Eric Kenneth Acton

GENERALIZED DISTRIBUTIVITY AND NONMAXIMALITY IN PLURAL DEFINITE NOUN PHRASES IN SUBJECT POSITION*

1. Introduction

In the last couple of decades there has been a great deal of literature on how to account for the various readings of sentences with a plural definite subject NP such as the following:

(1) The volunteers made a quilt.

A collective reading of sentence (1) requires that all the volunteers together made a single quilt. In the distributive reading of (1), each individual volunteer made a quilt by himself or herself. Recently—as I will do in this paper—many linguists (Lin (1998), Brisson (2003) et al.) have adopted Roger Schwarzschild's (1996) generalized distributivity operator (or 'D-operator') account of sentences such as (1). In Schwarzschild's account, the verb phrase hosts a D-operator that distributes over subsets of the denotation set of the subject NP. Whether a sentence has a distributive or collective reading (or some intermediate reading) depends on a context-determined 'cover' of the universe of discourse.

Another phenomenon associated with sentences like (1) that deserves attention is what Brisson (2003) calls 'nonmaximality'—the fact that a predicate may be considered true (or, as we will see later, 'close enough to true,') of the denotation set of a subject NP

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even if it is not true of every single member of that set. For instance, under certain circumstances most native English speakers would not consider a speaker of (1) to be lying even if a couple of the volunteers did not actually participate in quilt-making.

In this paper we will examine two different accounts of nonmaxmality—the first offered by Brisson (2003) and the second offered by Lasersohn (1999). Brisson's (2003) account employs Schwarzschild's (1996) analysis of the generalized D-operator. According to Brisson—building off of Schwarzschild—nonmaximality occurs as a result of certain kinds of covers. Lasersohn's (1999) account of nonmaximality, on the other hand, is framed within a larger analysis of the "…truism that people speak loosely" (522). Under his analysis, hearers assign to every complex expression a context-dependent 'pragmatic halo'—a partially ordered set containing all sets that the hearer considers to be, as Lasersohn says, 'close enough to true for practical purposes.' Based on this assumption, Lasersohn is able to formally define the conditions under which a hearer considers a sentence to be 'close enough to true.' Like Brisson's approach, Lasersohn's does successfully account for nonmaximality in sentences like (1). Unlike Brisson's approach, however, it contains no discussion of generalized distributivity.

Our goal in this paper then, is an attempt to determine/develop the analysis that best accounts for both distributivity and nonmaximality in sentences like *the volunteers made a quilt*. Sections 2, 3, and 4 of the paper serve to define the linguistic phenomena and the kind of sentence that we are concerned with, and provide an overview of some attempts to account for these phenomena. Section 2 is on Schwarzschild's (1996) account of the generalized D-operator, Section 3 is on Brisson's (2003) account of nonmaximality, and Section 4 is on Lasersohn's (1999) account of 'pragmatic slack,' focusing on nonmaximality. Then, in Section 5, is a detailed explanation of a way to incorporate Schwarzschild's D-operator analysis into Lasersohn's framework, and a demonstration of some of the advantages of combining the two analyses.

Later, in Section 6, we examine a couple of critical problems associated with the analyses of Schwarzschild and Brisson, then present an innovation to Schwarzschild's analysis that—when coupled with Lasersohn's account of pragmatic slack—precludes these problems while still accounting for nonmaximality. Finally, in Section 7, we compare Brisson's account of distributivity and nonmaximality to the account that combines the analyses of Schwarzschild and Lasersohn, and argue for the latter.

2. Schwarzschild and Generalized D-operators

In chapter five of his book *Pluralities* (Kluwer, 1996), Roger Schwarzschild lays out an analysis to account for collective and distributive readings of sentences with plural definite NPs. One of his example sentences is the following:

(2) John and Mary made $1,000.

which is true in the case that John and Mary made $1,000 together, and in the case that each of them made $1,000. To account for the fact that both readings are possible, Schwarzschild uses the set theoretic concept of a 'cover' for the universe of discourse. The formal definition of a cover—as stated by Schwarzschild—is given below.

(3) C covers A if:
a. C is a set of subsets of A[1]
b. Every member of A belongs to some set in C.
c. ∅ is not in C.

[1] Note that this does not require that C be the set of *all* subsets of A. That is, C is not necessarily the power set of A. (In fact, part three of this definition makes it impossible for C to be the power set of A.) Rather, part one of the definition of a cover requires that C be a set whose elements are *some* of the subsets of A.

Schwarzschild says that, in English, a cover of the set of everything in the universe of discourse is associated with each plural VP in a sentence, and is determined by the hearer based on context.

To make this clearer I offer the following example (in a style similar to one given by Brisson (2003)):

(4) Let[2] U be the universe, where

$$U = \{i, j, k, l, m, n\}$$

And let

$$\| \text{John} \| = \{j\}$$
$$\| \text{Mary} \| = \{m\}$$

with possible covers of the universe, C_1 and C_2, where

$$C_1 = \{\{i\}, \{j\}, \{k\}, \{l\}, \{m\}, \{n\}\}$$
$$C_2 = \{\{l, k\}, \{i, n\}, \{j, m\}\}$$

As the reader will notice, both C_1 and C_2 satisfy our definition of a cover of U. So, what makes a cover contextually relevant? Well, suppose that (2) were part of the answer to one of the following questions:

(5) How much money did each worker make?

(6) How much money did each couple make?

It should be clear that the presence of *each* in (6) suggests individuals, while *couple* in (7) suggests pairs of individuals. Thus (6) would correspond to a cover like C_1, whose elements are one-member sets (singletons), while (7) would correspond to a cover like C_2, whose elements are two-member sets (duples).

[2] Note that in this we use subscript numbers to index two distinct covers corresponding to the same VP. Schwarzschild does the same, but only when talking about two distinct covers each associated with a different VP within a sentence.

Having applied the notion of covers to semantics, Schwarzschild incorporates covers formally via the following two rules:

(7) Plural VP rule:
If α is a singular VP with translation α', then the corresponding plural VP is translated as Part(Cov)(α').
(1996, p.73)

(8) $x \in \| \text{Part(Cov)}(\alpha) \|$ if and only if

$$\forall y[\ (y \in \| \text{Cov} \| \wedge y \subseteq x) \rightarrow y \in \| \alpha \|\ ]$$

(1996, p.71)

'Part' is Schwarzschild's name for his D-operator, which, in effect, serves to distribute the VP predicate over subsets of the denotation of an NP. And "Cov is a free variable over sets of sets," whose value is a cover of the universe of discourse—determined by the hearer according to "…the linguistic and non-linguistic context" (70). With rules (7) and (8) established, we can state the truth conditions for a sentence with a plural definite subject NP and a plural VP:

(9) Let $\varepsilon\ \alpha$ be a sentence, where ε is a definite plural NP and α is plural VP. Then $\varepsilon\ \alpha$ is true iff

$$\| \varepsilon \| \in \| \text{Part(Cov)}(\alpha) \|.$$

We can elucidate this formalism by continuing to flesh out the example we were working with above. We assume, as Schwarzschild does, that the denotation set of *John and Mary* is the set {j, m}. Let's take each cover in turn. With C_1 we get that (2) is true if and only if || John and Mary || is an element of || Part (C_1)(made $1,000) ||. That is, (2) is true if and only if

(10) $$\forall y[\ (y \in \{\{i\}, \{j\}, \{k\}, \{l\}, \{m\}, \{n\}\} \wedge y \subseteq \{j, m\}) \rightarrow y \in \| \text{made } \$1{,}000 \|\ ]$$

Further simplification of (11) states that, if C_1 is selected, (2) is true if and only if {j} is in the denotation set of *made $1,000* and {m} is in the denotation set of *made $1,000*.

Hence, we have the distributive reading, requiring Mary and John to have each made \$1,000.

Now consider the case that the hearer assigns C_2. (2) is then true if and only if || John and Mary || is an element of || Part (C_2)(made \$1,000) || or, equivalently

(11) $\forall y[\ (y \in \{\{l, k\}, \{i, n\}, \{j, m\}\} \wedge y \subseteq \{j, m\}) \rightarrow y \in \|\ \text{made } \$1{,}000\ \|\ ]$

Thus, if C_2 is selected, (2) is true only if and only if the set {j, m} is in the denotation set of *made \$1,000*. That is, we have the collective reading, requiring John and Mary to have made \$1,000 together.

Thus we have seen that Schwarzschild's analysis provides an effective and fairly straightforward way of accounting for distributive and collective readings of sentences with plural definite subject NPs. In the next section we examine how his analysis contributes to Brisson's (2003) analysis of nonmaximality.

3. Brisson's Account of Nonmaximality in Plural Noun Phrases

In her 2003 article 'Plurals, *All*, and the Nonuniformity of Collective Predication', Christine Brisson utilizes Schwarzschild's generalized D-operator account in order to explain nonmaximality in plural definite NPs. She claims that "speakers and hearers are always engaged in a process of guessing about what each other have in mind" (138) and thus must always allow for what she calls 'ill-fitting' covers:

(12) A cover C is *ill-fitting* with respect to a denotation set D if there is no union X of elements of C such that X = D.

In order to clarify this definition, let's return to our first sentence, (1). To get truth conditions for (1), we need—in addition to the denotation set of *the volunteers*—a universe of discourse and a cover of that universe:

(13) Let U' be the universe, where

$$U' = \{i, j, k, l, m, n\}$$

And let

$$\| \text{the volunteers} \| = \{j, k, l\}$$

with possible covers of the universe, C_1' and C_2', where

$$C_1' = \{\{i\}, \{j\}, \{k\}, \{l\}, \{m\}, \{n\}\}$$
$$C_2' = \{\{i\}, \{j\}, \{k\}, \{l, m, n\}\}$$

Now consider the truth conditions derived from assigning each of the two covers, C_1' and C_2'. In the case of C_1', we get the straightforward distributive case, in which each of the volunteers made a quilt. C_2', on the other hand, is a bit more tricky. Recall that, according to (7) and (8), the predicate *made a quilt* need only be true of those sets that are both elements of C_2'and subsets of the denotation set of *the volunteers*:

(14) $$\forall y[\, (y \in \{\{i\}, \{j\}, \{k\}, \{l, m, n\}\} \wedge y \subseteq \{j, k, l\}) \rightarrow y \in \| \text{made a quilt} \| \,]$$

Therefore, the relevant sets are only {j} and {k}. That is, if the hearer of (1) allows for C_2', then (1) is true so long as {j} and {k} are elements of the denotation set of *made a quilt* and as Brisson would say, doesn't care about whether l made a quilt, even though $l \in \|$ the volunteers $\|$. This somewhat odd result comes from the fact that C_2'is in fact an ill-fitting cover, for no union of elements of C_2' is equivalent to $\|$ the volunteers $\|$.

