn, with some modification, from an article that appeared on espn.com on 2 November 2003.

does not make an existential claim. The intuition of many hearers of (7) that it is actually false with respect to Situation 1 can be attributed to scalar implicature, as discussed in section 4.1.

In Situation 2, one student passed the exam. The professor's statement is true. Again, one might argue that at least two students had to have passed the exam for the professor's statement to be true. However, since, "Few students passed. In fact, only one did," is not a contradiction, this intuition cannot be attributed to the truth conditions associated with *few*. This conclusion is exactly analogous to the discussion with *a few* in section 2.1.1, which concluded that use of *a few* does not preclude the truth of *all*.

In Situation 3, two students passed the exam. The professor's statement is true here.

In Situation 4, twelve students passed the exam. The truth of the professor's statement is a bit unclear in this situation. In this case, the truth of the sentence seems at least somewhat dependent on someone's expectations or desires regarding the performance of the class on the exam. Under "normal" circumstances, this sentence would be interpreted as false. There might, however, be circumstances under which the statement is construed as true (perhaps the professor thought the exam was easy and expected all the students to get A's).<sup>5</sup>

In Situation 5, fifteen students (out of fifteen) passed the exam. 'Few students passed; in fact, they all did' appears to be a contradiction. Therefore, the professor's statement is false.

Based on these observations, the only definitive observation that can be made, it seems, is that *few* may be equivalent to "not all". This conclusion, however, contrasts with the intuition that *few* has an equivalent meaning to *not many*. In addition, it seems possible that sentences (10) and (11) below appear to falsify the conclusion that *few* means, roughly, "not all."

(10) Few California condors are living.

<sup>&</sup>lt;sup>5</sup> To be sure, imagining a situation in which 12 out of 15 counts as 'few' is difficult. Such situations are, however, plausible. The rather frustrating "slipperiness" of *few* is discussed below in section 5.1.

(11) Few truly great books are in written form.

Suppose there are roughly 200 California condors remaining. All of them are alive.

Nonetheless, it seems as if (10) should be considered true. This directly contradicts the earlier claim that *few* is equivalent in its truth conditions to "not all". However, all is not lost for the "not all" analysis of *few*. If we expand the model to include all California condors that have ever existed (most of which have now died and decomposed), it is no longer the case that all California condors are alive. Thus, under the informal definition of *few* as "not all," (10) is true, just as our intuitions would predict.

Things get considerably hairier, however, with (11). In this case, no matter what the model, all "truly great books" are in written form, since being in written form is an inherent property of books. Yet, assuming there were some objective criterion for what constituted a truly great book, it seems entirely plausible that a speaker who thought there should be more truly great books would declare (11). This is rather damning evidence against the idea that *few* means "not all."

This leaves us with the intuition that *few* means, roughly, "not many." However, without a clear definition of *many*, a comparative definition for *few* is practically useless. One approach to follow would be to attempt to determine an upper limit on *few* based on cardinality. Based on the discussion above, we know that, at the very least, such an upper limit would have to be above two. Let's go higher by several degrees of magnitude. Suppose that in the fictional city of Florin (population: 500,000), there are 1,000 people who are unable to read. What is the truth of (12)?

- (12) Few Florinese are illiterate.
- (12) seems to be true. Therefore, an absolute upper limit on few must be above 1,000.

Let's go higher still. Suppose there are 250 U.S. citizens and 1 million of them speak French. Is (13) true?

- (13) Few Americans speak French.
- (13) also seems to be true. Thus, the upper limit of few must be above 1 million.

It should be apparent that it is very difficult (if not impossible) to establish an upper cardinal limit on few that holds in all situations. The truth of (12) and (13) seem quite dependent on the number of Florinese and the number of Americans, respectively. This suggests that a "proportional" approach to few would be more fruitful. However, we've already established that, in some cases, sentences in the form of "Few A are B" can be true in the case that all A are B (see the discussion on (11) and truly great books above). We seem to be at an impasse.

There is, however, a solution. Returning to (13), there also seems to be a good argument that (13) is actually false: a million Americans speak French; that's a huge number, certainly more than could ever be called 'few.' It is important to note here that this disagreement regarding the truth of (13) is not based on different expectations of how many Americans speak French, but rather whether the truth of (13) is dependent on the number of Americans who speak French or the proportions of Americans who speak French.

It should be apparent, therefore, that *few* actually has two senses. Aldrige (1982: 242) argues against absolute readings of determiners like *many*, but the observation that the truth of (13) can be dependent on the absolute number of Americans who speak French supports the existence of an absolute or cardinal sense of *few*. Even stronger evidence for the existence of a cardinal (i.e. non-proportional) sense of *few* is the evidence suggested by (11), namely that some *few*-sentences make assertions about the size of a set. One of these senses corresponds to the position that (13) is true on the grounds that a small percentage of Americans speak French. The

second sense corresponds to the argument that (13) is false since a large number (a million!) of Americans speak French. The two proposed senses of few, then, are the proportional  $(few_1)$  and the cardinal  $(few_2)$ . When appropriate, the two senses of few will from here on be treated separately.

# 2.3 Few<sub>1</sub>

#### 2.3.1 Informal discussion

The 'not all' restriction on few discussed above applies only to this, the proportional sense. This restriction can be explained as follows:  $few_1$ -sentences are true just in case the percentage of entities fulfilling the restriction who also fulfill the nuclear scope is small. When all the entities that fulfill the restriction also fulfill the nuclear scope, that percentage is 100%, and there is no way in which 100% can be understood as small.

