Interesting relationships between plasma lengths

Important length scales in plasma physics can often be represented as a ratio of a velocity to a frequency. For the ions with thermal speed $v_i$ we have for the Larmor radius and ion inertial length:

$$
\rho_i = \frac{v_i}{\omega_{ci}}
$$

$$
\delta_i = \frac{v_A}{\omega_{ci}} = \frac{c}{\omega_{pi}}
$$

From this we immediately find:

$$
\frac{\rho_i}{\delta_i} = \frac{v_i}{v_A} = \sqrt{\beta}
$$

From the ion inertial length equations we get:

$$
\frac{v_A}{c} = \frac{\omega_{ci}}{\omega_{pi}}
$$

The ratio of cyclotron to plasma frequency for the ions isn’t so interesting. More useful is that ratio for the electrons:

$$
\frac{\omega_{ce}}{\omega_{pe}} = \sqrt{\frac{M_i}{m_e}} \frac{\omega_{ci}}{\omega_{pi}}
$$

For SSX at $10^{15}$ and 0.1 T we have:

$$
\frac{\omega_{ce}}{\omega_{pe}} = \frac{1}{100} = \frac{v_A}{c} \sqrt{\frac{M_i}{m_e}} = \frac{70 \text{ km/s}}{300,000 \text{ km/s}} \times 43
$$

Since $\omega_{pe} \gg \omega_{ce}$, SSX is referred to as “over-dense”.

For the electrons with thermal speed $v_e$ we have for the Larmor radius, electron inertial length, and Debye length:

$$
\rho_e = \frac{v_e}{\omega_{ce}}
$$

$$
\delta_e = \frac{v_{Ae}}{\omega_{ce}} = \frac{c}{\omega_{pe}}
$$

$$
\lambda_D = \frac{v_e}{\omega_{pe}}
$$

The electron Alfven speed isn’t so interesting but from the inertial and Debye length formulas we get:

$$
\left( \frac{\lambda_D}{\delta_e} \right)^2 = \frac{kT_e}{m_e c^2}
$$