As Brisson (2003) points out, the possibility of ill-fitting covers led Lasersohn (1995) to criticize Schwarzschild's generalized D-operator account. Allowing for ill-fitting covers could lead to finding (2) true even if John didn't make any money at all. Schwarzschild (1996) argues that such covers are ruled out by pragmatics, while Brisson says that they are sometimes determined intentionally. "In fact," says Brisson, "it is quite clear that there are circumstances in which both speaker and hearer share the assumption that one (or more) individuals who are part of the denotation of the definite plural is (are) excluded from the domain of the D operator" (137). Suppose, for example, there is a group of students and among them is Chris, who is lactose intolerant. Then a hearer of *the students ate ice cream* might throw Chris into what Brisson calls a 'junkpile'—just as l was in C_2'—since the hearer knows that Chris never eats dairy products. In general, says Brisson, "speakers and hearers must always make room for the *possibility* of ill-fitting covers" (138).

However, Brisson never actually explains how problems like the one pointed out by Lasersohn above are precluded in her analysis. Furthermore, it seems that if speakers and hearers must always make room for the possibility of ill-fitting covers, then because the truth conditions of a sentence are dependent on the cover that the hearer assigns, they are never uniquely determined. Thus, while Brisson's analysis does account for nonmaximality, these issues—as well as others to be discussed later in the paper—suggest that we ought to try to find a *better* way to account for nonmaximality. One approach is Lasersohn's, which is discussed in the following section.

4. Lasersohn's Pragmatic Halos

As Peter Lasersohn (1999) points out, there are many cases in which people do not mean exactly what they say. For instance, the sentence

(15) Coleman left at noon.

when in fact he departed at even, say, 12:01 p.m., is considered close enough to the truth in most circumstances. Nonmaximality in NPs is another example of what Lasersohn calls 'pragmatic slack'. As we've discussed, our original sentence (1) would be considered true for practical purposes even in many circumstances under which not every volunteer made a quilt.

There are of course certain circumstances that don't allow so much leeway. If I were conducting a psychological experiment on human sleep patterns and told

(16) The subjects are sleeping.

I would be very upset if I noticed that one subject was actually awake. But it is not only pragmatic situations that determine the amount of licensed leeway; words within a sentence are also relevant.

(17) Coleman left at exactly noon.

(17) is more strict than (15). The presence of *exactly* allows for less pragmatic slack. *All* acts in a similar manner in

(18) All the volunteers made a quilt/The volunteers all made a quilt.

Lasersohn calls words like *exactly* and *all* 'slack regulators' because their presence allows for less pragmatic slack. The important points here are a) that sentences are considered close enough to the truth if they are "close enough not to obscure pragmatically relevant details or distinctions" (525) and b) that both linguistic and paralinguistic factors determine how much a speaker is licensed to approximate the truth.

Having discussed the nature of the phenomenon as Lasersohn sees it, the next step is to see how he models pragmatic slack and slack regulation. First we will build intuition of the concept, then provide the formalism. Lasersohn's proposed device is what he calls a 'pragmatic halo', which has two components. The first component of a pragmatic halo of a given linguistic expression is the set H_C containing the denotation of the expression and, in addition, all objects of the same logical type as the denotation of the expression that are close enough—relative to the context C—to the denotation for practical purposes. So the set H_C of the expression *the volunteers* in (1) would be the set whose members are the denotation set of *the volunteers* and all sets of individuals that are pragmatically close enough to that denotation set. The second component is a partial or complete ordering relation on H_C, based on the closeness of elements of H_C to the actual denotation. Again for *the volunteers*, this ordering would be determined by how closely a set of individuals in H_C approximates the denotation set of *the volunteers*. Let's now write this formally (paraphrased from Lasersohn (1999: p. 548):

(19) The pragmatic halo of a basic expression α with respect to a context C and model M is a partially ordered set $< H_C(\alpha), \leq_{\alpha,C} >$ where

$H_C(\alpha)$ is the set of objects of the same logical type as $\| \alpha \|^{M,C}$ that differ from $\| \alpha \|^{M,C}$ only in ways that are pragmatically ignorable in C.

$\leq_{\alpha,C}$ is a partial ordering of $H_C(\alpha)$ according to similarity to $\| \alpha \|^{M,C}$. And if v, w $\in H_C(\alpha)$ and v is more similar to $\| \alpha \|^{M,C}$ than w is, then we write

$$v \leq_{\alpha,C} w$$

Let's work out an example, taking the model given in (13):

(20) Let U' be the universe, where

$$U' = \{i, j, k, l, m, n\}$$

And let || the volunteers || = {j, k, l}

The pragmatic halo of *the volunteers* with respect to a context C would be something like

$<H_C(\textit{the volunteers}), \leq_{\textit{the volunteers},C}>$, where

$H_C(\textit{the volunteers}) = \{\{j, k, l\}\ \{j, k\}, \{j, l\}, \{l, k\}, \{j\}\}$

First of all, note that the value we have assigned to H_C(*the volunteers*) is just an example. A different context could call for an entirely different value of H_C(*the volunteers*). This is merely an example for clarification. That said, it should be clear that the selection we made for the elements of H_C(*the volunteers*) 'makes sense' in that they are relatively close to being the actual denotation of *the volunteers*. Also, this example should suggest an intuitive, inherent partial ordering to H_C(*the volunteers*). The sets {j, k}, {j, l}, and {l, k} are all equally close to the denotation set of *the volunteers* since they are each missing one member, so they are on the same level in terms of ordering. {j}, however is less similar than the others because it is missing two members, and {j, k, l} is more similar because it is exactly the denotation set of *the volunteers*[3]. We say then, for instance, that in this example

(21) $\{j, k, l\} \leq_{\textit{the volunteers},C} \{j,k\} \leq_{\textit{the volunteers},C} \{j\}$

Now that we know how to find the pragmatic halo of a basic expression, it is easy to understand how to derive the pragmatic halo of a complex expression—operating compositionally[4]. Suppose for example that α is a complex expression made up of β and γ, where β denotes a one place predicate f (a function), γ denotes an individual c and $\| \alpha \| = \| \beta \|(\| \gamma \|)$. Then H_C (β) would be a set of functions, say {f, g, h} and H_C (γ) would be a set of individuals, say {a, b, c}. H_C (α) comes from composing H_C (β) and H_C (γ), and would thus be {f(a), f(b), f(c), g(a), g(b), g(c), h(a), h(b), h(c)}. This composition lends itself to a partial ordering, based both on how close the individual is to c and how close the function is to f. Lasersohn formalizes as follows:

(22) $H_C(F_i(\alpha_1,\ldots, \alpha_n)) = \{ G_i(x_1,\ldots, x_n)) : x_1 \in H_C(\alpha_1),\ldots, x_n \in H_C(\alpha_n)\}$
The ordering relation is preserved in the composition:

(23) If $y, z \in H_C(\alpha_k)$ and $y \leq_{\alpha,C} z$, then

[3] We should note that Lasersohn makes the requirement, as we do, that $\| \alpha \|^{M,C} \in H_C(\alpha)$ and, fittingly $\| \alpha \|^{M,C}$ is the element y of $H_C(\alpha)$ such that for all $x \in H_C(\alpha)$, $y \leq_{\alpha C} x$.
[4] This explanation comes almost directly from Lasersohn (1999).

$$G_i(x_1,\ldots, y,\ldots, x_n) \leq_{\alpha,C} G_i(x_1,\ldots,z,\ldots, x_n)$$

(1999, p.548)

The two key points to keep in mind are a) pragmatic halos of complex expressions are constructed compositionally from the pragmatic halos of basic expressions and, crucially, b) pragmatic halos are not constructions that change truth conditions; rather, they indicate what conditions are close enough to the truth conditions for practical purposes. Taking all this together, as Lasersohn puts it, "We will count a sentence as 'close enough to true for context *C*' iff its halo relative to *C* contains at least one non-empty element" (528).

This last part deserves some explanation. To explain, we will go through one of Lasersohn's examples. We work with the following sentence from Lasersohn:

(24) Mary arrived at three o'clock.
(1999, p.528)

First of all it should be noted that sentences are merely complex expressions that are composed of basic expressions. Thus, a sentence should have a corresponding pragmatic halo, constructed in the same way as the halo of any other complex expression. Secondly, in Lasersohn's framework, sentences denote sets of eventualities[5] "(that is, events, states and processes)" rather than truth values, and a sentence is considered true if and only if its denotation set is non-empty (527). (24), for instance, would denote the set of Mary's arrivals at three o'clock, and would be true if and only if that set were non-empty.

[5]We refrain from including events in our framework, except in explaining Lasersohn's analysis here. Thus, for the purposes of this paper, we treat sentences as denoting sets of individuals rather than sets of eventualities—so that the pragmatic halo of a sentence is a set of sets of individuals. This is not to say, however, that our analysis is incompatible with event-based semantics. For an event-based analysis of distributivity see Lasersohn (1998).

For this example in particular, Lasersohn takes *three o'clock* to denote a specific time i, and has the halo of *three o'clock* be the set of times pragmatically close enough to i—for example, {i, j, k}. He takes the other halos in the sentence in this example to be trivial, so that the halo of *Mary* is { || *Mary* || } etc. The denotation of (24) as we said, is the set of Mary's arrivals at 3 o'clock—or, in set notation, {x: x is an arrival by Mary at i}. The halo of (24) is thus { {x: x is an arrival by Mary at i}, {x: x is an arrival by Mary at j}, {x: x is an arrival by Mary at k}}. Now suppose, says Lasersohn, that Mary arrived only at k, and call this event of Mary's arrival at k, *e*. Then the halo of (24) is {∅, ∅, {*e*}}. So while (24) is not true, it is close enough to true for practical purposes because it's halo is non-empty. In this manner, under Lasersohn's analysis we can determine whether any sentence is pragmatically close enough to true.

5. Extending Lasersohn's Account: Incorporating Schwarzschild's Account of Distributivity

While Lasersohn may be critical of Schwarzschild's account of distributivity, we have yet to see any evidence that the two cannot coexist. In fact I believe that we ought to adopt both. In this section we will outline simple a proposal to do just that.

First, we assume the validity of Schwarzschild's generalized D-operator analysis, as it intuitively and successfully accounts for generalized distributivity associated with the kind of sentence we're examining. That said, our first objective in this section is to justify an approach that would assign pragmatic halos to VPs as they are interpreted under Schwarzschild's analysis. To do so, let's start by recalling Schwarzschild's formal definition of the denotation of a plural VP:

(7) Plural VP rule:
If α is a singular VP with translation α', then the corresponding plural VP is translated as Part(Cov)(α').

