Another approach to finding Moment of Inertia

Let \( I_z \) be the moment of inertia around the \( z \)-axis. This is what we have computed so far, but in this problem we will also compute \( I_y \) and \( I_x \). Of course, for a ball or sphere with center at the origin they will all be the same by symmetry. This observation makes this method computationally simpler than the original method.

Let a point \( dM \) of mass be at position \((x, y, z)\). Then what we have written as

\[
\,dI_z = r^2 \,dM
\]

could just as well be written

\[
\,dI_z = (x^2 + y^2) \,dM,
\]

since the distance \( r \) of the point-mass from the \( z \)-axis is \( r = \sqrt{x^2 + y^2} \).

Similarly, you should write down expressions using \((x, y, z)\) coordinates for \( I_y \) and \( I_x \).

Now, let \( a = \sqrt{x^2 + y^2 + z^2} \). Thus \( a \) is the distance of the point mass from the origin. We would normally call this distance \( r \) or \( \rho \) or \( R \), but all these letters are already taken!

Anyway, if we had to integrate \( a^2 \,dM \), that would be a lot easier (for either a ball or a sphere) than integrating \( r^2 \,dM \). Why?

Last piece of the puzzle: What is a simplified integral formula for \( I_x + I_y + I_z \), and what is the relationship between \( I_x + I_y + I_z \) and \( I_z \)?