Ball and Beam Control Theory Demonstrator

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Motivation and Objectives:

To create a control theory demonstrator upon which common techniques of control can be experimented. The ball and beam is a very common control theory example problem, and it provides for simple modeling and low project cost.

The ball and beam setup is as displayed in Figure 1. The ball moves freely along the length of the beam. Sensors are placed on one side of the beam to detect the position of the ball. An actuator must drive the beam to a desired angle, either by applying a torque at the center, or in our case, a force at one of the ends.

The objective here was to make a standalone, fully functioning apparatus that would only require one input and one output from a computer (or other) controller. A secondary objective was to program a controller to prove the usefulness of the device. The hope was that this work would in some way contribute to the department's control theory laboratory curriculum.

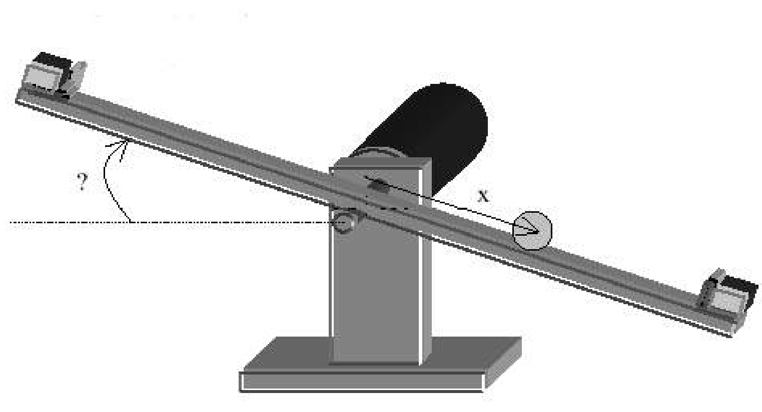

Figure 1

Introduction/Theory:

Classical Control Theory

Control Theory comes in many flavors, but the one we're most familiar with and that I concerned myself with exclusively was classical closed loop system design. In the most basic format, the control loop has only three important elements: the actuator, the sensor, and the comparator. The actuator is some sort of device used to attain some desired result within the system environment. The sensor is the way in which the performance of the actuator is measured. Lastly, the comparator is where the current state of the system is weighed against the referent standard; that is, the desired state of the system. Figure 2 depicts these relationships graphically.

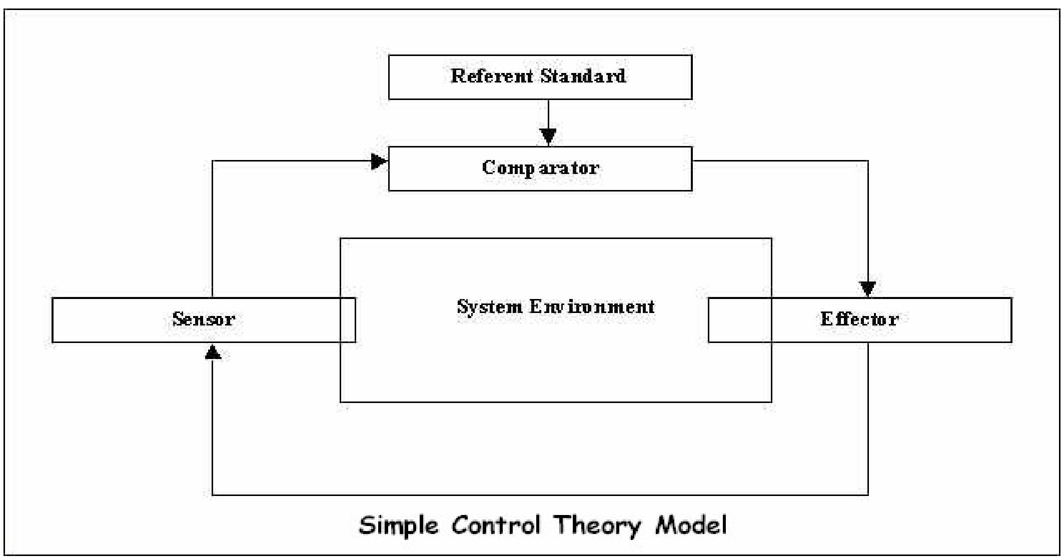

Figure 2

One of the simplest kinds of controllers (comparators) is the PID, which stands for proportional-integral-derivative. Each of the three elements is multiplied by its own constant, and then the sum is used to determine the new inputs for the actuator. The PID controller has the advantage of not requiring an explicit theoretical modeling of the system, and so is a good initial solution.