Unfortunately, it turns out to be impossible to stipulate unchanging truth conditions for  $few_1$ , since what constitutes "small" is context- and speaker-dependent. This phenomenon (rare among determiners) is discussed below in section 5.1. The only statement that can be formalized with universal application is the "falsity condition" for  $few_1$ .

### 2.3.2 Formal definition

(14) Let A and B be subsets of U, the universe. Assuming standard set notation,

**(FEW<sub>1</sub>)** (A) (B) = F if 
$$|A| \ge 1$$
 and  $A \subset B$ .

In other words, if A has at least one member<sup>6</sup> and all of the members of A are also members of B, sentences of the form "A few As B"<sup>7</sup> will be false. An example follows.

 $\underline{\mathbf{M}^2}$ 

<sup>&</sup>lt;sup>6</sup> This condition arises to account for the fact that *few*-sentences are always true when no individuals fulfill the restriction and the nuclear scope.

<sup>&</sup>lt;sup>7</sup> See Appendix A for a more comprehensive listing of the possible forms of *few*-sentences.

U:

 ${a, b, c, d, e, f, g, h, i, j, k}$ 

STUDENT:

 ${a, c, e, g, i, k}$ 

**VEGETARIAN:** 

 ${a, c, e, g, i, k}$ 

(15) Few students are vegetarians.

The set STUDENT has six members. All members of the set STUDENT are also members of the set VEGETARIAN. Therefore, relative to  $M^2$ , (15) is false when  $few_1$  is used. The following example shows that the condition described in (14) is not the only condition in

which few-sentences can be false.

 $\underline{\mathbf{M}^3}$ 

U:

 ${a, b, c, d, e, f, g, h, i, j, k}$ 

STUDENT:

 $\{a, g, i, k\}$ 

**VEGETARIAN:** 

 ${a, c, e, i, k}$ 

In M<sup>3</sup>, the set STUDENT has four members (and thus satisfies the first part of the falsity condition for *few*-sentences). However, not all members of the set STUDENT are members of the set VEGETARIAN (namely, g). Therefore, relative to M<sup>3</sup>, (15) is potentially true; in other words, the falsity of this statement is not guaranteed. However, it is still possible that this sentence is false. That falsity would come about due to the proportion of students who were also vegetarians being considered too large compared to the expectation of the speaker. Judgments of this sort are context-dependent and will be discussed later.

It is quite difficult to formalize the precise conditions under which  $few_1$ -sentences are true. This difficult arises from the "slipperiness" of few that has been mentioned above and will be discussed in section 5.1. The closest possible approximation of the formal truth conditions for  $few_1$  follows in (16).

(16) Let A and B be subsets of U, the universe. Assuming standard set notation,  $(\mathbf{FEW_1}) (A) (B) = T \text{ iff } (|A \cap B| / |A|) < (|A \cap B|_{exp.} / |A|) \text{ where } |A \cap B|_{exp.} \text{ is the expected size of the intersection of sets A and B.}$ 

#### 2.4 Few<sub>2</sub>

There is little to add to the previous discussions of the truth conditions of  $few_2$ . Sentences with  $few_2$  are true when the number of individuals satisfying both the restriction and the nuclear scope is lower than expected. This truth condition is "formalized" in (17).

# 2.4.1 Formal definition

(17) Let A and B be subsets of U, the universe. Assuming standard set notation,  $(\mathbf{FEW_2}) (A) (B) = T \text{ iff } |A \cap B| < |A \cap B|_{\text{exp.}} \text{ where } |A \cap B|_{\text{exp.}} \text{ is the expected size of the intersection of sets A and B.}$ 

# 3 Semantic Properties

Besides the truth conditions already discussed, few and a few (like all determiners) have a variety of semantic properties. Discussion of these properties can be found in B&C, K&S, and Keenan (2001). The following section explores those properties using the diagnostics suggested by those sources in an attempt to further characterize the meanings associated with few and a few.

#### 3.1 Conservativity

K&S define conservative functions as follows:

(18) A function f is conservative iff for all properties  $p, q p \in f(q)$  iff  $(p \land q) \in f(q)$ 

Conservativity can be understood as stipulating that elements in the domain that do not satisfy the restriction are not relevant to determining the truth value of the sentence. K&S propose the following test for checking on whether a determiner (*Det*) is conservative.

- (19a) Det doctor is a vegetarian.
- (19b) Det doctor is both a doctor and a vegetarian.

If (19a) and (19b) always have the same truth condition, *Det* is conservative. K&S propose that the vast majority of (if not all) natural language determiners are conservative (275-276).

#### 3.1.1 A few

- (20a) A few doctors are vegetarians.
- (20b) A few doctors are both doctors and vegetarians.

The two sentences in (20) will always have identical truth values. If there are two or more doctors that are vegetarians, (20a) will be true. (20b) has the same truth condition. Therefore, *a few* is conservative.

#### 3.1.2 Few

- (21a) Few doctors are vegetarians.
- (21b) Few doctors are both doctors and vegetarians.

The two sentences in (21) will always have identical truth values. If, relative to a particular model, the number of doctors who are also vegetarians is "small" enough to count as "few," (21a) will be true, as will (21b). Otherwise, both (21a) and (21b) will be false. Therefore, *few* is conservative.

