SSX scalings

In the spirit of my last note on the dimensionless Ohms law, here’s a set of scalings for various parameters of interest for SSX. The idea is to have a set of formulae for quick calculations. These are all worth checking on your own. Let’s set some nominal values for SSX:

\[ \ell_{0.1m} \equiv 0.1 \, m \]
\[ B_{0.1T} \equiv 0.1 \, T \]
\[ T_{10} \equiv 10 \, eV \]
\[ n_{14} \equiv 10^{14} \, cm^{-3} \]

Using these, we can write down a few simple formulae:

\[ \rho_i = 0.32 \, cm \frac{T_{10}^{1/2}}{B_{0.1T}} \]
\[ \rho_e = 0.075 \, mm \frac{T_{10}^{1/2}}{B_{0.1T}} \]
\[ \lambda_D = 2.35 \, \mu m \frac{T_{10}^{1/2}}{n_{14}^{1/2}} \]
\[ \beta \equiv \frac{W_{\text{kin}}}{W_{\text{mag}}} = 0.03 \frac{n_{14} T_{10}}{B_{0.1T}^2} \]
\[ N_{\text{particles}} \simeq 10^{19} n_{14} \]
\[ W_{\text{mag}} \simeq 0.4 \, kJ \, B_{0.1T}^2 \]
\[ f_{ci} = 1.52 \, MHz \, B_{0.1T} \]
\[ f_{ce} = 2.8 \, GHz \, B_{0.1T} \]
\[ f_{pe} = 90 \, GHz \, n_{14}^{1/2} \]
What this means is that typically $\rho_i \approx 0.32 \text{ cm}$ in SSX. If $T_i$ is $4 \times$ larger, ie. $T_i = 40 \text{ eV}$, then $\rho_i$ is $2 \times$ larger, ie $0.64 \text{ cm}$. Note that $\rho_i$ also scales like $\sqrt{M_{\text{ion}}}$ so the carbon ions we’re measuring have a gyroradius $\sqrt{12} = 3.5 \times$ larger than protons at the same temperature. A typical DeBye length in SSX is a few microns and higher densities than $10^{14}$ only make it smaller.

$$v_A = 22 \frac{\text{cm}}{\mu s} \frac{B_{0.1T}}{n_{14}^{1/2}}$$

where $22 \frac{\text{cm}}{\mu s} = 220 \frac{\text{km}}{s}$

$$\tau_A = \frac{\ell}{v_A} = 0.46 \mu s \frac{\ell_{0.1m}n_{14}^{1/2}}{B_{0.1T}}$$

$$v_i = \sqrt{\beta v_A}$$

$$\delta_i = \frac{c}{\omega_{pi}} = 2.3 \text{ cm} \frac{\sqrt{n_{14}}}{\tau_A}$$

$$\delta_e = \frac{c}{\omega_{pe}} = 0.53 \text{ mm} \frac{\sqrt{n_{14}}}{\tau_A}$$

Now, if we invoke classical Spitzer resistivity, which is based on Coulomb collisions between ions and electrons, we have:

$$\eta = 5.15 \times 10^{-5} \frac{Z \ln \lambda}{T_e^{3/2}} \Omega \text{ m}$$

where $\ln \lambda$ is about 10 for SSX, $Z$ is about unity, and $T_e$ is expressed in eV. Plugging in the values for SSX at 10 eV we get a scaled resistivity:

$$\eta_{10}^{SSX} = 1.6 \times 10^{-5} T_e^{-3/2} \Omega \text{ m}$$

Using this, we can calculate all the SSX parameters that depend on resistivity.

$$\tau_{\text{res}} \equiv \frac{\mu_0 \ell^2}{\eta} = 770 \mu s \ell_{0.1m} T_{10}^{3/2}$$

$$S \equiv \frac{\tau_{\text{res}}}{\tau_A} = 1674 \frac{\ell_{0.1m} T_{10}^{3/2} B_{0.1T}}{n_{14}^{1/2}}$$

$$R_m = 76 \frac{v_{cm}}{\mu s} \ell_{0.1m} T_{10}^{3/2}$$
\[
\tau_{\text{whistler}} = \frac{\ell^2}{\delta_e^2 \omega_{ce}} = 2 \mu s \frac{n_{14}}{B_{0.1T}}
\]

Stuff you can do if you have these formulae memorized include quickly comparing our parameters to others. Our \(\delta_i = 2.3 \, cm\). The magnetosphere plasma density is \(10 \, cm^{-3}\) or \(10^{13}\) smaller than ours. Since \(\sqrt{10^{13}} \approx 3 \times 10^6\), the magnetosphere ion inertial scale (important for reconnection there) must be \(3 \times 10^6\) bigger than ours or about \(10^7 \, cm\) or \(100 \, km\) (which is right). Spacecraft measurements (Polar and Cluster, see Mozer PRL) show a reconnection layer in the magnetosphere about that scale.