The general procedure for designing a control system is as follows

- Create design analytically and by simulation
- Choose sensors to measure plant output
- Choose actuators to drive the plant
- Develop the plant, actuator, and sensor models
- Test the controller on the physical system

System Dynamics:

The only theory needed for the physical system designed here is the kinematics of rigid bodies, specifically rolling ball motion. The equations of motion are simplified drastically if we assume linear motion with no slippage or loss of contact with the track. The free body diagram (disregarding the inertial terms) is displayed in Figure 3.

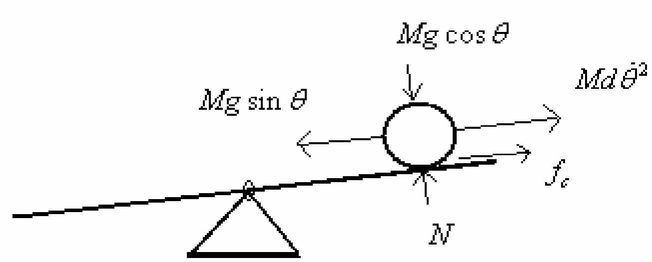

Figure 3

The Md*thetadot-squared term is from the centripetal effect. The typical form this is mv^2/r, but in this case v=d*thetadot, and r=d.

The derivation for the equation of motion of the ball is as follows:

$$\sum F_T \Rightarrow M\left|d\right|\dot{\theta}^2 - Mg\sin\theta + f_c = M\ddot{d}$$
$$\sum M_C \Rightarrow f_c r = -J\ddot{\phi}$$

$$Mr\left|\phi\right|\dot{\theta}^2 - Mg\sin\theta + f_c = Mr\ddot{\phi}$$
$$f_c r = -\frac{2}{5}Mr^2\ddot{\phi} \Rightarrow f_c = -\frac{2}{5}M\ddot{\phi}$$

$$x = r\phi\cos\theta$$
$$y = r\phi\sin\theta + r + h$$

$$Mr\left|\phi\right|\dot{\theta}^2 - Mg\sin\theta - \frac{2}{5}M\ddot{\phi} = Mr\ddot{\phi}$$
$$(r + \frac{2}{5})\ddot{\phi} = r\left|\phi\right|\dot{\theta}^2 - g\sin\theta$$

$$\ddot{\phi} = \frac{r}{r + 2/5}\left|\phi\right|\dot{\theta}^2 - \frac{g}{r + 2/5}\sin\theta$$

This equation was used to simulate the behavior of the ball and beam using the MATLAB/Simulink software suite (see Appendix for details).

Methods and Materials:

This lab involved a multitude of small tasks, mostly related to the building of the system. The first step was deciding on the basic physical properties of the system. It was determined early on that the structure itself didn't have to be particularly elegant, so most decisions were based on the easiest solution available.

An aluminum structural angle with the dimensions 1-1/2 x 1-1/2 x 3/16 x 36-3/8" was determined to be the track. This rests on a wooden block with two nylon screws supporting the sides to maintain the angle's balance as well as limit lateral motion.

The other preliminary decision was the selection of a ball. A lacrosse ball was chosen as a compromise between volume, mass, contact area (and therefore friction and ease of control), and affordability. The ball's diameter is 2.6" and the mass is 170 grams.

The next step was to determine what kind of sensing technology was to be used. After considering a number of other methods, including measuring a voltage on a conductive element effected by the ball position (requires too much sensitivity), and optical detection (requires heavy processing), sonar technology was settled on as a starting point. After ordering and testing a Devantech SRF04 sonar rangefinder, it was decided that it would be a suitable sensor for this system.

This particular sonar has a 0.866 cm wavelength, a 0.03-3 meter range of detection, and a sticker price of $37-$42US. The sonar requires a 5V pulse input and returns another 5V pulse output, with the length of the pulse dependent on the amount of time elapsed before a sound wave returns to the receiver. A 166Hz 50/50 pulse train was therefore applied continuously to the device. The period of the signal was determined based on the maximum length of return pulse that the sensor could be responding with.