3.2 The Strong/Weak Distinction

B&C divides determiners into "weak" and "strong" (strong is further divided into positive strong and negative strong) (182). Following Milsark (1977), one test in diagnosing weak determiners as "those that create noun-phrases which sound good after *there is* or *there are*. Besides this diagnostic, B&C proposes the following test: form a sentence in the form of (22) below. If the sentence is automatically valid (a tautology), the determiner in question is positive strong. If the sentence is automatically false (a contradiction), the determiner is negative strong. If the truth of the sentence is contingent on the model, the determiner is weak.

(22) D N is a N/are Ns.

#### 3.2.1 A few

Applying the test described above, we get the following sentence.

(23) A few apples are apples.

The validity of this sentence is dependent on the model. In models where there are no apples, (23) evaluates as false. In models where there are at least two apples, (23) evaluates as true. Therefore, *a few* is a weak determiner. This conclusion fits in with the observation that a sentence such as "There are a few apples," sounds fine.

#### 3.2.2 Few<sub>1</sub>

Applying B&C's test for determiner strength, we get (24).

(24) Few apples are apples.

It's a bit harder to get a firm grasp on the validity of this sentence. Recalling the discussion regarding the truth of *few*-sentences is useful here. It was previously concluded that when the cardinality of the set denoted by "few Ns" for a given model is zero, sentences that have "few

Ns" will always be true with respect to that model (recall this is true for both  $few_1$  and  $few_2$ ). Thus, in a model where there are no apples, (24) is true for both senses of few.

With respect to few1 (the proportional sense), a given few-sentence will always be false in the case that the non-empty set denoted by "few Ns" is identical to the set denoted by "all Ns." Since it is the case that all apples are, in fact, apples, (24) will always be false with respect to  $few_1$ . Therefore,  $few_1$  is a weak determiner. This coincides with the observation that "There are few apples" sounds fine.

3.2.3 Few<sub>2</sub>

Sentences with few2 (the cardinal sense) are true in the case that the number of individuals that have both properties A and B is small relative to the expected number of individuals that have both properties. This rather loose definition makes it difficult to evaluate the truth of a sentence like (25) with respect to few2, since (25) appears to be asserting the number of apples that are apples is small, a rather bizarre assertion to make. The best, though still unsatisfying, non-bizarre paraphrase seems to be "There are few apples." The truth of that statement is dependent on how many apples there are and could conceivably be true or false. Therefore, few2 should be classified a weak determiner, but only hesitantly.

3.3 Definteness B&C (183-184) and K&S (296-298) both provide discussions definite determiners. Following K&S:

(26) A determiner Det is definite iff Det is non-trivial and whenever "Det A" is defined, it is the intersection of one or more individuals.

In other words, definite noun phrases (formed by the combination of a definite determiner and a common noun phrase) always pick out a unique set of individuals. B&C demonstrates that all definite determiners are also positive strong (210). Since neither *a few* or *few* are positive strong determiners, it follows that both are indefinite determiners.

#### 3.4 (Right) Monotonicity

B&C (184-5) define monotone quantifiers as follows:

- (27) A quantifier Q is monotone increasing if  $X \in Q$  and  $X \subseteq Y$  implies  $Y \in Q$ .
- (28) A quantifier Q is monotone decreasing if  $X \in Q$  and  $Y \subseteq X$  implies  $Y \in Q$ . These definitions are best understood through an example. The following sentences follow B&C.
  - (29) Some Republicans entered the race early.
  - (30) Some Republicans entered the race.

Translating these sentences into B&C's test for monotonicity, we get "Some Republicans" for Q, "entered the race early" for X, and "entered the race" for Y. (29) asserts that "some Republicans" are elements of "entered the race early." It is obvious that the entities satisfying "entered the race early" form a subset of those satisfying "entered the race. Since (29) quite clearly implicates (30), *some* is a monotone increasing determiner.

# 3.4.1 A few

- (31) A few Republicans entered the race early.
- (32) A few Republicans entered the race.

Following the discussion of *some* above, it becomes apparent that *a few* triggers the same implications. In other words, (31) entails (32)<sup>8</sup>. Therefore, *a few* is a right monotone increasing determiner.

- 3.4.2 Few<sub>1</sub>
  - (33) Few Republicans entered the race early.
  - (34) Few Republicans entered the race.

With respect to  $few_1$ , (33) and (34) can be paraphrased as follows.

- (35) A small percentage of Republicans entered the race early.
- (36) A small percentage of Republicans entered the race.

It is readily apparent that (35) does not entail (36); it might very well be the case that all Republicans entered the race, they simply did not do so early. However, (36) does entail (35), for if a small percentage of Republicans entered the race, it follows that a small percentage of Republicans entered the race early. Therefore,  $few_1$  is a right monotone decreasing determiner.

3.4.3 Few<sub>2</sub>

With respect to  $few_2$ , (33) and (34) can be paraphrased as follows.

- (37) The number of Republicans who entered the race early is small.
- (38) The number of Republicans who entered the race is small.
- (37) does not entail (38). It might be the case that lots of Republicans entered the race, they simply did not do so early. However, (38) does entail (37), for if a small number of Republicans entered the race, it is impossible that a large number of Republicans entered the race early.

Therefore, few2 is a monotone decreasing determiner.

<sup>&</sup>lt;sup>8</sup> This entailment pattern still holds even if all Republicans entered the race. Recall that *a few* sets no upper limit truth conditionally.

3.5 Negative Polarity Items

According to the Ladusaw-Fauconnier Generalization (Ladusaw 1982) negative polarity items occur within arguments of monotonic decreasing functions but not within arguments of monotonic increasing arguments. According to the LFG, then, few (a monotone decreasing determiner) should allow negative polarity items and a few (a monotone increasing determiner) should not.