(8) $x \in \| \text{Part(Cov)}(\alpha) \|$ if and only if

$$\forall y[\ (y \in \| \text{Cov} \| \wedge y \subseteq x) \rightarrow y \in \| \alpha \|\ ]$$

An equivalent way of writing (8) is

(25) $\| \text{Part(Cov)}(\alpha) \| = \{x : \forall y[\ (y \in \| \text{Cov} \| \wedge y \subseteq x) \rightarrow y \in \| \alpha \|\ ]\}$

By writing it this way we are highlighting the fact that the denotation set of plural VP is a set of sets. This will be useful in providing a halo for a plural VP, for we want the elements of the halo to be of the same logical type as the denotation of the plural VP.

In keeping with Lasersohn's analysis, we construct the halo of Part(Cov)(α) compositionally, incorporating the halos of its relevant subparts, namely the VP, α, and Cov. α in this framework denotes the set of sets of individuals that have the α-property. Thus the halo of α is a set of sets of sets of individuals that are close enough to $\| \alpha \|$ for practical purposes. To clarify this recursive mess, let's work with the following example, building off of (20):

(26) Let U' be the universe, where

U' = {i, j, k, l, m, n}

And let || the volunteers || = {j, k, l}
|| made a quilt || = {{j}, {l}, {m}}, where

H_C(*made a quilt*) = { {{j}, {l}, {m}}, {{j}, {l}, {i}} }

The pragmatic halo H_C(*made a quilt*) satisfies the requirement that its elements be of the same logical type as || made a quilt ||, and it has a partial ordering, defined by an element's closeness to || made a quilt ||.

This part was fairly straightforward, but we have yet to account for Cov. The question is, can/should we provide a pragmatic halo for Cov? And, if so, how?

My answer to the first question is yes. Covers are determined, in part, by linguistic content. Certain words, such as *couple*, *team*, and *individual* cause a hearer to select a particular cover. So the value of Cov, || Cov ||, is partially determined by the denotation of basic expressions within a sentence and across sentences within a discourse. Furthermore, we can say—as we have shown at several points—that || Cov || has significant bearing on the interpretation of a sentence of the form given in (9). And thus—since Lasersohn's analysis requires that the halo of a complex expression be composed of the halos of those elements that contribute to the meaning of the complex expression—it seems clear that we should allow for covers to have halos, since they both contribute to meaning and are partially determined by other linguistic expressions.

In fact, there are cases where a pragmatic halo for a cover would be extremely helpful. Consider the role of *each* in the following:

(27) The children each painted a picture.

In addition to contributing universal quantification, *each* seems to suggest that every student painted his or her own painting, rather than contributing to one group painting. Within Schwarzschild's framework, that means that in this sentence *each* serves to select a cover that is the set of singletons. But consider the following case. Suppose you are a preschool teacher and you have a class of, say, twenty children and that yesterday the children were all to make a self-portrait using finger-paints. One of the younger children, Ben, was having trouble, so you had an older child, Lil, help him paint his picture. Thus what actually happened is that every child painted his or her own portrait by himself or herself, except for Ben who received help from Lil. However, given these circumstances, most English speakers would judge (27) as close enough to true for practical purposes.

The disparity between the requirements of the truth conditions and what is actually the case comes solely from the cover. In our example scenario it is true that every child participated in painting a picture, but it is not true that every child did so on his or her own. The cover corresponding to this scenario is one in which every student is in his or her own singleton set, except for Ben who is in a set with Lil. Therefore, in deeming (27) to be close enough to the truth for practical purposes, one is actually saying that the cover described above is pragmatically close enough to the cover whose elements are all singletons. And so we have evidence that pragmatic halos for covers are not only admissible, but in some cases *necessary*.

The pragmatic halo H_C(Cov) of a cover Cov is thus the set of all sets close enough to || Cov || for practical purposes. Let's continue build on our working example by adding

(28) Let Cov* be the context-dependent cover assigned to (1), where

$$\text{Cov*} = \{\{i\}, \{j\}, \{k\}, \{l\}, \{m\}, \{n\}\} \text{ and}$$

$$H_C(\text{Cov}) = \{\ \text{Cov*}, \|\ c_1\ \| = \{\{i\}, \{j\}, \{k\}, \{l, m\}, \{n\}\}, \|\ c_2\ \| = \{\{i\}, \{j\}, \{k\}, \{l\}, \{m, n\}\}\ \}$$[6]

Now that we've established that we have pragmatic halos both for singular VPs and for covers, we can formally create a pragmatic halo for plural VPs as they are interpreted within Schwarzschild's framework. First, as a reminder, is the set version of || Part(Cov)(α) ||, then the formalism of the halo:

(25) $\|\ \text{Part(Cov)}(\alpha)\ \| = \{x : \forall y[\ (y \in \|\ \text{Cov}\ \| \wedge y \subseteq x) \rightarrow y \in \|\ \alpha\ \|\ ]\}$

(29) Let Part(Cov)(α) be the translation of a plural VP α where

[6] We introduce the c_i's here for notational brevity later on.

$\| \text{Cov} \| = \text{Cov}^*$. Then the pragmatic halo of the plural VP with respect to context C is

$$\langle H_C(\text{Part}(\text{Cov})(\alpha)), \leq_{\text{Part}(\text{Cov})(\alpha),C} \rangle \text{ where}$$

$$H_C(\text{Part}(\text{Cov})(\alpha)) = \{ \| \text{Part}(c)(\delta) \| : \| c \| \in H_C(\text{Cov}) \wedge \| \delta \| \in H_C(\alpha) \}$$

$$= \{ \{x : \forall y[(y \in \| c \| \wedge y \subseteq x) \rightarrow y \in \| \delta \|]\} : \| c \| \in H_C(\text{Cov}) \text{ and } \| \delta \| \in H_C(\alpha) \}$$

and $\leq_{\text{Part}(\text{Cov})(\alpha),C}$ is partial ordering based on how pragmatically close $\| c \|$ is to Cov* and how pragmatically close $\| \delta \|$ is to $\| \alpha \|$ for each element $\| \text{Part}(c)(\delta) \|$ of $H_C(\text{Part}(\text{Cov})(\alpha))$.

The first line of the formalism says that the halo of Part(Cov)(α) is the set of all sets of the form Part(c)(δ), such that $\| c \|$ is an element of the halo of Cov and $\| \delta \|$ is an element of the halo of α. So how would this look with our ongoing example based on (1)? Recall that for our example

(30) The universe, U' = {i, j, k, l, m, n}

$\|$ the volunteers $\|$ = {j, k, l}
$\|$ made a quilt $\|$ = {{j}, {l}, {m}}
$\|$ Cov $\|$ = Cov* = {{i}, {j}, {k}, {l}, {m}, {n}}

H_C(*made a quilt*) = { {{j}, {l}, {m}}, {{j}, {l}, {i}} }

H_C(Cov) = { Cov*, $\| c_1 \|$ = {{i}, {j}, {k}, {l, m}, {n}},
$\| c_2 \|$ = {{i}, {j}, {k}, {l}, {m, n}} }

Knowing the halo for α—which in this case, is *made a quilt*—and knowing the halo for Cov, we are able to define the halo of Part(Cov)(*made a quilt*):

(31) H_C(Part(Cov)(*made a quilt*)) = { $\|$ Part(Cov)(*made a quilt*) $\|$,
$\|$ Part(Cov)({{j}, {l}, {i}}) $\|$,
$\|$ Part(c_1)(*made a quilt*) $\|$,
$\|$ Part(c_1)({{j}, {l}, {i}}) $\|$,
$\|$ Part(c_2)(*made a quilt*) $\|$,
$\|$ Part(c_2)({{j}, {l}, {i}}) $\|$ }

The halo of Part(Cov)(*made a quilt*) thus has six elements, corresponding to the six possible pairings of an element from the halo of Cov with an element from the halo of *made a quilt*. The representation in (31) can be further expanded as follows:

(32) H_C(Part(Cov)(*made a quilt*)) =

$$
\begin{array}{l}
\{\ \{x : \forall y[\ (y \in \mathrm{Cov}^* \wedge y \subseteq x) \rightarrow y \in \|\ \textit{made a quilt}\ \|\ ]\},\\
\ \ \{x : \forall y[\ (y \in \mathrm{Cov}^* \wedge y \subseteq x) \rightarrow y \in \{\{j\}, \{l\}, \{i\}\}]\},\\
\ \ \{x : \forall y[\ (y \in \|\ c_1\ \| \ \wedge y \subseteq x) \rightarrow y \in \|\ \textit{made a quilt}\ \|\ ]\}\\
\ \ \{x : \forall y[\ (y \in \|\ c_1\ \| \ \wedge y \subseteq x) \rightarrow y \in \{\{j\}, \{l\}, \{i\}\}]\},\\
\ \ \{x : \forall y[\ (y \in \|\ c_2\ \| \ \wedge y \subseteq x) \rightarrow y \in \|\ \textit{made a quilt}\ \|\ ]\},\\
\ \ \{x : \forall y[\ (y \in \|\ c_2\ \| \ \wedge y \subseteq x) \rightarrow y \in \{\{j\}, \{l\}, \{i\}\}]\}\ \}
\end{array}
$$

Finally, substituting in for the values of Cov*, || c_1 ||, || c_2 ||, and || *made a quilt* || we get

(33) H_C(Part(Cov)(*made a quilt*)) =

$$
\begin{array}{l}
\{\ \{x : \forall y[\ (y \in \{\{i\}, \{j\}, \{k\}, \{l\}, \{m\}, \{n\}\} \wedge y \subseteq x) \rightarrow\\
\qquad\qquad\qquad\qquad y \in \{\{j\}, \{l\}, \{m\}\}]\},\\
\{x : \forall y[\ (y \in \{\{i\}, \{j\}, \{k\}, \{l\}, \{m\}, \{n\}\} \wedge y \subseteq x) \rightarrow\\
\qquad\qquad\qquad\qquad y \in \{\{j\}, \{l\}, \{i\}\}]\},\\
\{x : \forall y[\ (y \in \{\{i\}, \{j\}, \{k\}, \{l, m\}, \{n\}\} \wedge y \subseteq x) \rightarrow\\
\qquad\qquad\qquad\qquad y \in \{\{j\}, \{l\}, \{m\}\}]\}\\
\{x : \forall y[\ (y \in \{\{i\}, \{j\}, \{k\}, \{l, m\}, \{n\}\} \wedge y \subseteq x) \rightarrow\\
\qquad\qquad\qquad\qquad y \in \{\{j\}, \{l\}, \{i\}\}]\},\\
\{x : \forall y[\ (y \in \{\{i\}, \{j\}, \{k\}, \{l\}, \{m, n\}\} \wedge y \subseteq x) \rightarrow\\
\qquad\qquad\qquad\qquad y \in \{\{j\}, \{l\}, \{m\}\}]\},\\
\{x : \forall y[\ (y \in \{\{i\}, \{j\}, \{k\}, \{l\}, \{m, n\}\} \wedge y \subseteq x) \rightarrow\\
\qquad\qquad\qquad\qquad y \in \{\{j\}, \{l\}, \{i\}\}]\}\ \}
\end{array}
$$

Thus we have the fully expanded form of the halo[7] of Part(Cov)(*made a quilt*).