The performance of the sonar device when attached to the track was variable based on its orientation. The two orientations where the transmitter and receiver are vertically aligned proved to be the most reliable, producing a relatively error-free, linear relationship between distance and voltage.

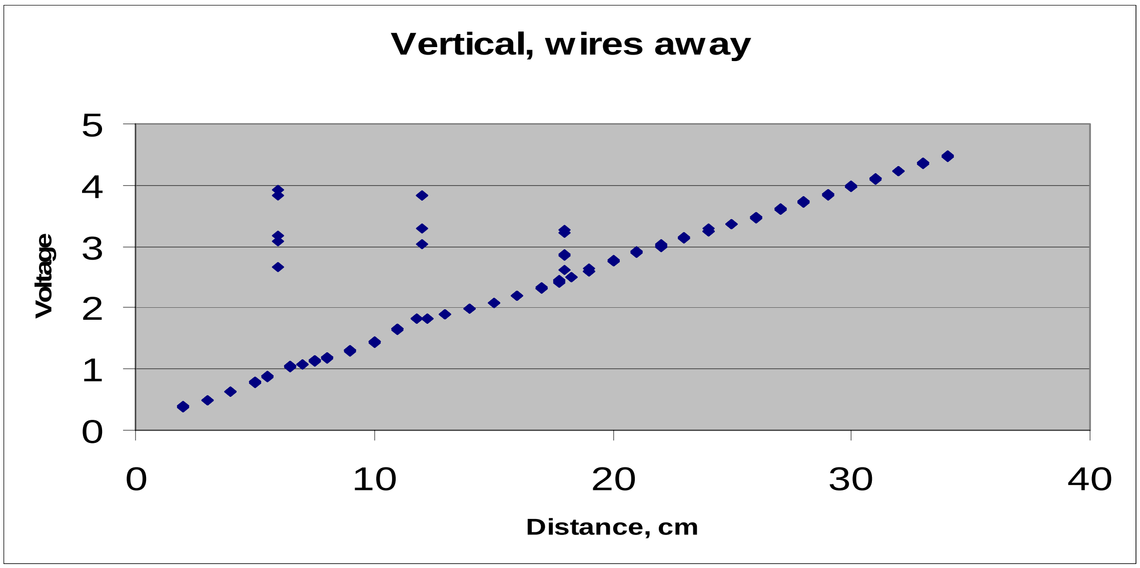

Figure 4

Various problems would later arise in the operation of the sonar. Late in the project cycle the device had to be replaced, raising questions of reliability. Nevertheless, the sensor did a reasonable job of reporting the position of the ball.

The next aspect to consider was the actuator. In reality, it was conceived very early on that a servomotor would be used in this system. Still, a particular device had to be selected and its properties ascertained. In the end the Futaba 3305 servomotor was chosen due to its moderate price of ~$35US (this project never intended to be reproduced on a large scale), ample torque, and reasonable speed.

Specifically, the motor runs at a constant 0.25 sec/60 degree clip when fed 4.8V. It is capable of 7.1kg-cm of torque at that same voltage, and draws a maximum of 1.3A current when fed 5V power. Like all other servomotors, this one has an internal control loop that translates an input voltage into an angle, rather than a torque as would a normal motor.

Figure 5

The servomotor is also driven by a pulse train, but of a lower frequency and duty cycle. The typical servomotor, asks for a 1-2ms pulse every 10-20ms. The length of the pulse determines the angle the motor will turn to.

With all of the integral parts decided upon, the next step was to build the apparatus. Figure 6 shows some of the measurements that went into creating the physical system.