- (39) Few students here have ever been to Moscow.
- (40) \*A few students here have ever been to Moscow.

The grammaticality judgments for (39) and (40) are as predicted; few and a few fit the LFG.

3.6 Persistence (Left Monotonicity)

Like right monotonicity, diagnosing persistence of determiners relies on entailment patterns. However, the relevant superset-subset relationship for persistence deals with the quantifier itself rather than the predicate of the sentence. B&C (193) define persistence as follows:

- (41) A determiner D is persistent (left monotone increasing) if for all  $A \subseteq B, X \in \|D\|(A)$ entails  $X \in ||D||(B)$ .
- (42) A determiner D is anti-persistent (left monotone decreasing) if for all  $A \subseteq B, X \in$ ||D||(B) entails  $X \in ||D||(A)$ .

Again, this property is best illustrated through example.

- (43) No man that left the party before 10 P.M. went home.
- (44) No man that left the party before 9 P.M. went home.

"No" is D, "man that left the party before 9 P.M." is A, "man that left the party before 10 P.M." is B, and "went home" is X. "Man that left the party before 9 P.M." is a subset of "man that left the party before 10 P.M.," since all the men that left the party before 9:00 also left the party before 10:00. (43) entails (44) since it impossible that no man who left before 10:00 went home and some man who left before 9:00 went home. Therefore, the determiner *no* is anti-persistent.

#### 3.6.1 A few

- (45) A few men that left before 9:00 went home.
- (46) A few men that left before 10:00 went home.

It is readily apparent that (45) entails (46); it is impossible for (45) to be true and (46) false. Since "A few men that left before 9:00" is a subset of "A few men that left before 10:00," a few is a persistent determiner.

# 3.6.2 Few<sub>1</sub>

- (47) Few men that left before 9:00 went home.
- (48) Few men that left before 10:00 went home.

With respect to  $few_1$ , (47) and (48) can be paraphrased as follows.

- (49) A small percentage of the men who left before 9:00 went home.
- (50) A small percentage of the men who left before 10:00 went home.

There is no entailment pattern between (50) and (49). Suppose one hundred men left before 10:00, five of which went home. Only five men left before 9:00, all of whom went home (the very same five just mentioned. Under these circumstances, (50) is true and (49) is false. Suppose, however, that ten men left before 9:00, none of whom went home and that one hundred men left before 10:00, ninety of whom went home (i.e. all but those who left before 9:00).

Under these circumstances, (49) is true and (50) is false. Therefore,  $few_1$  is neither persistent nor anti-persistent.

- 3.6.3 Few<sub>2</sub>
  - With respect to  $few_2$ , (47) and (48) can be paraphrased as follows.
  - (51) The number of men who left before 9:00 and went home is small.
  - (52) The number of men who left before 10:00 and went home is small.

In this case, (52) entails (51). Since the men that left before 10:00 and went home is a superset of the men that left before 9:00 and went home, if the number of men who left before 10:00 and went home is small, it follows that the number of men who left before 9:00 and went home is also small. Therefore, followed few2 is anti-persistent.

B&C's conclusion regarding the persistence of few is a bit muddled. In section 4.7 and Appendix D, they claim that few is anti-persistent (albeit questionably so), while they present few as persistent in Appendix B. This latter conclusion appears to be based on the assumption that few is equivalent to not many. It may be that this confusion arises from their failure to recognize the two senses of few.

3.7 Intersectivity

Keenan (2001) describes the property of intersectivity for a given determiner in terms of what knowledge is need to determine the truth of a sentence with that determiner. For intersective determiners, the only relevant entities are those that fulfill both the restriction and the nuclear scope. For example, some is judged to be intersective, since the truth of (53) is dependent solely on students that are vegetarians; students who aren't vegetarians and nonstudents are irrelevant.

(53) Some students are vegetarians.

A formal definition of intersective determiners follows:

(54) Intersective determiners are those whose denotation D satisfies D(A)(B) = D(X)(Y) whenever  $A \cap B = X \cap Y$ .

Keenan (2001) proposes the following diagnostic for evaluating the intersectivity of determiners: if the truth conditions of (55a) and (55b) are identical, the determiner in question is intersective.

- (55a) Det students are vegetarians.
- (55b) Det individuals are both students and vegetarians.
- 3.7.1 A few
  - (56a) A few students are vegetarians.
  - (56b) A few individuals are both students and vegetarians.

(56a) and (56b) are true in the exact same circumstances: (56a) is true only in the case that there exist at least two students that have the property of being vegetarians. (56b) is true only in the case that there exist at least two individuals have the property of being students (i.e. are students) and have the property of being vegetarians. Therefore, *a few* is intersective.

- 3.7.2 Few<sub>1</sub>
  - (57a) Few students are vegetarians.
  - (57b) Few individuals are both students and vegetarians.
- (57a) is true when the percentage of students that are vegetarians is judged "small" for a particular context. (57b) is true when the percentage of individuals that are students and vegetarians is judged "small" for a particular context. The model below is just one case in which the truth values of (57a) and (57b) are likely to be different.

 $M^4$ 

U:

 ${a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q}$ 

STUDENT:

 $\{a, b, c\}$ 

**VEGETARIAN:** 

 $\{a, b, c\}$ 

With respect to M<sup>4</sup>, (57a) is false; all the students are vegetarians (recall the 'not all' restriction on  $few_1$ ). (57b), however, is true, since only three of the seventeen individuals are both students and vegetarians. Since the truth values of (57a) and (57b) do not necessarily match,  $few_1$  is nonintersective.