Now that we have provided an account of halos of plural VPs as they are treated in Schwarzschild's analysis, we are ready to tackle the most important question: How do we determine whether a sentence of the form given in (9) is close enough to true for practical purposes?

---

[7] Note that the set is, in accordance with our framework, partially ordered. || Part(c_1)(*made a quilt*) ||, for example, is closer to || Part(Cov)(*made a quilt*) || than is || Part(c_1)({{j}, {l}, {i}}) ||.

As explained in the last section, a sentence in Lasersohn's account is judged close enough to true if its halo has a member that is nonempty. Thus, in order for us to operate within the framework of Lasersohn, all sentences must have pragmatic halos. And for a sentence to have a pragmatic halo, it must denote a set. Therefore, we must take sentences in Schwarzschild's account to be set denoting. The following proposal shows how to do just that.

Recall our rule from Section 2 that stated the truth conditions of the kind of sentences that we are examining:

(9) Let $\varepsilon\ \alpha$ be a sentence, where ε is a definite plural NP and α is plural VP. Then $\varepsilon\ \alpha$ is true iff

$$\|\ \varepsilon\ \| \in \|\ \text{Part(Cov)}(\alpha)\ \|.$$

We wish to put a different slant on this rule, so as to make the analysis compatible with Schwarzschild's. Thus we have

(34) Let $\varepsilon\ \alpha$ be a sentence, where ε is a definite plural NP and α is plural VP. Then

$$\|\ \varepsilon\ \alpha\ \| = \{z\colon z = \|\ \varepsilon\ \| \wedge z \in \|\ \text{Part(Cov)}(\alpha)\ \|\}$$

where $\varepsilon\ \alpha$ is true if $\|\ \varepsilon\ \alpha\ \|$ is nonempty and false if $\|\ \varepsilon\ \alpha\ \|$ is empty.

This rule may seem a bit odd/indirect, but it provides exactly what we want. A sentence now explicitly denotes a set and the sentence's truth conditions are based on whether that set is empty. And, crucially, (34) provides equivalent truth conditions to those stated in (9). That is, $\|\ \varepsilon\ \| \in \|\ \text{Part(Cov)}(\alpha)\ \| \Leftrightarrow \{z\colon z = \|\ \varepsilon\ \| \wedge z \in \|\ \text{Part(Cov)}(\alpha)\ \|\}$ is nonempty. The proof is essentially trivial, and given below:

(35) $\|\ \varepsilon\ \| \in \|\ \text{Part(Cov)}(\alpha)\ \| \Leftrightarrow \exists x = \|\ \varepsilon\ \|$ such that $x \in \|\ \text{Part(Cov)}(\alpha)\ \|$

$$\Leftrightarrow \{z\colon z = \|\ \varepsilon\ \| \wedge z \in \|\ \text{Part(Cov)}(\alpha)\ \|\}$$
is nonempty

Our Schwarzschild-based framework is now fully capable of being incorporated into Lasersohn's account. We can now provide a formalism for halos of sentences of the kind we have been examining:

(36) Let ε α be a sentence, where ε is a definite plural NP and α is plural VP. Then the pragmatic halo for ε α is

$$\langle H_C(\varepsilon\ \alpha), \leq_{\text{Part}_{\varepsilon\ \alpha}, C} \rangle \text{ where}$$

$$H_C(\varepsilon\ \alpha) = \{\ \{z\text{: } z = \|\ \eta\ \| \wedge z \in \|\ \upsilon\ \|\}\text{: } \|\ \eta\ \| \in H_C(\varepsilon) \wedge \|\ \upsilon\ \| \in H_C(\text{Part(Cov)}(\alpha))\}$$

and $\leq_{\varepsilon\ \alpha, C}$ is partial ordering based on how pragmatically close $\|\ \eta\ \|$ is to $\|\ \varepsilon\ \|$ and how pragmatically close $\|\ \upsilon\ \|$ is to $\|$ Part(Cov)(α) $\|$ for each element $\{z\text{: } z = \|\ \eta\ \| \wedge z \in \|\ \upsilon\ \|\}$ of $H_C(\varepsilon\ \alpha)$.

The sentence is then judged close enough to true for practical purposes if $H_C(\varepsilon\ \alpha)$ contains at least one nonempty element.

This formalism may seem complicated but it's not as bad as it looks. As usual, we clarify with an example. Let's work again with sentence (1). We will adopt the following model for this example[8]:

(37) Let M be a model such that

The universe, U'' = {i, j, k, l, m, n, o}
|| the volunteers || = {j, k, l}
|| made a quilt || = {{j}, {l}, {m}}
|| Cov || = Cov* = {{i}, {j}, {k}, {l}, {m}, {n}}

H_C(*the volunteers*) = { {j, k, l}, {j, l} }

H_C(*made a quilt*)[9] = { {{j}, {l}, {m}} } and

H_C(Cov) = { Cov*, c_1 = {{i}, {j}, {k}, {l, m}, {n}} } so

H_C(Part(Cov)(*made a quilt*)) =
{ {x : ∀y[(y ∈ {{i}, {j}, {k}, {l}, {m}, {n}} ∧ y ⊆ x) →

[8] Note that this is a different model than the one we were working with before when discussing sentence (1).
[9] Note that the halo of *made a quilt* is trivial, for the sake of simplicity.

$$
\begin{aligned}
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad y \in \{\{j\}, \{l\}, \{m\}\}]\},\\
&\{x : \forall y[\ (y \in \{\{i\}, \{j\}, \{k\}, \{l, m\}, \{n\}\} \wedge y \subseteq x) \rightarrow\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad y \in \{\{j\}, \{l\}, \{m\}\}]\}\ \}
\end{aligned}
$$

First of all, we should note that, with respect to model M, this sentence is false. For, in this model, the truth conditions of *the volunteers made a quilt* require that {k} be in the denotation set of *made a quilt*, which it is not. However, taking (36) and (37) together, we have

(38) H_C(*the volunteers made a quilt*) =

$$
\begin{aligned}
\{\ &\{z: z = \{j, k, l\} \wedge z \in \{x : \forall y[\ (y \in \{\{i\}, \{j\}, \{k\}, \{l\}, \{m\}, \{n\}\} \wedge\\
&\qquad\qquad y \subseteq x) \rightarrow y \in \{\{j\}, \{l\}, \{m\}\}]\}\},\\
&\{z: z = \{j, k, l\} \wedge z \in \{x : \forall y[\ (y \in \{\{i\}, \{j\}, \{k\}, \{l, m\}, \{n\}\} \wedge\\
&\qquad\qquad y \subseteq x) \rightarrow y \in \{\{j\}, \{l\}, \{m\}\}]\}\},\\
&\{z: z = \{j, l\} \wedge z \in \{x : \forall y[\ (y \in \{\{i\}, \{j\}, \{k\}, \{l\}, \{m\}, \{n\}\} \wedge\\
&\qquad\qquad y \subseteq x) \rightarrow y \in \{\{j\}, \{l\}, \{m\}\}]\}\},\\
&\{z: z = \{j, l\} \wedge z \in \{x : \forall y[\ (y \in \{\{i\}, \{j\}, \{k\}, \{l, m\}, \{n\}\} \wedge\\
&\qquad\qquad y \subseteq x) \rightarrow y \in \{\{j\}, \{l\}, \{m\}\}]\}\}\ \}
\end{aligned}
$$

with respect to the model M.

The set H_C(*the volunteers made a quilt*) is exactly as we want it in accordance with (36). Notice that it has four members because H_C(*the volunteers*) has two members and H_C(Part(Cov)(*made a quilt*)) has two members. Further simplification of (38) yields the following:

(39) H_C(*the volunteers made a quilt*) $= \{\varnothing, \varnothing, \{\{j, l\}\}, \{\{j, l\}\}\}$
$= \{\varnothing, \{\{j, l\}\}\}$

Thus, with respect to model M given in (37), the halo of *the volunteers made a quilt* has a nonempty element, namely {{j, l}}. Thus, although *the volunteers made a quilt* is technically false, it is close enough to true for practical purposes. Note also that the third set in H_C(*the volunteers made a quilt*) demonstrates that in this analysis nonmaximality can come strictly from the pragmatic slack associated with the subject NP and need not rely on ill-fitting covers. For in the case of this third set of

H_C(*the volunteers made a quilt*), the cover is not ill-fitting, and yet the set is nonempty.

What we have done here effectively is provide an alternative to Brisson's account of nonmaximality in NPs, while still respecting Schwarzschild's account of generalized distributivity. We will from here on out refer to this approach as the Lasersohn-Schwarzschild approach to generalized distributivity and nonmaximality[10]. In summary, in the Lasersohn-Schwarzschild approach the truth conditions of a sentence of the form described in (9) are derived by assigning a particular value to the denotation of the subject NP, the VP, and Cov. From there—given a context *C*—we can define the pragmatic halo of the sentence, from which we can determine whether the sentence is pragmatically close enough to true.

6. Some Problems with Schwarzschild's as Interpreted by Brisson and a Solution

Before moving on to a comparison of the two approaches to nonmaximality and distributivity I would like to illustrate a couple of major problems with Schwarzschild's (1996)/Brisson's (2003) analysis. Then, following the presentation of the problems, I will offer an innovation that precludes them. The innovation is afforded to us by the fact that in the Lasersohn-Schwarzschild approach we don't need to rely on 'ill-fitting covers' as the source of nonmaximality. For, as we have seen, under the Lasersohn-Schwarzschild approach nonmaximality can come solely from the pragmatic slack associated with the denotation of the subject NP.

---

[10] In adopting the term 'Lasersohn-Schwarzschild approach' I do not mean to suggest any agreement between Lasersohn and Schwarzschild on the matter of distributivity and/or nonmaximality. Rather, this name is merely to suggest that the approach is one based primarily on a combination of Schwarzschild's account of distributivity and Lasersohn's account of 'pragmatic slack.'

Problems with Schwarzschild's analysis—especially as interpreted by Brisson—occur in the case of 'ill-fitting covers.' Recall the definition of an ill-fitting cover, given in (12):

(12) A cover C is *ill-fitting* with respect to a denotation set D if there is no union X of elements of C such that X = D.

So, for instance || Cov || is an ill-fitting cover of || John and Mary || in the following:

(40) Let U = {j, k, l, m}
|| John and Mary || = {j, m}
|| Cov || = {{m}, {j, k, l}}

Ill-fitting covers like || Cov ||, as identified by Lasersohn (1995), can lead to sentences being judged true in circumstances under which we wouldn't want them to be. Lasersohn points out, as explained in Schwarzschild (1996) and as we've already mentioned, that given a cover like the one in (40), the truth conditions of a sentence like *John and Mary left* depend solely on whether or not Mary left, so that the sentence can still be considered true even if John didn't leave. Schwarzschild says that such a 'pathological' cover should not concern us and "…should be ruled out pragmatically" (77). And, presumably, Schwarzschild is right—it is highly unlikely that a hearer of *John and Mary left* would disregard whether John actually left.