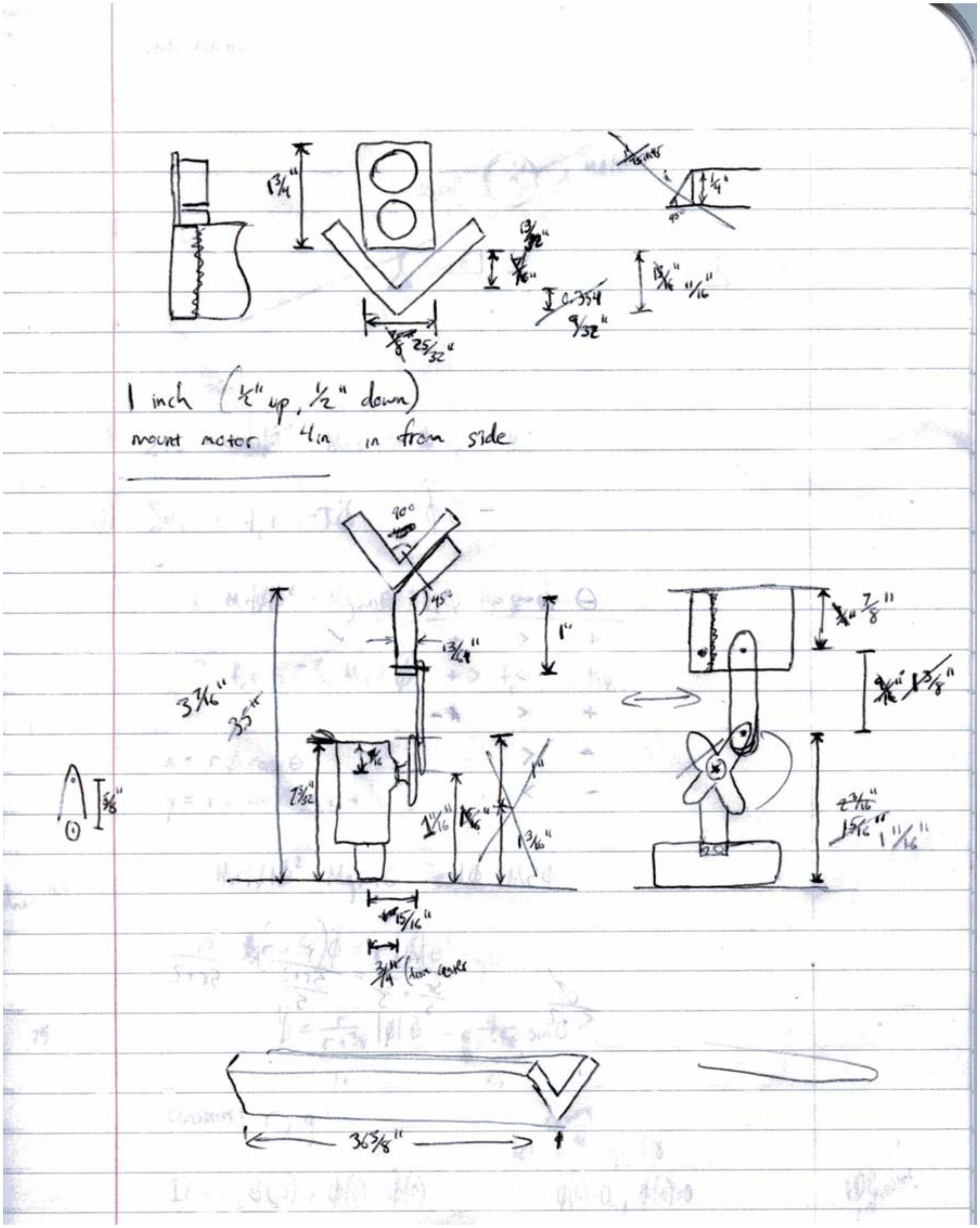

Figure 6

The final step in building the apparatus was making the circuitry to connect the individual parts together. Figure 7 is a flowchart of the basic connections made here.