3.7.3 Few<sub>2</sub>

Unlike sentences with  $few_1$ , the truth  $few_2$ -sentences is not dependent on the proportion of individuals that fulfill the restriction and nuclear scope relative to those that fulfill the nuclear scope. (57a) is true when the number of students who are vegetarians is small. (57b) is true when the number of individuals who are both students and vegetarians is small. Both (57a) and (57b) are true with respect to M<sup>4</sup>. It turns out that (57a) and (57b) are true in exactly the same circumstances when few2 is used.

All pairs in the form of (57a) and (57b) will, in fact, have the same truth conditions since the truth of sentences with  $few_2$  is dependent on the size of the intersection of properties A and B. In (57a), A corresponds to the property STUDENT and B corresponds to the property VEGETARIAN. The relevant set for determining the truth of (57a), then, is STUDENT \(\cappa\) VEGETARIAN. In (57b), A corresponds to the property INDIVIDUAL, that is the property of existence and B corresponds to the property STUDENT & VEGETARIAN. The set used in determining the truth of (57b) is INDIVIDUAL \(\cappa\) STUDENT & VEGETARIAN. The property STUDENT & VEGETARIAN is nothing more than the property held by members of the

intersection of the property STUDENT and the property VEGETARIAN. Generalizing from this, it can be said that the relevant sets for (57a) and (57b) are  $A \cap B$  and  $C \cap A \cap B$ , respectively (where C represents the property INDIVIDUAL). Since the property INDIVIDUAL is the property of existence, it is a property shared by all entities in the model and is insignificant in differentiating various subsets of the model. What remain as the relevant sets for (57a) and (57b), then, are  $A \cap B$  and  $A \cap B$ . Since these two sets are identical, (57a) and (57b) have identical truth conditions. Therefore,  $few_2$  is intersective.

3.8 Cardinality

Keenan (2001: 3-4) describes cardinality as being a stronger condition than intersectivity. In other words, only determiners that are intersective can be cardinal. The truth value of sentences with cardinal determiners is dependent on the exact number of objects that lie in the intersection of the restriction and the nuclear scope. Cardinality is defined as follows:

(58) A possible Det denotation D is cardinal iff for all properties A,B,X,Y,  $D(A)(B) = D(X)(Y) \text{ if } |A \cap B| = |X \cap Y|.$ 

3.8.1 A few

The discussion of the truth conditions of a few above revealed that sentences with a few were true just in case at least two entities satisfied both the restriction and the nuclear scope. Therefore, a few is a cardinal.

3.8.2 Few<sub>1</sub>

Since  $few_1$  is non-intersective, it is necessarily non-cardinal.

3.8.3 Few<sub>2</sub>

Few<sub>2</sub> was described above as intersective and could, therefore, be cardinal. The slipperiness of few prevents it from meeting the rigorous definition for cardinality described above. Suppose, for example, that ten Americans speak French, and ten people in the world speak fifty languages.

- (59) Few Americans speak French.
- (60) Few people speak fifty languages.
- (59) is certainly true under these circumstances. The truth of (60) is a bit harder to get at, but it seems plausible that (60) could be false; given the expectation that no one speaks fifty languages, ten such people would be an astoundingly high number of people who did so. Then, (FEW2) (AMERICANS) (SPEAK-FRENCH) = T and (FEW2) (PEOPLE) (SPEAK-50-LANGUAGES) = F. |AMERICANS ∩ SPEAK-FRENCH| = |PEOPLE ∩ SPEAK-50-LANGUAGES|. Given the definition of cardinality in (58), few2 is non-cardinal. It seems, however, that few2 is in fact cardinal, but not in the strict sense defined above. Rather, the threshold for few is dependent on context but is nonetheless a cardinal threshold.

3.9 Proportionality

As described by Keenan (2001: 6-7), proportional and intersective determiners are mutually exclusive; an intersective determiner cannot be proportional, and vice versa. Sentences involving proportional determiners rely, unsurprisingly, on the proportion of entities that fulfill the restriction and nuclear scope to those that fulfill the restriction to yield their truth conditions. A definition of proportional determiners follows.

(61) D is proportional iff D(A)(B) = D(X)(Y) whenever  $|A \cap B|/|A| = |X \cap Y|/|X|$ 

In other words, each proportional determiner has a percentage "threshold" at which sentences with that determiner are always true; below that threshold, such sentences are always false.

3.9.1 A few

Since a few has already been shown to be intersective, it follows that it must be nonproportional.

3.9.2 Few<sub>1</sub>

At first glance, since  $few_1$  is apparently non-intersective, it seems likely that  $few_1$  is proportional.9 However, as the following discussion demonstrates, few does not fit the rigorous definition of proportionality given above.

 $M^5$ 

U:

 $\{a,\,b,\,c,\,d,\,e,\,f,\,g,\,h,\,i,\,j,\,k,\,l,\,m,\,n,\,o,\,p,\,q\}$ 

STUDENT:

 ${a, b, c, d, e, f}$ 

PASSED-EXAM:

 $\{a, b, c\}$ 

PROFESSOR:

 ${g, h, i, j, k, l, m, n, o, p}$ 

STINGY:

 $\{g, h, i, j, k\}$ 

- (62) Few students passed the exam.
- (63) Few professors are stingy.

