

A Sovereign Debt Model with Trade Credit and Reserves

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Abstract

This paper analyzes sovereign debt in an economy in which the availability of short-term trade credit reduces international trade transaction costs. The model highlights the distinction between gross and net international reserve positions. Borrowed reserves provide net wealth and liquidity services during a negotiation, as long as they are not fully attachable by creditors. Moreover, reserves strengthen the bargaining position of a country by shielding it from a cut-off from short-term trade credits thereby diminishing its degree of impatience to conclude a negotiation. We show that competitive banks do lend for the accumulation of borrowed reserves, which provide partial insurance.

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1 Introduction

Access to short-term trade credits has often been identified as key to understanding why countries repay their debts, if not for reputational considerations alone. Describing the debt crisis of the 1980s, Ron Chernow notes that

“The collective power of the commercial banks kept the lid on the pressure cooker in a way impossible in earlier times. These giant global banks had many more levers than the investment banks of the 1920s with which to keep debtors from outright repudiation. Among other things, they could cancel the trade credits of defaulting countries or reduce their “interbank” credit lines. In consequence, as the 1980s progressed the banks were able to boost their loan-loss reserves and weather the crisis, while living standards tumbled in indebted Latin American countries” (1990, p. 647).

Kenneth Rogoff is no less emphatic in a 1999 survey of the global financial architecture: “The strongest weapon of disgruntled creditors, perhaps, is the ability to interfere with short-term trade credits that are the lifeblood of international trade” (1999, p. 31). Yet short-term trade credits have not been formally incorporated into the sovereign debt literature.¹ This paper attempts to bridge this gap.

¹In a seminal paper that introduced retaliatory trade measures into the sovereign debt literature, Bulow and Rogoff (1989a) refer to the importance of trade credits but do not formally model their role.

As a form of debt transaction, trade credit can be thought of as providing some combination of investment finance, consumption smoothing, and risk-sharing. But the quotations above point to a more distinctive role: trade credits reduce the transactions costs associated with international trade. Capturing this liquidity role is central to our analysis. Puzzles immediately ensue, however, once this liquidity role is recognized: in particular, sovereign borrowers routinely hold large stocks of gross international reserves, which pay very low interest rates relative to the rates payable on long-term debts, and highly-indebted countries are often reluctant to use reserves to retire outstanding debts, even at the discounted rates available on the secondary market. What justifies the accumulation and retention of what are, in effect, borrowed international reserves, if liquidity is available in the form of undrawn trade credit lines? An adequate account of debt and trade credits, in our view, will have to be a unified account of debt, trade credits, and international reserves.

By tackling this problem directly we obtain a set of important insights regarding the role of international reserves. First, while gross reserves are dominated by undrawn credit lines under conditions of perfect creditworthiness, they constitute a superior form of liquidity under conditions of debt distress. Figure 1 tracks the evolution of trade credit lines for the 19 countries that defaulted after 1990 and the countries that did not, according to Standard & Poor's. Bankers significantly reduced their exposure to

countries in default: trade credit lines to the median defaulting country were 48% lower than those in the control group four years after the default. In the cases of Venezuela (1995), Ukraine (1998) and Ecuador (1999), the cuts exceeded 50 percent after 24 months. In our analysis, trade credits dry up in a situation of serious arrears. International reserves, in contrast, receive substantial protection during debt distress, particularly if the debtor country is involved in a good-faith negotiation. Legal protections provide one line of defense; central bank assets held in the USA, for example, are protected by the Foreign Sovereign Immunities Act. But reserves can also be repatriated or moved to third-party countries, and aggressive action by lending-country banks or governments is restrained by reputational concerns, given inter-bank competition for deposits. The record, in any case, is clear: there have been very few successful freezes of reserves in association with debt difficulties.

If gross reserves are to be available when trade lines disappear, they must be accumulated in advance, during normal times when long-term debts are being serviced and trade credits are freely available. Gross reserves may prove inefficient ex post, because with some probability the country will avoid a situation of debt distress. But ex ante, reserves are not dominated by credit lines unless the probability of debt distress is negligible.² The point that

²Anecdotal evidence confirms the imperfect substitutability of reserves and short-term credits during debt difficulties. In 2002, for example, Brazilian Central Bank Governor Arminio Fraga added \$30bn of IMF funds to international reserves, after securing the IMF's agreement that these balances might be used to extend short-term credits. Fraga explained that "It is much easier to negotiate the lowering of the net reserves limit [with the IMF] so that funds can be used to intervene in the foreign exchange market or fund

credit lines are imperfect substitutes for gross reserves is surely more general, since the former may be subject to market contagion or other phenomena unrelated to the borrowing country's economic performance.