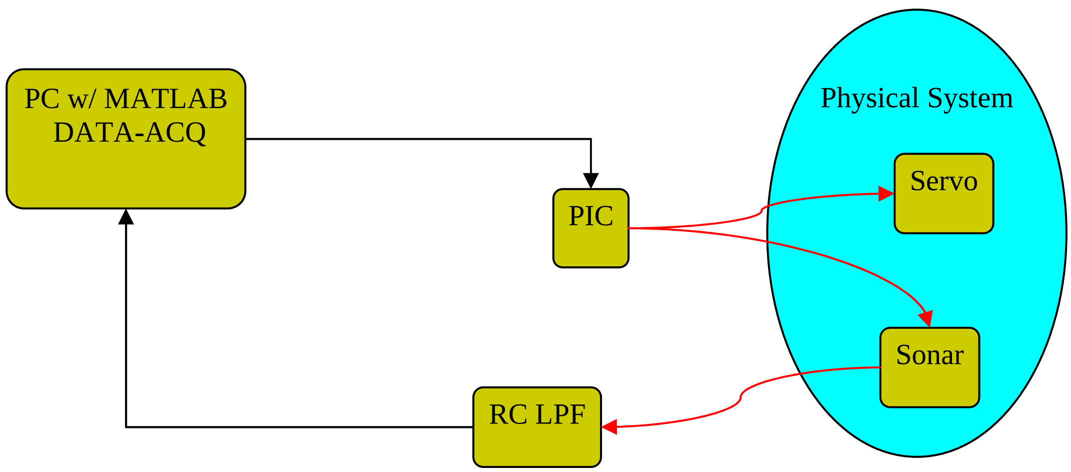

Figure 7

The following photographs show the finished product.