Suppose that, in previous years, all the students have always passed this exam. With respect to M<sup>5</sup>, (62) seems to be true (based on the expectation that all the students would pass the exam) and (63) false. However, the proportion of students who passed is exactly the same as the proportion of professors who are stingy, 50%. Since the definition for proportionality requires

<sup>&</sup>lt;sup>9</sup> Keenan (2001: 5-6) also discusses "co-intersective" or generalized universal determiners. However, since cointersective determiners are described as "basically just the universal quantifier with exceptions phrases," it is clear that few is not co-intersective.

that proportional determiners have an unchanging "threshold" and few evidently lacks this threshold, it seems necessary, intuitions aside, to classify few as non-proportional. It seems that one sense of few is in fact proportional, but not in the strict sense defined above. Rather, the threshold for few is dependent on context.

3.9.3 Few<sub>2</sub>

Since few2 has already been shown to be intersective, it follows that it must be nonproportional.

3.10 Pronominal Reference

The following sentences demonstrate one property of a few and few in discourse.

- (64) [A few students]<sub>i</sub> are in the library. [They]<sub>i</sub>'re studying for an exam.
- (65) \*[Few students]<sub>i</sub> are in the library. [They]<sub>i</sub>'re studying for an exam.
- (64) shows that individuals introduced into discourse in noun phrases using a few can serve as antecedents for pronouns later in the discourse. This is not the case with few. In (65), it is impossible for the they in the second sentence to be co-referenced with the students in the library introduced in the first sentence. This is not to say, however, that pronominal reference is completely "blocked" by few-sentences.
- (66) [Few students] $_k$  are in the library. [They] $_l$ 're studying for an exam somewhere else. This mini-discourse is entirely comprehensible on its own; the referent of they in the second sentence is not left hanging. What's interesting here is what they is co-referenced with. It is not, as discussed above, the few students in the library. Instead, it is those students who are not in the library. In other words, they refers to individuals who have not been explicitly introduced into

the discourse and are, in fact, the "mirrors" of those individuals who have been introduced into the discourse.10

Moxey and Sanford (1993) describes the above phenomenon as "compset reference." The compset is understood to be the complementary set of the set referenced in the first sentence. For example, in sentences (63)-(65), the compset is the set of students who are not in the library. While Moxey and Sanford (1993: 60-62)'s experiments suggest that few sentences can provide "direct" antecedents for pronouns in future sentences, compset reference occurs somewhere between 65% and 90% of the time. Thus, the rule suggested above may be a bit of an overstatement in its excludion of the possibility of few allowing "direct" reference. In typical usage, however, few sentences provide for compset reference in later sentences in the discourse.

4 Pragmatics

There is more to the meanings of few and a few than the truth conditions and semantic properties discussed above. While certain meanings associated few and a few are universal, other meanings rely heavily on contrast and usage. Some of those meanings are discussed below.

4.1 Scalar Implicature

Recall that (4), reproduced below for convenience seemed odd in the case that all students passed the exam.

- (4) A few students the exam.
- (5) was just as odd in the case that no students passed the exam.
  - (5) Few students passed the exam.

<sup>&</sup>lt;sup>10</sup> Bill Ladusaw pointed out this phenomenon to me in a conversation in October 2003.

In spite of this oddness, it was observed that both (4) and (5) were true in the contexts given. How then can we account for the argument that (4) seems to be saying that not all students passed the exam and (5) that at least some students passed the exam?

Horn (1972) builds on the work of Grice (1975) to explain this phenomenon. Grice (1975) posits a number of conversational maxims. The relevant maxims for the current discussion are that of quantity and quality. Quantity has two sub-maxims: 1) Make your contribution as informative as is required (for the current purposes of the exchange) and 2) Do not make your contribution more informative than is required. Quality also has two sub-maxims: 1) Do not say what you believe to be false and 2) Do not say that for which you lack adequate evidence. When these maxims are being followed, a speaker will provide as much information as they deem both necessary and true and the hearer will assume that the information they are receiving is both the appropriate quantity and true. The key feature of Grice's account is that more is meant in conversation than is actually said. When a professor writes a recommendation for a student that goes something like "Dear Sir, Mr. X's command of English is excellent, and his attendance at tutorials has been regular. Yours, etc." (from Grice 1975: 52), all that is said is that the student is competent in English and attends class on a regular basis. What is meant is that the student in question doesn't have much else going for him besides these two facts.

Horn (1972: 59-62) extends this analysis by arguing for the phenomenon of scalar implicature. In this account, a claim Q is weaker than claim P if P entails Q and Q does not entail P. In other words, P provides more information than Q. Use of claim Q implicates the negation of P, since under the maxims of quantity and quality, a speaker provides as much information as they know to be true; if the speaker knew P to be true, they would say P. In terms of quantifiers (or determiners in the present account of *few* and *a few*), "a quantifier q<sub>i</sub>

conversationally implicates that, as far as the speaker knows, no stronger quantifier  $q_j$  could be substituted for  $q_i$ ." Horn goes onto provide two scales of quantifiers, one for positive quantifiers and one for negative quantifiers. They are as follows:

- (67) one ---- some/a few ---- many ---- half ---- most/a majority ---- all/every
- (68) not all ---- not half ---- a minority ---- not many/few ---- no/none<sup>11</sup>

For both scales, each quantifier entails those to the left of it and implicates the negation of those to the right of it.