But Schwarzschild fails to specify when, if ever, 'pathological'/ill-fitting covers are allowed[11]. And the example above is not the only case in which ill-fitting covers lead to trouble. There are also cases in which—under Brisson's analysis based on

[11] If anything, by applying the term 'pathological' to an ill-fitting cover in one of his examples, Scwarzschild is suggesting that all ill-fitting covers are "ruled out pragmatically" (1996: 77). In this case, Brisson's account of nonmaximality has no grounding because it relies on ill-fitting covers as the source of nonmaximality.

Schwarzschild's—certain covers can lead to a sentence being evaluated as false, when we would want it to be evaluated as true, (or at least *close enough to true*.)[12]

To illustrate let's return to sentence (1). Let our universe of discourse, U, be defined as follows:

(41) $U = \{a_1, a_2, \ldots, a_{50}, b_1, b_2, \ldots, b_{49}\}$
$\|$ the volunteers $\| = \{ a_1, a_2, \ldots, a_{50}\}$

And suppose that the hearer knows that each volunteer, a_i, was paired up with a corresponding expert quilt-maker, b_i, to help him make a quilt—except for a_{50}, who did not participate in the quilt-making program at all. Furthermore, assume that in this scenario each of the individuals $a_1, a_2, \ldots, a_{49}$ did in fact make a quilt with his assigned expert. Under these circumstances, we would most likely consider (1) at least close enough to true for practical purposes. But suppose that the hearer, knowing that all of the volunteers except a_{50} were paired up with expert quilt-makers, assigns the following cover to the universe of discourse:

(42) $\|$ Cov $\| = \{\{a_1, b_1\}, \{a_2, b_2\}, \ldots, \{a_{49}, b_{49}\}, \{a_{50}\}\}$

Remember that (1) is true if and only if every element of the cover that is a subset of $\|$ the volunteers $\|$ is also an element of $\|$ made a quilt $\|$. In this case, the only element of the cover that is also a subset of $\|$ the volunteers $\|$ is a_{50}. And since a_{50} did not make a quilt, a_{50} is not an element of $\|$ made a quilt $\|$ and (1) would be judged as strictly false.

So, in summary, under the Brisson (2003) approach we have seen circumstances under which sentences that ought to be judged close enough to true are evaluated as false, and vice versa. Furthermore, neither Brisson nor Schwarzschild provides a straightforward and complete account of this problem—they do not definitively explain

[12] This problem was pointed out to me by Ted Fernald.

when ill-fitting covers are licensed nor how the licensing is determined. And even the Lasersohn-Schwarzschild approach, as it currently stands, is susceptible to the same problems.

Thus we need an amendment to the analyses that will preclude these undesirable results. One thing we don't want to do is prohibit ill-fitting covers. After all, it is likely that in certain cases—like the one described above corresponding to sentence (1)— the speaker and/or hearer may group individuals that are in the denotation set of the subject NP with individuals that are not in the denotation set of the subject NP. The cover given in (42) is a perfect example of this. So, we must find some other way to rule out the objectionable results we saw above. Schwarzschild offers the following (but doesn't adopt it himself):

(43) Alternative Semantics for the Part operator:
a. For any Y, a set of sets of individuals, and any y, a set of individuals, Y/x is the largest subset of Y that covers x, if there is one, otherwise it is undefined.
b. $x \in \|\, \text{Part}(\text{Cov})(\alpha) \,\|$ if and only if
$\forall y[\, (y \in \|\, \text{Cov} \,\| /x) \rightarrow y \in \|\, \alpha \,\| \,]$
(1996, p. 77)

As we said, Schwarzschild feels that 'pathological' covers should be ruled out by pragmatics and not semantics, so he is not in favor of adopting this rule. And neither are we, for under this rule $\|\, \text{Cov} \,\| /x$ is undefined in certain circumstances, leaving us no way to evaluate the truth of some sentences.

Under our innovation given below, however, $\|\, \text{Part}(\text{Cov})(\alpha) \,\|$ is always defined. First, we provide a crucial definition:

(44) Let V be a cover of Z and let X be a subset of Z, where X and Z are sets of individuals and V is a set of sets of individuals. Then

$$V{\uparrow}X = \{v \cap X : v \in V \wedge v \cap X \neq \varnothing\}.$$

To illustrate this definition, consider the following example:

(45) Let $C_1 = \{\{i\}, \{j\}, \{k\}, \{l\}, \{m\}, \{n\}\}$
$C_2 = \{\{i\}, \{j\}, \{k, l, m, n\}\}$
$\| \varepsilon \| = \{j, k, l\}$

$$\begin{aligned}
\text{Then } C_1 \uparrow \| \varepsilon \| &= \{v \cap \| \varepsilon \| : v \in C_1 \ \wedge v \cap \| \varepsilon \| \neq \varnothing\} \\
&= \{v \cap \{j, k, l\} : v \in \{\{i\}, \{j\}, \{k\}, \{l\}, \{m\}, \{n\}\} \wedge \\
&\qquad\qquad v \cap \{j, k, l\} \neq \varnothing\} \\
&= \{\{j\}, \{k\}, \{l\}\}
\end{aligned}$$

$$\begin{aligned}
\text{and } C_2 \uparrow \| \varepsilon \| &= \{v \cap \| \varepsilon \| : v \in C_2 \wedge v \cap \| \varepsilon \| \neq \varnothing\} \\
&= \{v \cap \{j, k, l\} : v \in \{\{i\}, \{j\}, \{k, l, m, n\}\} \wedge \\
&\qquad\qquad v \cap \{j, k, l\} \neq \varnothing\} \\
&= \{\{j\}, \{k, l\}\}
\end{aligned}$$

We are now prepared to introduce our innovation—a new definition of the interpretation of $\|$ Part(Cov)(α) $\|$:

(46) $x \in \| \text{Part(Cov)}(\alpha) \|$ if and only if

$$\forall y[\, (y \in \| \text{Cov} \| \uparrow x) \rightarrow y \in \| \alpha \| \,]$$

This innovation is highly powerful for three major reasons. First of all—for V, X and $V \uparrow X$ as defined in (44)—$V \uparrow X$ is always necessarily a cover of X[13]. Thus $\| \text{Cov} \| \uparrow x$ in

[13] The proof of this statement is as follows: We must show that $V \uparrow X$ satisfies the three necessary and sufficient conditions defining covers, as laid out in (3):

(3) C covers A if:
a. C is a set of subsets of A
b. Every member of A belongs to some set in C.
c. $\varnothing$ is not in C.

We take each condition in turn.
A) $V \uparrow X$ is a set if sets, each of which is of the form $v \cap X$ where $v \in V$. By definition of intersection, $v \cap X \subseteq X, \forall v \in V$. Thus $V \uparrow X$ is a set of subsets of X.

B) V is a cover of Z. Therefore, by definition, every element of Z belongs to some set in V. And since $X \subseteq Z$, we know that every element of X also belongs to some set in V. That is, $\forall x \in X \ \exists v \in V$ such that $x \in v$. Thus, $\forall x \in X \ \exists v \cap X \in V \uparrow X$ such that $x \in v \cap X$. That is, every member of X belongs to some set in $V \uparrow X$.

C) By definition of $V \uparrow X$, $V \uparrow X$ does not contain the empty set.

(46) is necessarily a cover of x. This means that our formalism still allows for all possible collections of subsets of the denotation set of the subject NP, (provided that every individual in the denotation set is in one of those subsets.) As a result, the formalism permits distributive readings, collective readings, and any sort of intermediate readings—just as before.

The second key advantage of this innovation is that it precludes the problems associated with ill-fitting covers in Schwarzschild's approach and Brisson's approach. Under the old definition of $\|$ Part(Cov)(α) $\|$, certain covers, (like those in the examples we've seen so far in this section,) lead to problematic truth conditions—ones that don't take certain members of the subject NP's denotation set into account. The new definition of $\|$ Part(Cov)(α) $\|$, however, makes sure that in order for a sentence to be true, *every* individual in the denotation of the NP—either in its own singleton set or as a member of a larger set—must be a member of some set in the denotation of the VP. To see this, first recall (9), which needs no amendment under our innovation:

(9) Let ε α be a sentence, where ε is a definite plural NP and α is plural VP. Then ε α is true iff

$$\| \varepsilon \| \in \| \text{Part(Cov)}(\alpha) \|.$$

By combining (9) and (46) we have:

(47) Let ε α be a sentence, where ε is a plural NP and α is plural VP. Then ε α is true iff

$$\forall y[\ (y \in \| \text{Cov} \| \uparrow \| \varepsilon \|) \rightarrow y \in \| \alpha \|\ ]$$

The truth conditions defined in (47) require that every single element of $\|$ Cov $\|\uparrow\|$ ε $\|$ be in $\|$ α $\|$. And—as we've already shown—no matter what the value of Cov is

$\|$ Cov $\|\!\uparrow\!\|$ ε $\|$ necessarily covers $\|$ ε $\|$, so every member of $\|$ ε $\|$ is a member of some set in $\|$ Cov $\|\!\uparrow\!\|$ ε $\|$. Furthermore, $\|$ Cov $\|\!\uparrow\|$ ε $\|$ is a set of subsets of $\|$ ε $\|$. Thus, taking all of this together, we know that a sentence ε α is true only if[14] for every member of $\|$ ε $\|$ there exists some subset of $\|$ ε $\|$ that is a member of $\|$ α $\|$—regardless of the choice of cover made by the hearer.