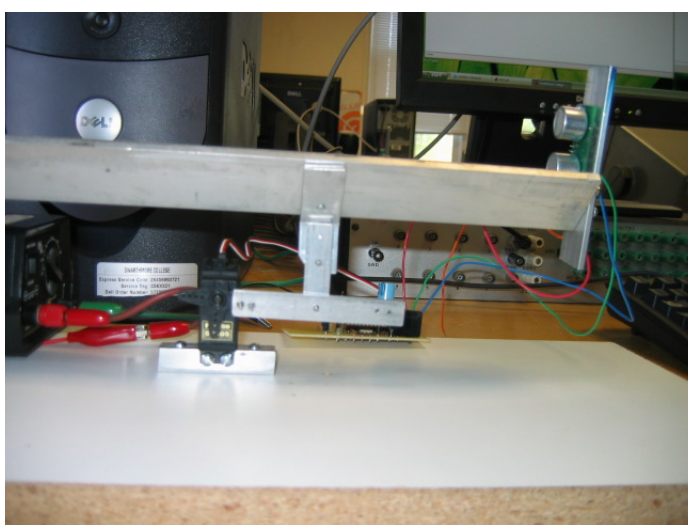

Figure 8

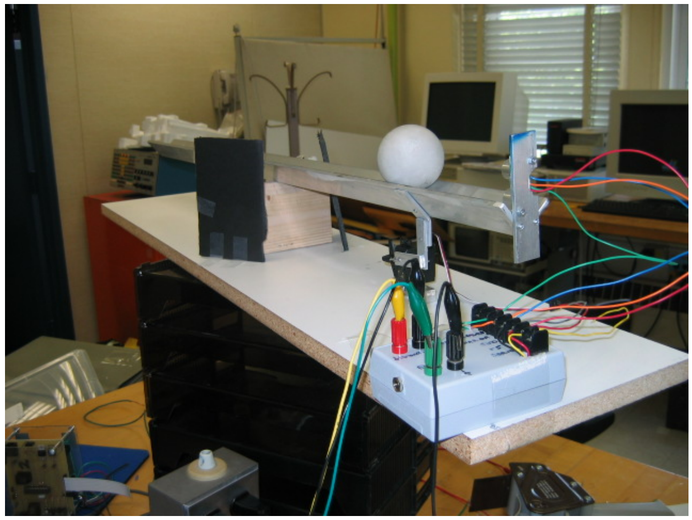

Figure 9

Results

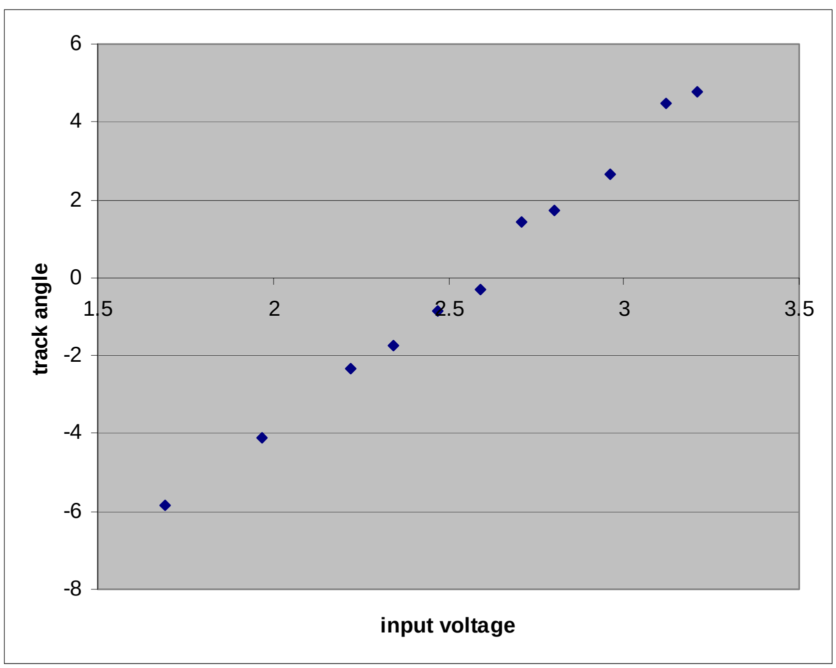

Figure 10: Servo Input v. Track Angle (degrees)

The results displayed in Figure 10 are important because they establish that the input voltage for the system has a linear relationship with the track angle within the operating range of our servomotor as currently configured.
After that had been established the design of a controller was the next step. Continuing from the example of the Simulink model, a PD controller was quickly implemented. Unfortunately the derivative term was quite noisy, which meant it had to be amplitude-limited. Also, its coefficient was scaled down drastically. All this led to a very underdamped solution. The other problem lied in the static friction of the ball resting on the track. When the angle was slight, the ball would tend to rest, and so the controller would get stuck before it reached its destination. The solution to this was to add an integral term, which builds the longer the ball stays on one side of the position it is supposed to be. This had the effect of exciting the ball out of these points of stagnation, and often to their desired location. The following figures depict various runs using this PID controller.

(Vtrack refers to the desired position, Vin refers to the actual position, and Vout refers to the servo control signal. Error is the difference between Vin and Vtrack, Deriv is the velocity of the ball, and Intgrl is the accumulated error.)

