Please explicitly state any results you use in answering the following questions.

1. Denote the subset $\{(x, y) : x^2 + y^2 = 1\}$ of the plane by S^1. Show that S^1 is connected.

2. Is $C([0, 1])$ compact with the metric
$$d(f, g) = \max_{p \in [0,1]} \{|f(p) - g(p)|\}?$$
Why or why not?

3. Prove that the function
$$f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$
is differentiable but that
$$f(x) \neq \int_0^x f'(x) dx.$$

4. Approximate
$$\int_0^{.1} \sin(x^2) dx$$
up to three decimal places.

5. Suppose C is a convex, closed curve in the plane. Namely, if you pick any two points on the curve and join them by a line, the line lies entirely inside the closed curve. (Like an oval.) Assume the curve is smooth.

    Let P be an arbitrary point on the curve. Prove that it is possible to find two other points Q and R on the curve so that PQR is an equilateral triangle.

6. (a) Give the definition of a k-manifold in $\mathbb{R}^n$, $0 < k \leq n$.
    (b) Is $I = [0, 1]$ a 1 manifold in $\mathbb{R}$? Why or why not?
    (c) Is $I^2 = I \times I = [0, 1] \times [0, 1]$ a 1 manifold in $\mathbb{R}^2$? Why or why not?

7. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ given by
$$f(x, y) = (e^x \cos y, e^x \sin y).$$
Prove that
(a) f is C^∞.
(b) f is locally one-to-one.
(c) f is not one-to-one.