Let us now illustrate the utility of this innovation by demonstrating how it 'fixes' the problems we saw earlier in this section. The first sentence we discussed was *John and Mary left*. Recall that the problematic model we examined was as follows:

(40) Let U = {j, k, l, m}
$\|$ John and Mary $\|$ = {j, m}
$\|$ Cov $\|$ = {{m}, {j, k, l}}

As we explained, under the old definition of $\|$ Part(Cov)(α) $\|$, the truth conditions of this sentence don't take into John into account:

(48) *John and Mary left* is true iff

$$\begin{aligned}
& \|\text{ John and Mary }\| \in \|\text{ Part(Cov)}(\alpha)\| \\
\Leftrightarrow\quad & \forall y[\,(y \in \|\text{ Cov }\| \wedge y \subseteq \|\text{ John and Mary }\|) \rightarrow y \in \|\text{ left }\|\,] \\
\Leftrightarrow\quad & \forall y[\,(y \in \{\{m\}, \{j, k, l\}\} \wedge y \subseteq \{j, m\}) \rightarrow y \in \|\text{ left }\|\,] \\
\Leftrightarrow\quad & \{m\} \in \|\text{ left }\|
\end{aligned}$$

However, under the new definition of $\|$ Part(Cov)(α) $\|$, we avoid this problem completely:

(49) *John and Mary left* is true iff

$$\begin{aligned}
& \|\text{ John and Mary }\| \in \|\text{ Part(Cov)}(\alpha)\| \\
\Leftrightarrow\quad & \forall y[\,(y \in \|\text{ Cov }\|\!\uparrow\!\|\text{ John and Mary }\|) \rightarrow y \in \|\text{ left }\|\,] \\
\Leftrightarrow\quad & \forall y[\,(y \in \{\{m\}, \{j, k, l\}\}\!\uparrow\!\{j, m\}) \rightarrow y \in \|\text{ left }\|\,] \\
\Leftrightarrow\quad & \forall y[\,(y \in \{v \cap \{j, m\} : v \in \{\{m\}, \{j, k, l\}\} \wedge v \cap \{j, m\} \neq \varnothing\}) \rightarrow \\
& \qquad\qquad\qquad\qquad y \in \|\text{ left }\|\,] \\
\Leftrightarrow\quad & \forall y[\,(y \in \{\{j\}, \{m\}\}) \rightarrow y \in \|\text{ left }\|\,] \\
\Leftrightarrow\quad & \{j\}, \{m\} \in \|\text{ left }\|
\end{aligned}$$

[14] Note here that we say 'only if' *not* 'if *and* only if.'

Thus, the new definition of $\|$ Part(Cov)(α) $\|$ requires that both John and Mary left in order for the sentence to be considered true.

Let's now turn to the second problem we saw in this section, involving sentence (1). Remember that under Schwarzschild's analysis and Brisson's analysis in the following model:

(50) $U = \{a_1, a_2, \ldots, a_{50}, b_1, b_2, \ldots, b_{50}\}$
$\|$ the volunteers $\| = \{ a_1, a_2, \ldots, a_{50}\}$
$\|$ Cov $\| = \{\{a_1, b_1\}, \{a_2, b_2\}, \ldots, \{a_{50}, b_{50}\}\}$

the sentence was considered utterly false. This is because 1) the sentence's truth conditions aren't satisfied in this model and 2) the sentence can't be considered 'close enough to true for practical purposes' because the notion of 'close enough' doesn't even exist in Schwarzshild's/Brisson's account. Another problem here is that the truth conditions only take one out of the 50 volunteers into account under the old analysis. These results are undesirable in many circumstances. That is, there are certainly times when we would want *the volunteers made a quilt* to be close enough to true if 49 out 50 volunteers did indeed participate in quilt-making. And we virtually always want our semantics to take more than just one of the 50 volunteers into account in determining whether the sentence is close enough to true.

Before showing how our innovation precludes the kind of problem exemplified by this sentence we must first introduce the innovation's third compelling advantage—namely, that it requires very little change to the Lasersohn-Schwarzschild approach outlined in the previous section. We demonstrate this by recalling each formalism we introduced in the previous section, revising the formalisms where necessary.

The first major formalism introduced was the definition of the pragmatic halo of $\|$ Part(Cov)(α) $\|$:

(29) Let Part(Cov)(α) be the translation of a plural VP α, where $\|$ Cov $\|$ = Cov*. Then the pragmatic halo of the plural VP with respect to context C is

$$<H_C(\text{Part(Cov)}(\alpha)), \leq_{\text{Part(Cov)}(\alpha),C}> \text{ where}$$

$$\begin{aligned} H_C(\text{Part(Cov)}(\alpha)) &= \{ \| \text{Part(c)}(\delta) \| : \| c \| \in H_C(\text{Cov}) \wedge \\ &\qquad \| \delta \| \in H_C(\alpha) \} \\ &= \{ \{x : \forall y[(y \in \| c \| \wedge y \subseteq x) \rightarrow y \in \| \delta \|]\} : \\ &\qquad \| c \| \in H_C(\text{Cov}) \text{ and } \| \delta \| \in H_C(\alpha) \} \end{aligned}$$

and $\leq_{\text{Part(Cov)}(\alpha),C}$ is partial ordering based on how pragmatically close $\|$ c $\|$ is to Cov* and how pragmatically close $\|$ δ $\|$ is to $\|$ α $\|$ for each element $\|$ Part(c)(δ) $\|$ of H_C(Part(Cov)(α)).

Because we have changed the definition of $\|$ Part(Cov)(α) $\|$, we need to make one small change to the formalism. Instead of saying

(51)
$$\begin{aligned} &= \{ \| \text{Part(c)}(\delta) \| : \| c \| \in H_C(\text{Cov}) \wedge \| \delta \| \in H_C(\alpha) \} \\ &= \{ \{x : \forall y[(y \in \| c \| \wedge y \subseteq x) \rightarrow y \in \| \delta \|]\} : \\ &\qquad \| c \| \in H_C(\text{Cov}) \text{ and } \| \delta \| \in H_C(\alpha) \} \end{aligned}$$

we want

(52)
$$\begin{aligned} &= \{ \| \text{Part(c)}(\delta) \| : \| c \| \in H_C(\text{Cov}) \wedge \| \delta \| \in H_C(\alpha) \} \\ &= \{ \{x : \forall y[(y \in \| c \| \uparrow x) \rightarrow y \in \| \delta \|]\} : \\ &\qquad \| c \| \in H_C(\text{Cov}) \text{ and } \| \delta \| \in H_C(\alpha) \} \end{aligned}$$

so that we have the following for our definition of the halo of $\|$ Part(Cov)(α) $\|$:

(53) Let Part(Cov)(α) be the translation of a plural VP α, where $\|$ Cov $\|$ = Cov*. Then the pragmatic halo of the plural VP with respect to context C is

$$<H_C(\text{Part(Cov)}(\alpha)), \leq_{\text{Part(Cov)}(\alpha),C}> \text{ where}$$

$$\begin{aligned} H_C(\text{Part(Cov)}(\alpha)) &= \{ \| \text{Part(c)}(\delta) \| : \| c \| \in H_C(\text{Cov}) \wedge \\ &\qquad \| \delta \| \in H_C(\alpha) \} \\ &= \{ \{x : \forall y[(y \in \| c \| \uparrow x) \rightarrow y \in \| \delta \|]\} : \\ &\qquad \| c \| \in H_C(\text{Cov}) \text{ and } \| \delta \| \in H_C(\alpha) \} \end{aligned}$$

and $\leq_{\mathrm{Part(Cov)}(\alpha), C}$ is partial ordering based on how pragmatically close || c || is to || Cov || and how pragmatically close || δ || is to || α || for each element || Part(c)(δ) || of H_C(Part(Cov)(α)).

The only other formalism introduced in the previous section was the definition of the halo of a sentence of the form ε α, repeated below:

(36) Let ε α be a sentence, where ε is a plural NP and α is plural VP. Then the pragmatic halo for ε α is

$$\langle H_C(\varepsilon\ \alpha), \leq_{\mathrm{Part}_{\varepsilon\ \alpha}, C} \rangle \text{ where}$$

$$H_C(\varepsilon\ \alpha) = \{ \{z: z = \|\ \eta\ \| \wedge z \in \|\ \upsilon\ \|\}|: \|\ \eta\ \| \in H_C(\varepsilon) \wedge \|\ \upsilon\ \| \in H_C(\mathrm{Part(Cov)}(\alpha))\}$$

and $\leq_{\varepsilon\ \alpha, C}$ is partial ordering based on how pragmatically close || η || is to || ε || and how pragmatically close || υ || is to || Part(Cov)(α) || for each element $\{z: z = \|\ \eta\ \| \wedge z \in \|\ \upsilon\ \|\}$ of H_C(ε α).

The sentence is then judged close enough to true for practical purposes if H_C(ε α) contains at least one nonempty element.

Notice that here no change is necessary. We must simply remember that the definition of || Part(Cov)(α) || and H_C(Part(Cov)(α)) have been changed due to the innovation. Thus, all in all our innovation required only two changes to our entire framework, including the change that was the innovation itself.

Now we can demonstrate how our innovation accounts for the problems that occur when a model like the one defined in (41) and (42) is applied to a sentence like (1). We begin by reviewing the model we were working with:

(54) $U = \{a_1, a_2, \ldots, a_{50}, b_1, b_2, \ldots, b_{50}\}$
|| the volunteers || $= \{ a_1, a_2, \ldots, a_{50}\}$
|| Cov || $= \{\{a_1, b_1\}, \{a_2, b_2\}, \ldots, \{a_{50}, b_{50}\}\}$

Recall also that in our model, each volunteer a_i had been paired off with an expert quilt-maker b_i, and that every volunteer—except for a_1—did in fact make a quilt both by himself and with his assigned expert, so that

(55) $\|\text{ made a quilt }\| = \{\{a_2\},\ldots, \{a_{50}\}, \{b_1\}, \{b_2\},\ldots, \{b_{50}\},$
$\{a_2, b_2\},\ldots, \{a_{50}, b_{50}\}\}$

Given this model, we can see below that the truth conditions of (1) are not satisfied:

(56) *The volunteers made a quilt* is true iff

$\|\text{ the volunteers }\| \in \|\text{ Part(Cov)(}\textit{made a quilt}\text{) }\|$
$\Leftrightarrow \{ a_1, a_2,\ldots, a_{50}\} \in \{x : \forall y[(y \in \|\text{Cov}\|\uparrow x) \rightarrow y \in \|\text{made a quilt}\|]\}$
$\Leftrightarrow \{ a_1, a_2,\ldots, a_{50}\} \in \{x : \forall y[(y \in\{\{a_1,b_1\}, \{a_2,b_2\},\ldots, \{a_{50},b_{50}\}\}\uparrow x)$
$\rightarrow y \in \|\text{made a quilt}\|]\}$
$\Leftrightarrow \forall y[(y \in\{\{a_1,b_1\}, \{a_2,b_2\},\ldots, \{a_{50},b_{50}\}\}\uparrow\{a_1, a_2,\ldots, a_{50}\})$
$\rightarrow y \in \|\text{made a quilt}\|]$
$\Leftrightarrow \forall y[(y \in\{\{a_1\}, \{a_2\},\ldots, \{a_{50}\}\}) \rightarrow y \in \|\text{made a quilt}\|]$
$\Leftrightarrow \{a_1\}, \{a_2\},\ldots, \{a_{50}\} \in \{\{a_2\},\ldots, \{a_{50}\}, \{b_1\}, \{b_2\},\ldots, \{b_{50}\},$
$\{a_2, b_2\},\ldots, \{a_{50}, b_{50}\}\}$

Thus, since $\{a_1\} \notin \{\{a_2\},\ldots, \{a_{50}\}, \{b_1\}, \{b_2\},\ldots, \{b_{50}\}, \{a_2, b_2\},\ldots, \{a_{50}, b_{50}\}\}$, we know that *The volunteers made a quilt* is false in this model—and rightly so. If one of the volunteers didn't make a quilt—in this case a_1—then we don't want the sentence to be considered true. For as Kroch (1974:190-91, as cited in Lasersohn (1999)) points out, if we allow for such truth conditions we end up with sentences like

(57) Although the volunteers made a quilt, some of them didn't make a quilt.

which sounds contradictory.

