Worksheet 2 - Practice with Integration by Substitution

1. Compute the following integrals.

a) $\int \cos 3x \, dx$ b) $\int \frac{1}{\sqrt[3]{4x+7}} \, dx$ c) $\int_1^2 xe^{x^2} \, dx$

d) $\int e^x \sin(e^x) \, dx$ e) $\int_1^e \frac{(\ln x)^3}{x}$ f) $\int \tan x \, dx$ (Hint: $\tan x = \frac{\sin x}{\cos x}$)

g) $\int \frac{x}{x^2+1}$ h) $\int \frac{\arcsin x}{\sqrt{1-x^2}} \, dx$ i) $\int_0^1 (x^2+1)\sqrt{2x^3+6x} \, dx$

2. Find and correct the mistakes in the following "solutions" to these integration problems.

a) $\int \sin 5x \, dx; \quad u = 5x, \; du = 5dx$

$$\int \sin 5x \, dx = \int 5 \sin u \, du = -5 \cos u + C = -5 \cos 5x + C$$

b) $\int \frac{\cos x}{1+\sin^2 x} \, dx; \quad u = \sin x, \; du = \cos x dx$

$$\int \frac{\cos x}{1+\sin^2 x} \, dx = \int \frac{du}{1+u^2} = \ln|1+u^2| + C = \ln|(1+\sin^2 x)| + C$$

c) $\int_0^1 (4x+1)(2x^2+x)^{\frac{5}{3}} \, dx; \quad u = 2x^2+x, \; du = 4x+1$

$$\int_0^1 (4x+1)(2x^2+x)^{\frac{5}{3}} \, dx = \int_0^1 u^{\frac{5}{3}} \, du = \left[\frac{3}{8}u^{\frac{8}{3}}\right]_0^1 = \frac{3}{8}$$

3. Evaluate four of the following five integrals (one cannot be done using methods we know so far):

a) $\int \sin(7x) dx$

b) $\int \frac{1}{\cos^2 x} \, dx$

c) $\int x \sin x \, dx$

d) $\int \frac{\cos(2x)}{[\sin(2x)]^3} dx$

e) $\int \sin^{27} x \cos x dx$

4. Yes, more integrals to solve!

a) $\int \frac{\tan x}{\cos x} \, dx$ b) $\int \frac{1}{x\sqrt{\ln x}} \, dx$ c) $\int e^{e^x} e^x \, dx$

5. Show that:

a) $\int_0^1 f(x) \, dx = \int_0^1 f(1-x) \, dx$

b) $2\int_a^b f(x)f'(x) \, dx = (f(b))^2 - (f(a))^2$