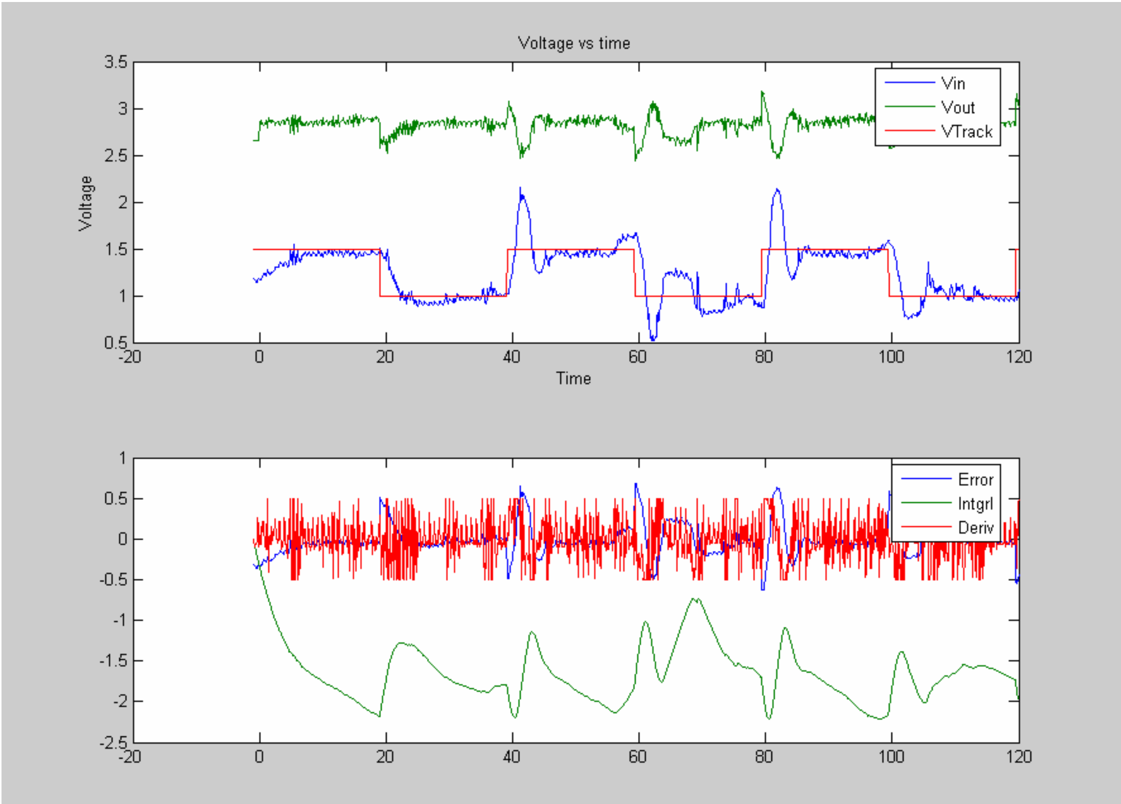

Figure 11: 1.5-1V desired position, 2.5V center

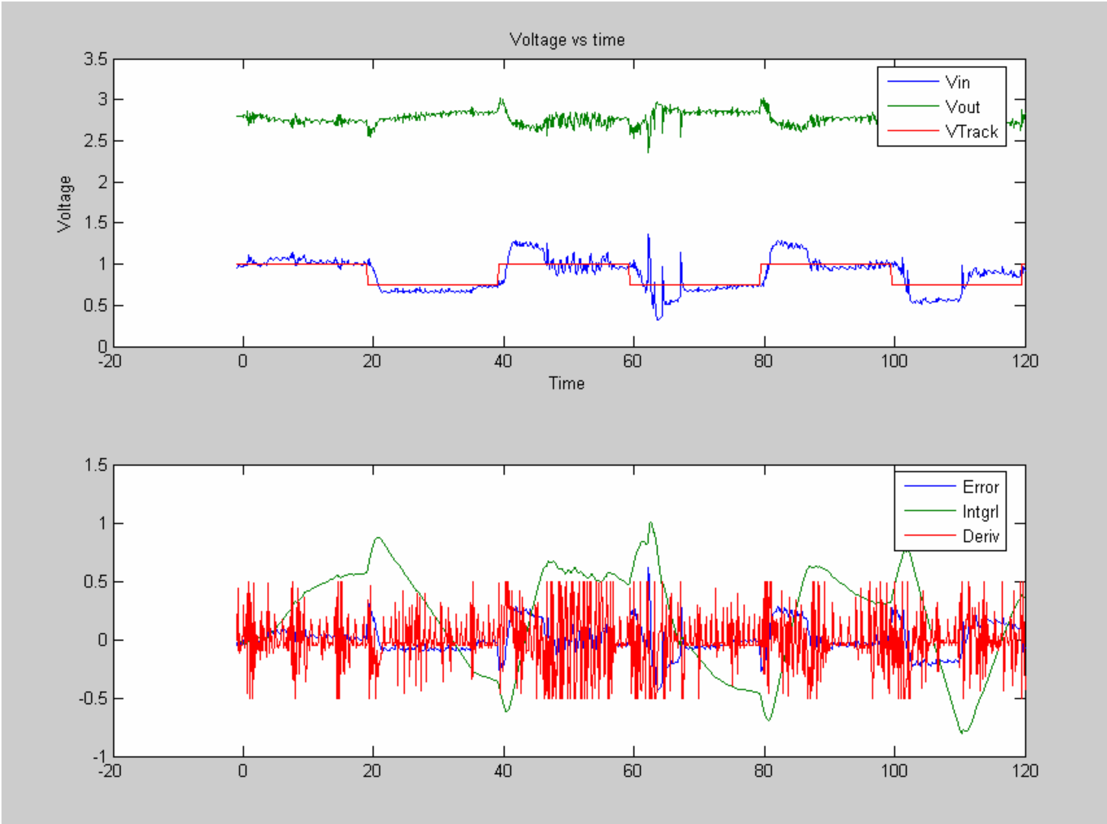

Figure 12: 1-0.75V desired position, 2.8V center

Conclusions

The project was a success in that there is a working system with a sample controller.

Future Work

Many improvements could be made to this design, some of which can be implemented in software. Filter the derivative input in an intelligent manner could provide for a solution closer to critical damping, as could the design of a different type of controller.

A different beam, with a slight curvature would create an interesting unstable system to contend with. Also a different ball material with smaller contact area would present a new set of challenges and benefits.