Second, we find a theoretical underpinning for anecdotal evidence suggesting that the terms of rescheduling agreements may be sensitive to the ability of creditors and borrowers to 'wait out' a bargaining process. From the borrower's side, time pressure comes from the disappearance of trade credit lines during the default period. Pre-existing international reserves alleviate this time pressure by providing an interim source of trade finance. The terms of repayment therefore shift in favor of the debtor, and by a larger amount the less impatient the debtor is to reach agreement. From the lenders' perspective, time pressure may take the form of regulatory deadlines for declaring delinquent loans non-performing (Dornbusch 1984). For any given level of debtor impatience, the effect of such deadlines is to shift repayment terms further in favor of the borrower.

Finally, reserves may improve the borrower's outside option in a debt negotiation. In our analysis, outright default is inefficient and is not observed in equilibrium, but bargaining outcomes may be affected by the threat of the borrower or lender to terminate negotiation, if this threat is credible. This introduces a final role for reserves. Borrowed reserves are offset by external liabilities and therefore do not constitute net wealth ex ante. But commercial trade lines, than to negotiate a new financial assistance package."

unless they are fully attachable in default (a situation distinct from good-faith negotiation), the non-attachable portion does represent net wealth in the event of a repudiation. A higher stock of reserves therefore increases the credibility of the borrower's threat to walk away. If this outside option is binding, the impact is again to shift bargaining power towards the borrower.

Taking the liquidity and net wealth roles together, we find that reserves may allow the debtor to shift consumption from a high-consumption state in which debt is repaid to a low-consumption state in which debt is rescheduled (van Wijnbergen 1990). We show that competitive banks will end up lending for reserve accumulation, suggesting that borrowed reserves may be interpreted, in part, as a mechanism for shifting risk from risk-averse borrowers to risk-neutral lenders.

While our model allows for the possibility that higher reserve holdings lead to higher rather than lower debt repayments, our main contribution is to explain why countries with sizeable foreign reserves sometimes obtain favorable concessions from creditors or show reluctance to spend reserves on debt buyback operations. A recent negotiation between the Russian and German governments provides an example. In the beginning of 2001, after its reserves had tripled to \$24 billion, the Russian government adopted a hard line with its creditors, of which Germany was the principal, by declaring a "technical delay of repayments." The timing coincided with ongoing negotiations with the German government over some \$6.4 billion worth of "transfer

roubles” – an artificial currency used for trade in Soviet times.³ Following discontinuation of debt service, Germany responded by withholding new export credit guarantees to Russia. Although the overall success of the Russian strategy is an open issue, Germany settled one year later for \$440 million, at the same time agreeing to raise the insurance cover of business with Russia.

We are not aware of a study that quantifies the relationships between trade finance and sovereign lending on a systematic basis. The limited available evidence suggests, however, that the effects may be of first order. Rose (2005) finds empirical support for the hypothesis that the downside of a non-repayment strategy comes through the trade channel: changes in international debt contracts are generally followed by substantial reductions in trade flows between the creditor and debtor country. While Rose mentions both retaliatory trade measures and reductions in the availability of trade credits as candidate explanations for this finding, the scope for the former appears to be rather narrow. An increasing number of countries are WTO members, and the GATT articles make no provision for non-repayment of debt in enumerating exemptions to the non-discrimination principle. No such legal impediment applies to trade credit, of course (as suggested by Figure 1): it is provided on a voluntary basis, often by private banks deeply

³The first creditor affected was the German export credit guarantee group Hermes, which did not receive repayments of Soviet-era borrowings. The sovereign debt analyst for the rating agency Standard and Poor’s commented on the Russian threat of default by noting that “They are not desperate for funds, [and] that obviously strengthens their position.” See “Russia’s Threat of Default,” *Financial Times*, January 5, 2001.

involved in other long-term lending operations, or by creditor-country government agencies.

Our own analysis generates a precise set of predictions regarding the determinants of sovereign debt ‘haircuts’ – the realized losses to private creditors in debt restructuring. Using the haircuts data compiled by Sturzenegger and Zettelmeyer (2005), we find broad support for the bargaining model developed here. These results suggest a high return to further econometric work on the relationships between debt, trade credits, and reserves.