Scalar implicature explains the oddness of (4) and (5) in the cases that all students passed the exam and no students passed the exam, respectively. With (4), use of *a few* implicates that use of any quantifiers stronger than *a few* (many, most, all) would cause the sentence to be false. In other words, *a few* implicates *not all*. The implicature can be cancelled by adding another sentence following (4), namely "In fact, you all did." In (5), use of *few* implicates that any stronger quantifiers are inappropriate. Since "no students passed the exam" is a stronger statement than "few students passed the exam," use of (5) implicates the falsity of "no students passed the exam." Again, this implicature can be cancelled by saying, "In fact, none of you did."

# 4.2 Tense and implicature

- (69) Few Republicans opposed the tax cuts.
- (70) Few Republicans will oppose the tax cuts.

Both (69) and (70) implicate that at least some Republicans opposed/will oppose tax cuts (see the previous section for an explanation of why this implicature arises). However, it seems that (69) makes this implicature far more strongly than (70). In other words, (69) sounds a whole lot

Horn's quantifer scales aren't perfect. As discussed below in section 5.1, the truth conditions for few are quite variable. It is plausible, for example, that in a given context, a minority of students like cheese be false and few students like cheese be true. In this case, few, though it is to the right of a minority on the scale, does not entail a minority. A similar problem arises with many and the positive quantifier scale.

worse in the case that no Republicans opposed the tax cuts than (70) sounds in the case that it turns out no Republicans oppose the tax cuts. This phenomenon arises from the relative confidence expected of a speaker of each of these sentences. Since the situation in (69) has already occurred, the typical listener would expect the speaker to have a firm grasp on the details of how many Republicans opposed the tax cuts; if no Republicans opposed the tax cuts and the speaker knew this, they would, by Grice's maxim of quantity have said so, rather than leaving open the possibility of a small number of Republicans opposing the tax cuts. With (70), however, the eventuality occurs in the future. Therefore, no one can reasonably expect a speaker to know the precise number of Republicans who will oppose the tax cuts. As such, the implicature that there are at least some Republicans who will oppose the tax cuts is not as strong; the speaker of (70) would not be chastised for deception if it later turned out that no Republicans opposed the tax cuts.

A similar pattern emerges with a few.

- (71) A few students passed the exam.
- (72) A few students will pass the exam.

As discussed above, the class would likely be a bit upset with a professor who said (71) with the full knowledge that all the students in the class passed the exam. However, it seems unlikely that anyone would accuse the professor of lying bat an eye if he/she said (72) and it turned out that everyone passes the exam. Again, this arises due to the level of confidence that speakers are expected to have with respect to certain claims. Sentences in the past tense are typically assigned a greater confidence level since the situation described has already occurred and its details are available to the speaker, while sentences in the future tense describe situations whose details might not yet be known and are typically capable of undergoing unexpected changes.

It is difficult to get a firm sense of the strength of implicatures in present-tense sentences. This difficulty arises largely from the fact that the English present tense sometimes describes habitual eventualities. In particular, verbal predicates classified as activities and accomplishments are generally understood as indicating habits when in the present tense (Dowty 1979: 51-71). In other words, a sentence like (73) is making a different sort of assertion than (71) or (72), namely that, from year to year, a few students typically pass a particular exam, rather than making any claim about how any particular students did on an exam from a specific year.

(73) A few students pass the exam.

The question of the strength of implicatures in present tense sentences is worthy of further exploration, but it is not directly relevant to this paper and must therefore be laid aside for now.

# 5 Another look at few

5.1 The Semantic Slipperiness of few

The vast majority of determiners have a meaning consistent across all models and contexts. A sentence like "All students love ice cream" is true just in case every individual that is a student also loves ice cream. A few has this property of semantic consistency; a sentence using a few will be true whenever at least two individuals have the properties A and B linked by a few.

Few is different. It is impossible to determine the truth of few-sentences across all models. In fact, even with respect to a single model, a given few-sentence might be true or false. Suppose that John and Mary are going out to a restaurant for dessert. Feeling adventurous, they decide to try all eight desserts on the menu and afterwards agree that five of the desserts were

delicious, and the other three weren't very good. After dessert they meet up with another friend at a bar. When asked how the restaurant was, John asserts (a bit awkwardly, perhaps), "Few desserts were good," to which Mary immediately responds, "What?! There were so many good desserts." After some further discussion, it becomes clear that they've had vastly different experiences with the restaurant in question. Every dessert that John has ever had at the restaurant has been delectable. Mary has had the opposite experience; she's never had a dessert there that she enjoyed. Thus, for John, the number of good desserts was well below his expectation, while for Mary, there were far more good desserts than she expected.

It seems, therefore, that *few*-sentences make a claim based on the expectations (or perhaps the desires) of the speaker. The truth of such sentences is not dependent on the way the world is, but rather how the way the world is relates to the world envisioned by the speaker.

5.2 Disambiguating the two senses of few

The means by which the applicable sense of *few* is recognized in discourse has not yet been addressed. One overt means of indicating which sense of *few* is intended is seen in (73).

(74) Few of the students passed the exam.

The few in (74) must be interpreted as  $few_1$ , the proportional sense of few. In other words, if it were the case that all the students passed the exam, (74) would necessarily be false. The partitive of-construction, then, indicates that the proportional sense of few is at play. This conclusion matches the understanding of partitive of-constructions as describing part-whole relationships.

Other methods of distinguishing the relevant sense of *few* are far harder to come by. It seems decisions on sentences without the partitive construction rely heavily on context for determining which sense of *few* is intended. Suppose, for example, Speaker A in a conversation

has expressed interest in the percentage of citizens of various countries who speak French. Speaker replies that, "Few Americans speak French." It is clear that few1 is intended here, with Speaker B indicating that a low percentage of Americans speak French. This interpretation arises because the topic of proportions of populations who speak French had already been discussed. If, however, Speaker A is wondering about the countries with the highest number of French speakers and Speaker B gives the same response, the expected interpretation, that the number of Americans who speak French is low, would be based on few2.