Appendix: Simulink Model

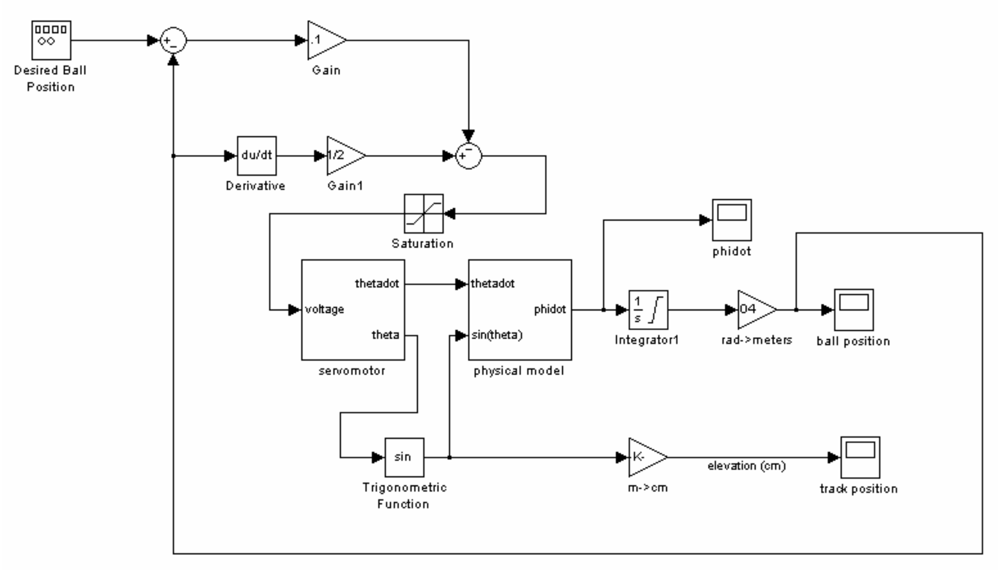

Figure 13: Complete system model in Simulink

Figure 13 shows the simulation model fabricated using the equation derived in the theory section of this report. The constants were chosen in an attempt to adhere to the design decisions of the real world project. The controller used only the proportional and derivative terms; this model did not need an integral term since the physical modeling was idealized. The results from this control loop are displayed in Figure 14.

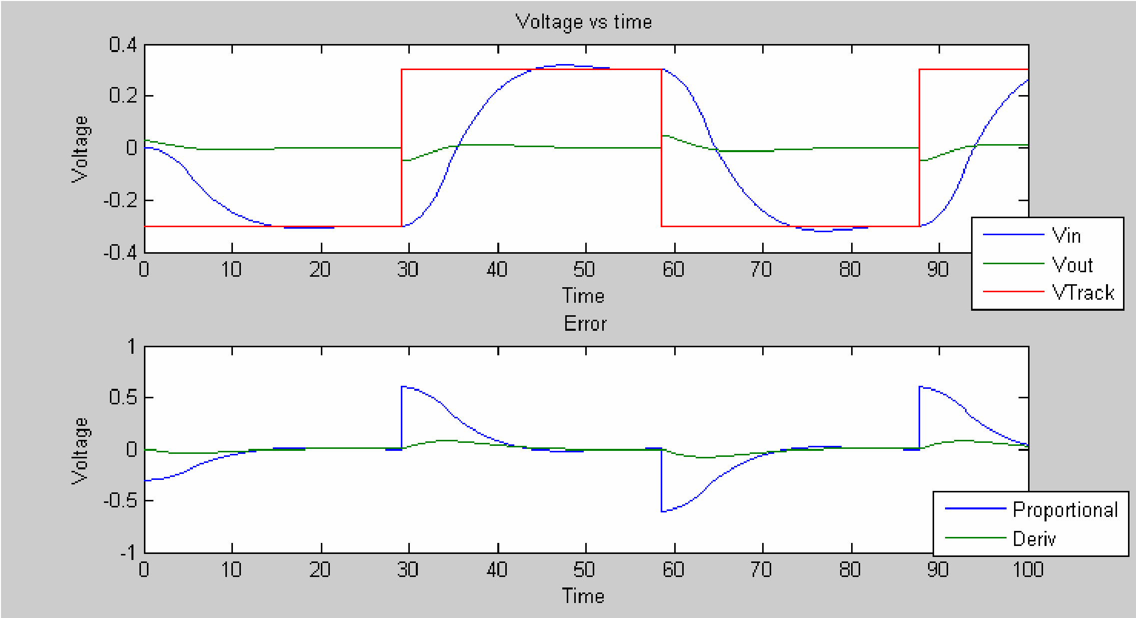

Figure 14: Simulated Results

The response is very close to being critically damped.