Despite the fact (1) is judged false in this model under the Lasersohn-Schwarzschild approach, it can still be considered close enough to true for practical purposes. As long as one of the elements of the pragmatic halo of (1) is nonempty, then the sentence will be considered practically true. Let's demonstrate how this plays out in our new framework. We will assume the following pragmatic halos[15] for the expressions that constitute (1):

(58) $H_C(\textit{the volunteers}) = \{ \{a_1, a_2,\ldots, a_{50}\}, \{a_2,\ldots, a_{50}\} \}$
$H_C(\textit{made a quilt}) = \{ \{\{a_2\},\ldots, \{a_{50}\}, \{b_1\}, \{b_2\},\ldots, \{ b_{50}\}, \{a_2, b_2\},\ldots, \{a_{50}, b_{50}\}\} \}$
$H_C(\text{Cov}) = \{ \{\{a_1, b_1\}, \{a_2, b_2\},\ldots, \{a_{50}, b_{50}\}\} \}$

Note that we made H_C(*made a quilt*) and H_C(Cov) trivial for simplicity's sake, so that

(59) $H_C(\text{Part(Cov)}(\textit{made a quilt})) = \{ \| \text{Part(Cov)}(\textit{made a quilt}) \| \}$

Thus the halo for (1) is

(60) $H_C(\textit{The volunteers made a quilt}) =$
$\{ \{z : z = \{a_1, a_2,\ldots, a_{50}\} \wedge z \in \| \text{Part(Cov)}(\textit{made a quilt}) \| \},$
$\{z : z = \{a_2,\ldots, a_{50}\} \wedge z \in \| \text{Part(Cov)}(\textit{made a quilt}) \| \} \}$

Let's see if either of the two sets in H_C(*The volunteers made a quilt*) is nonempty. If $\{a_1, a_2,\ldots, a_{50}\} \in \|$ Part(Cov)(*made a quilt*) $\|$ then the first set is nonempty, and if $\{a_2,\ldots, a_{50}\} \in \|$ Part(Cov)(*made a quilt*) $\|$ then the second set is nonempty. We already showed above that $\{a_1, a_2,\ldots, a_{50}\} \notin \|$ Part(Cov)(*made a quilt*) $\|$, so we only need to check to see if the second set is nonempty:

(61) $\{z : z = \{a_2,\ldots, a_{50}\} \wedge z \in \| \text{Part(Cov)}(\textit{made a quilt}) \| \}$ is nonempty iff

$\{a_2,\ldots, a_{50}\} \in \| \text{Part(Cov)}(\textit{made a quilt}) \|$
$\Leftrightarrow \forall y[(y \in \| \text{Cov} \| \uparrow \{a_2,\ldots, a_{50}\}) \rightarrow y \in \| \text{made a quilt} \|]$
$\Leftrightarrow \forall y[(y \in \{\{a_1, b_1\}, \{a_2, b_2\},\ldots, \{a_{50}, b_{50}\}\} \uparrow \{a_2,\ldots, a_{50}\}) \rightarrow y \in \| \text{made a quilt} \|]$
$\Leftrightarrow \forall y[(y \in \{\{a_2\},\ldots, \{a_{50}\}\}) \rightarrow y \in \| \text{made a quilt} \|]$
$\Leftrightarrow \{a_2\},\ldots, \{a_{50}\} \in \{\{a_2\},\ldots, \{a_{50}\}, \{b_1\}, \{b_2\},\ldots, \{b_{50}\}, \{a_2, b_2\},\ldots, \{a_{50}, b_{50}\}\}$

Thus, since $\{a_2\},\ldots, \{a_{50}\} \in \{\{a_2\},\ldots, \{a_{50}\}, \{b_1\}, \{b_2\},\ldots, \{b_{50}\}, \{a_2, b_2\},\ldots, \{a_{50}, b_{50}\}\}$, we know that $\{z : z = \{a_2,\ldots, a_{50}\} \wedge z \in \| \text{Part(Cov)}(\textit{made a quilt}) \| \}$ is nonempty. And

[15] Here we call a set $H_C(\alpha)$ a pragmatic halo, although technically when we write a pragmatic halo we include its partial ordering.

so sentence (1) in this model is judged not true, but close enough to true for practical purposes—which is exactly what we want.

To recap, in this section we first pointed out a pair of problems associated with Schwarzschild's (1996) analysis as interpreted by Brisson (2003), which are a result of the possibility of ill-fitting covers. We then presented a new definition for || Part(Cov)(α) || and showed how the Lasersohn-Schwarzschild approach—coupled with this new definition—prevents the problems associated with Schwarzschild's analysis. In the next and final major section we will contrast Brisson's approach and the Lasersohn-Schwarzschild approach[16], arguing for the latter.

7. The Two Approaches to Generalized Distributivity and Nonmaximality: A Final Comparison

Let's begin this section with a brief review of the way nonmaximality is accounted for in each approach. In Brisson's (2003) approach, nonmaximality is the result of covers that are ill-fitting with respect to the subject NP. The example we gave way back in Section 2 was based on the following universe of discourse:

(13) Let U' be the universe, where

$$U' = \{i, j, k, l, m, n\}$$

And let

$$\|\text{ the volunteers }\| = \{j, k, l\}$$

with possible covers of the universe, C_1' and C_2', where

$$C_1' = \{\{i\}, \{j\}, \{k\}, \{l\}, \{m\}, \{n\}\}$$
$$C_2' = \{\{i\}, \{j\}, \{k\}, \{l, m, n\}\}$$

[16] From here on out, when we refer to the 'Lasersohn-Schwarzschild approach' it is to be understood that our framework includes the new definition of || Part(Cov)(α) || introduced in this section, as opposed to Schwarzschild's original definition.

According to Brisson's account, if the hearer assigns the ill-fitting cover C_2' as the value Cov, the *truth conditions* of (1) depend on whether j and k made quilts, and 'doesn't care' whether l made a quilt—even though l is a volunteer:

(62) *The volunteers made a quilt* is true iff:

$$\forall y[\ (y \in \{\{i\}, \{j\}, \{k\}, \{l, m, n\}\} \wedge y \subseteq \{j, k, l\}) \rightarrow y \in \|\text{ made a quilt }\|\ ]$$

$$\Leftrightarrow \quad \{j, k\} \in \|\text{ made a quilt }\|$$

This is an example of how Brisson's analysis—though it does account for nonmaximality—is at the same time not completely satisfying. For under her account, the truth conditions of a sentence ε α can correspond to a nonmaximal reading. As we explained in the last section—in accordance with Kroch (1974:190-91, as cited in Lasersohn (1999))—this is rather undesirable, for if we allow for such truth conditions we end up with sentences like

(57) Although the volunteers made a quilt, some of them didn't make a quilt.

In addition to this problem, Brisson (2003) says that "…speakers and hearers must make room for the *possibility* of ill-fitting covers" (138), making it unclear as to whether the hearer ever actually arrives at a particular cover for a sentence, and whether any one cover is favored over another. Thus we have no explicit way of getting at the truth conditions of a sentence.

Under the Lasersohn-Schwarzschild approach, however, we avoid these problems. Let's review the Lasersohn's (1999) basic framework, which aims to account for the fact that "People speak with varying degrees of precision" (522). As we recall, under this approach every linguistic expression is assigned a context-dependent 'pragmatic halo'—a partially ordered set containing all sets pragmatically 'close enough' to the denotation set of the expression. In addition, every sentence is assigned a denotation set rather than a truth value, and considered true if and only if its denotation set is nonempty. At the same time, a sentence is considered 'close enough to true for practical purposes' with respect to a given context *C* and model *M* if and only if its pragmatic halo contains a set that is nonempty. And so a sentence can be considered close enough to true even if it is technically false.

To review how nonmaximality is accounted for within our Lasersohn-Schwarzschild approach, let's recall the formal definition of the halo of a sentence of the form $\varepsilon\ \alpha$:

(36) Let $\varepsilon\ \alpha$ be a sentence, where ε is a definite plural NP and α is plural VP. Then the pragmatic halo for $\varepsilon\ \alpha$ is

$$<H_C(\varepsilon\ \alpha), \leq_{\mathrm{Part}_{\varepsilon\ \alpha}, C}> \text{ where}$$

$$H_C(\varepsilon\ \alpha) = \{\ \{z\text{: } z = \|\ \eta\ \| \wedge z \in \|\ \upsilon\ \|\}\text{: } \|\ \eta\ \| \in H_C(\varepsilon) \wedge \|\ \upsilon\ \| \in H_C(\mathrm{Part(Cov)}(\alpha))\}$$

and $\leq_{\varepsilon\ \alpha, C}$ is partial ordering based on how pragmatically close $\|\ \eta\ \|$ is to $\|\ \varepsilon\ \|$ and how pragmatically close $\|\ \upsilon\ \|$ is to $\|\ \mathrm{Part(Cov)}(\alpha)\ \|$ for each element $\{z\text{: } z = \|\ \eta\ \| \wedge z \in \|\ \upsilon\ \|\}$ of $H_C(\varepsilon\ \alpha)$.

The sentence is then judged close enough to true for practical purposes if $H_C(\varepsilon\ \alpha)$ contains at least one nonempty element.