The problem of disambiguating the senses of few is not as serious as it might appear. There is a considerable amount of overlap between  $few_1$  and  $few_2$ . In other words, there are many times when the set denoted by  $few_1$  could also be denoted by  $few_2$  and vice versa. For example, a set of ten Americans could likely be referred to by both "few<sub>1</sub> Americans" (that is, a small percentage of Americans) and "few<sub>2</sub> Americans" (that is, a small number of Americans). It is only when the cardinality of the set in question becomes fairly large that the distinction between few1 and few2 becomes significant.

# 6 Conclusion

In spite of their superficial similarity, the meanings of the determiners few and a few are quite different. Both signify small quantities of individuals, but the ways in which that meaning is generated differ considerably. For few, that meaning is semantic, with large quantities falsifying few-sentences. With a few, the small quantity is only pragmatically implicated, as large quantities do not falsify sentences with a few but only make them somewhat bizarre. Scalar implicature associated with few has the opposite effect as that associated with a few; rather than implicating the existence of some individuals who do not fulfill the restriction and nuclear scope of the sentence (as a few does), few implicates that such individuals do exist.

Few and a few also differ in a number of semantic properties. These differences are summarized in Appendix B. Few also some interesting characteristics in its own right, namely that the precise quantity denoted by few can vary considerably by context and that there appear to be two senses (a proportional and a cardinal) of few whose meanings appear to overlap at small quantities but can lead to dramatically different interpretations of sentences at larger quantities.

By no means does this paper present an exhaustive examination of the meanings of *few* and *a few*. There are many issues that remain unexplored. One such issue is the interpretation of sentences that lack a common noun phrase following the initial determiner, like (75) and (76).

- (75) A few like pizza.
- (76) Few passed the exam.

The unresolved question for these sentences is "A few what?" Another issue, touched only briefly in section 5.2, is the disambiguation of  $few_1$  and  $few_2$ . It is hoped that this paper provides a framework for exploring these and other questions regarding the meaning of few and a few as well as other determiners.

# Appendix A

As discussed in section 2, A and B are treated as subsets of the universe. Subsets of the universe are typically understood as containing entities that share a particular property. The precise form of sentences whose denotations are written as (A FEW) (A) (B) is dependent on how the properties A and B are expressed. Examples follow.

(1) If A is expressed by a common noun and B is expressed by a common noun, the syntactic realization of (A FEW) (A) (B) is in the following form:

'A few As are Bs.'

Example: A few students are vegetarians.

(2) If A is expressed by a common noun and B is expressed by an adjective, the syntactic realization of (A FEW) (A) (B) is in the following form:

'A few As are B.'

Example: A few students are happy.

(3) If A is expressed by a common noun and B is expressed by a verbal predicate, the syntactic realization of **(A FEW)** (A) (B) is in the following form:

'A few As B.'

Example: A few students read books.

(4) If A is expressed by an adjective and B is expressed by a common noun, the syntactic realization of **(A FEW)** (A) (B) is in the following form:

'A few A individuals are Bs.'

Example: A few happy individuals are students.

(5) If A is expressed by an adjective and B is expressed by an adjective, the syntactic realization of **(A FEW)** (A) (B) is in the following form:

'A few A individuals are B.'

Example: A few happy individuals are hungry.

(6) If A is expressed by an adjective and B is expressed by a verbal predicate, the syntactic realization of (A FEW) (A) (B) is in the following form:

'A few A individuals B.'

Example: A few happy individuals read books.

(7) If A is expressed by a verbal predicate and B is expressed by a common noun, the syntactic realization of (A FEW) (A) (B) is in the following form:

'A few individuals that A are Bs.'

Example: A few individuals that read books are students.

(8) If A is expressed by a verbal predicate and B is expressed by an adjective, the syntactic realization of (A FEW) (A) (B) is in the following form:

'A few individuals that A are B.'

Example: A few individuals that read books are happy.

(9) If A is expressed by a verbal predicate and B is expressed by a verbal predicate, the syntactic realization of (A FEW) (A) (B) is in the following form:

'A few individuals that A B.'

Example: A few individuals that read books like semantics.

Appendix B			
Property	a few	few <sub>1</sub>	few <sub>2</sub>
Cardinality	cardinal	non-cardinal	cardinal*
Conservativity	conservative	conservative	conservative
Definiteness	indefinite	indefinite	indefinite
Intersectivity	intersective	non-intersective	intersective
(Left) Monotonicity	increasing	n/a	decreasing
(Right) Monotonicity	increasing	decreasing	decreasing
Pronominal Reference	direct reference	compset reference	compset reference
Proportionality	non-proportional	proportional**	non-proportional
Strength	weak	weak	weak (?)

Summary of the semantic properties of a few and few.

\* Under the strict definition of cardinality discussed in the body of the paper,  $few_2$  is not cardinal. However, that the precise cardinality associated with  $few_2$  changes with context does not change the fact that one of the key features of  $few_2$  is its cardinality

 $few_2$  is its cardinality.

\*\*\* Under the strict definition of proportionality discussed in the body of the paper,  $few_1$  is not proportional. However, that the precise porportionality associated with  $few_1$  changes with context does not change the fact that

one of the key features of few<sub>1</sub> is its proportionality.

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