As stated in the formalism, elements of the set $H_C(\varepsilon\ \alpha)$ are of the form $\{z\text{: } z = \|\ \eta\ \| \wedge z \in \|\ \upsilon\ \|\}$ where $\|\ \eta\ \|$ is an element of the halo of ε and $\|\ \upsilon\ \|$ is an element of the halo of Part(Cov)(α), and $\varepsilon\ \alpha$ is judged close enough to true for practical purposes if at least one of these sets is nonempty. Another way of thinking about this is to say that $\varepsilon\ \alpha$ is close enough to true for practical purposes if there exists $\|\ \eta\ \|$, an element of the halo of ε, and $\| \upsilon \|$, an element of the halo of α, such that $\| \eta \| \in \| \upsilon \|$. The following is a proof of this statement:

(63) A sentence ε α is judged close enough to true for practical purposes if

$$
\begin{array}{ll}
& H_C(\varepsilon\ \alpha) \text{ contains at least one nonempty element} \\
\Leftrightarrow & \{\ \{z\text{: } z = \| \eta \| \wedge z \in \| \upsilon \|\}\text{:} \\
& \qquad \| \eta \| \in H_C(\varepsilon) \wedge \| \upsilon \| \in H_C(\text{Part(Cov)}(\alpha))\} \\
& \quad \text{has at least one nonempty element} \\
\Leftrightarrow & \text{For some set } \{z\text{: } z = \| \eta \| \wedge z \in \| \upsilon \|\} \text{ where } \| \eta \| \in H_C(\varepsilon) \text{ and} \\
& \| \upsilon \| \in H_C(\text{Part(Cov)}(\alpha)) \text{, there exists a z such that } z = \| \eta \| \text{ and} \\
& z \in \| \upsilon \|. \\
\Leftrightarrow & \text{There exists some } \| \eta \| \in H_C(\varepsilon) \text{ and } \| \upsilon \| \in H_C(\text{Part(Cov)}(\alpha)) \\
& \text{such that } \| \eta \| \in \| \upsilon \|.
\end{array}
$$

This is exactly what we saw, for instance, in (60)-(61) in the previous section. The pragmatic halo for *the volunteers made a* quilt was the following:

(60) H_C(*The volunteers made a quilt*) =

$$
\begin{array}{l}
\{\ \{z : z = \{a_1, a_2,\ldots, a_{50}\} \wedge z \in \| \text{Part(Cov)}(\textit{made a quilt}) \|\ \}, \\
\qquad \{z : z = \{a_2,\ldots, a_{50}\} \wedge z \in \| \text{Part(Cov)}(\textit{made a quilt}) \|\ \}\ \}
\end{array}
$$

As we showed *the volunteers made a quilt* was judged close enough to true within the context and model we had assumed because the second set listed in H_C(*The volunteers made a quilt*) was nonempty. Said another way, there was an element $\| \eta \|$ of H_C(*the volunteers*), namely $\{a_2,\ldots, a_{50}\}$, and an element $\| \upsilon \|$ of H_C(Part(Cov)(*made a quilt*)), namely $\|$ Part(Cov)(*made a quilt*) $\|$, such that $\| \eta \| \in \| \upsilon \|$.

Thus, given a sentence ε α, if there exists some $\| \eta \|$ in the halo[17] of ε and some $\| \upsilon \|$ in the halo of Part(Cov)(α) such that $\| \eta \| \in \| \upsilon \|$ and $\| \eta \| \subset \| \varepsilon \|$, then the sentence will be evaluated as pragmatically close enough to true under a *nonmaximal* reading. For if $\| \eta \| \subset \| \varepsilon \|$ that means that the hearer is still counting the sentence as close enough to true even if not every member of $\| \varepsilon \|$—the denotation set of the NP—is contained in

[17] Keep in mind, of course, that halos are assigned by the hearer.

some member of the denotation of the VP, $\| \alpha \|$. Again, this is exactly what we saw in (60)-(61). $\{a_2,\ldots, a_{50}\} \in H_C$(*the volunteers*), $\{a_2,\ldots, a_{50}\} \in \|$ Part(Cov)(*made a quilt*) $\|$, and $\{a_2,\ldots, a_{50}\} \subset \|$ the volunteers $\|$. The sentence was thus considered close enough to true for practical purposes, even though it was not true that every single volunteer made a quilt.

The last important item to recall for now is that, as we showed in the previous section, the definition of $\|$ Part(Cov)(α) $\|$ associated with the Lasersohn-Schwarzschild approach doesn't allow for nonmaximality to come from ill-fitting covers. Rather, the truth conditions are explicitly defined by a *single* choice of a cover made by the hearer. And regardless of what that cover is, the truth conditions will require that every member of the NP's denotation set be in some member of the VP's denotation set. The cover merely serves to define the way in which members of subject NP's denotation set are grouped together.

Having reviewed all of this, we can lay out two of the major advantages of the Lasersohn-Schwarzschild approach over Brisson's. For one, the Lasersohn-Schwarzschild approach has much more precisely defined truth conditions for sentences of the form $\varepsilon\ \alpha$. Namely, that the hearer determines a cover $\|$ Cov $\|$—ill-fitting or not—based on context, and given this choice, the sentence is true if and only if $\| \varepsilon \| \in \|$ Part(Cov)(α) $\|$. On the other hand, Brisson (p.c.) says, "the listener…has to assume that the speaker might have in mind some ill-fitting cover and 'make room' for the possibility of a nonmaximal reading." And since the truth conditions of a sentence depend on its associated cover, this is equivalent to saying that the speaker must allow for multiple truth conditions—meaning that in Brisson's analysis, the truth conditions of a sentence ε α are not uniquely defined.

The second advantage to note here has to do with how the two approaches treat nonmaximality with respect to truth conditions. Brisson's account allows for a nonmaximal reading to correspond to a sentence's truth conditions, which permits equivocal, objectionable sentences like

(57) Although the volunteers made a quilt, some of them didn't make a quilt.

Under the Lasersohn-Schwarzschild account, however, nonmaximal readings never correspond to truth conditions, which precludes sentences like (57). A nonmaximal reading of a sentence, however, can be evaluated as pragmatically close enough to true under this approach—as we saw in (60)-(61)—whereas the notion of 'close enough to true for practical purposes' doesn't even exist in Brisson's analysis. We thus favor the Lasersohn-Schwarzschild approach because, unlike Brisson's, it clearly defines a sentence's truth conditions, while still allowing for false sentences to be judged close enough to true—where the judgment of what is in fact close enough to true depends on context and the discretion of the hearer.

The third substantial advantage of the Lasersohn-Schwarzschild approach was demonstrated in Section 6. Recall that because Brisson's account of nonmaximality relies on ill-fitting covers coupled with Schwarzschild's definition of $\| \text{Part}(\text{Cov})(\alpha) \|$, it is susceptible to two problematic results: 1) a sentence that ought to be judged false is judged to be true, and 2) a sentence that ought to be judged true (or, more accurately, close enough to true) is judged to be strictly false. The alternative definition of

|| Part(Cov)(α) || that we've adopted as part of the Lasersohn-Schwarzschild approach, however, takes care of these problems. For in this approach, a) a sentence's truth conditions never correspond to a nonmaximal reading and b) nonmaximality is not a result of ill-fitting covers, but rather a result of pragmatic slack associated with the subject NP[18].

The fourth major advantage of the Lasersohn-Schwarzschild approach is that it allows for 'approximation' with respect to covers. That is, it accounts for the fact that while certain contexts call for an obvious choice for the value of Cov—such as a cover containing only singleton sets—there are times when we ought to permit covers that are close but not equal to that obvious choice. As we explained in Section 5, covers are assigned pragmatic halos under the Lasersohn-Schwarzschild approach. This has powerful and advantageous ramifications. To show this, the example we used before was based on the following sentence:

(27) The children each painted a picture.

The presence of *each* in this sentence suggests that its truth conditions should correspond to a distributive reading. This means that the cover associated with this sentence should be a set of singleton sets. The argument made in Section 5, however, was that there might be times in which a reading of (27) that is not strictly distributive would be considered close enough to the truth conditions for practical purposes. The example we used was a model in which *the children* refers to a preschool class of children, all of whom are capable of painting a picture on his or her own—except Ben, who gets help from an older child in the class, Lil. Knowing this, a hearer's pragmatic halo for the cover of this sentence would likely include a set containing a singleton set for each child

[18] Please refer to Section 6 for a more thorough explanation of this.

in the class except for Ben, who is in a duple set with Lil. With this model and this pragmatic halo for Cov, sentence (27) would then be judged not true but close enough to true for practical purposes. Brisson's approach, on the other hand, does not present any way of accounting for this.

The final important benefit of the Lasersohn-Schwarzschild approach is that in it nonmaximality is accounted for as an instance of the much broader phenomenon of pragmatic slack in general. Thus nonmaximality can be accounted for in the same way as other instances of pragmatic slack—such as approximation regarding the time during which an event occurred. This way, more than one kind of pragmatic slack occurring within a single sentence can be accounted for. For instance, the Lasersohn-Schwarzschild approach we've outlined could easily be extended to account for sentences like

(64) The students arrived at noon.

Here there is the possibility of pragmatic slack associated with the subject NP and with the PP *at noon*. That is, it may be that the not every student arrived at noon, and that by noon the speaker did not mean exactly 12pm on the dot. Within Lasersohn's (1999) framework, both instances of pragmatic slack would be captured in the sentence's pragmatic halo. Brisson's approach, however, doesn't frame her analysis of nonmaximality as being part of the more general concept of approximation or 'speaking loosely.' That is, her analysis does not provide a uniform way of accounting for all kinds of pragmatic slack[19]. Therefore, we prefer the Lasersohn-Schwarzschild approach because it is able to account for far more than Brisson's approach.

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[19] This is not meant to imply that Brisson's account 'fails' in this respect. Her analysis simply doesn't attempt to provide a general account for pragmatic slack.

Given all the advantages that the Lasersohn-Schwarzschild approach to generalized distributivity and nonmaximality has over Brisson's it should be quite clear why we so strongly favor this approach. In addition to being more precise, it precludes certain problems associated with Brisson's approach, and accounts for phenomena that Brisson's approach, as it stands, is unable to account for.

9. Summary

In this paper we began by providing a detailed overview of some of the literature of Schwarzschild, Brisson, and Lasersohn as it related to generalized distributivity and nonmaximality in sentences with plural definite subject NPs. Brisson's account of nonmaximality, as we saw, depends on Schwarzschild's generalized D-operator analysis, and so it has a built-in account of generalized distributivity. On the other hand, the portion of Lasersohn's work we focused on doesn't provide any analysis of generalized distributivity. Thus we also introduced a way to incorporate Schwarzschild's generalized D-operator account into Lasersohn's analysis of pragmatic halos. This combination of analyses was dubbed the Lasersohn-Schwarzschild approach to generalized distributivity and nonmaximality.

After introducing this approach, we discussed some of the problems with Brisson's approach, then created a new definition for $|| \text{Part}(\text{Cov})(\alpha) ||$ that was amenable to the Lasersohn-Schwarzschild approach and precluded those problems. Finally, in the last section of the body of the paper, we made our final arguments for the Lasersohn-Schwarzschild approach as opposed to Brisson's. The main reasons we wish to adopt the former is that it accounts for more linguistic phenomena, is more precise, and precludes the problematic results associated with Brisson's approach. More specifically, the

Lasersohn-Schwarzschild approach does not allow for a nonmaximal reading of a sentence to correspond to its truth conditions, and instead considers nonmaximality to be a special case of the general pragmatic phenomenon of 'speaking loosely.'

Furthermore, what is judged close enough to true, crucially, is ultimately up to the context and the hearer. But the manner in which this judgment is made is formally defined to be consistent across hearers in our framework. Therefore, our move to adopt the Lasersohn-Schwarzschild approach implies that in order to have a complete understanding of how we derive meaning from natural language, our linguistic analyses must approach language study from the perspectives of both what is context-dependent and what is defined independent of context.

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