Relation to the sovereign debt literature

The sovereign debt literature has evolved around a controversy about the form of punishment that disgruntled creditors can impose on defaulting borrowers. We allow the country’s assets to be partially seized in the event of a repudiation, thereby adopting a framework that is closer to Bulow and Rogoff (1989a), who assume that a fraction of exports can be attached by lenders, than to Eaton and Gersovitz (1981) or Bulow and Rogoff (1989b), where non-repayment is punished with permanent exclusion from credit markets and reputational considerations alone support repayment. In practice, attachments have occasionally occurred during confrontational debt renegotiations; see Delaume (1994) for an extended discussion and Wright (2002, pp. 35-37) for an account of the recent legal battle between the Swiss Compagnie Noga d’Importacion and the Russian government. Given full information and rational expectations, asset seizures do not actually occur in our analy-

sis: deadweight losses are avoided in equilibrium (Eaton and Engers 1999). The possibility of attachment nonetheless conditions the bargaining outcome by defining the threat points.

In a related theoretical paper, Detragiache (1996) argues that international reserves increase rather than decrease international debt repayments. As in the reserve demand model of Aizenmann and Marion (2004), Detragiache relies on a combination of convex Barro-style tax distortions and the non-existence of domestic debt markets. The argument is that international reserves reduce the borrower's bargaining power by reducing the adjustment costs associated with repaying debt from current tax revenue. A limitation of this argument, however, is that as long as the borrower holds international reserves, it can choose the timing of default. If reserves increased negotiated repayments, the borrower would always choose – absent some other motivation for retaining reserves – to get rid of reserves just ahead of formally entering default.⁴ While in our model reserves may increase repayments to lenders, they unambiguously reduce the lenders' share of the surplus. Moreover, the welfare of a borrower in arrears is a monotonically increasing function of the stock of reserves. The model may therefore explain why we do not see debt buyback operations with greater frequency during

⁴Consider the strategy, for example, of using reserves to pay for government expenditures, whether domestic or imported, in advance. If the resulting decline in reserves reduces debt repayments, tax distortions fall, and this benefit is obtained with no impact on the time path of government spending. To eliminate this possibility, one must go further than ruling out domestic debt markets; there must be no intertemporal trade of any kind with suppliers.

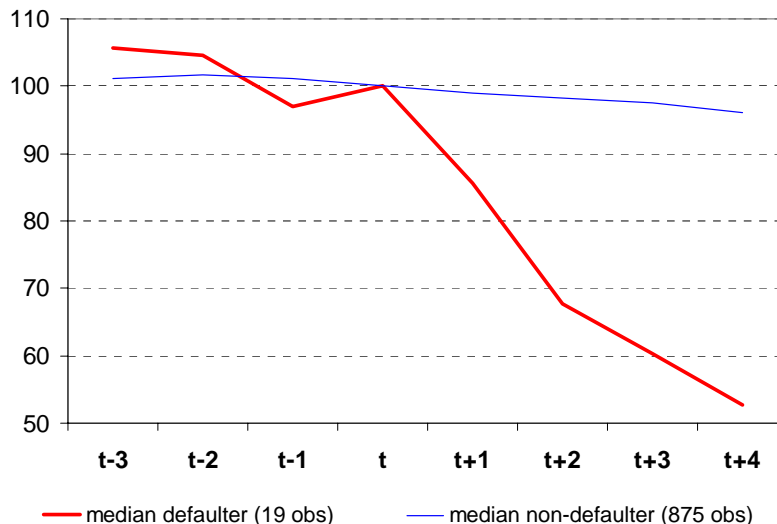


Figure 1: Bank Trade Credit Lines (BIS)

debt crises. It may also explain why most borrowers that do default do so with positive reserve holdings, a case that has been referred to as *strategic default* in the literature.

Outline

The paper is organized as follows. Section 2 outlines the model. Section 3 analyzes the bargaining game that ensues the moment output is realized and debt service is due. Following Rubinstein (1982), we find a unique subgame-perfect equilibrium by exploiting the relative impatience of the players and the requirement that only credible threats affect the play. Section 4 scrutinizes the borrower’s decision between repayment and rescheduling, condi-

tional on the anticipated bargaining outcome. Section 5 then studies the reserve accumulation process by endogenizing long-term borrowing in advance of a potential rescheduling. We show that competitive lenders provide finance not just for investment projects, but also for the accumulation of international reserves, and that reserves provide the borrowing country with partial insurance against randomness in the return to investment. We conclude by testing some empirical implications of the model and discussing directions for future research.

2 The Model

We begin by introducing a motivation for long-term debt, a source of potential repayment problems, and a characterization of the liquidity roles of reserves and short-term trade credits. These elements will lead us to an insurance model of reserves where insurance takes the form of state-dependent liquidity services.

The model is a hybrid of a two-period and an infinite-horizon model. At time zero the borrower enters a competitive loan market in which a large number of risk-neutral lenders competes to provide funds. Banks maximize expected profits, discounting at the rate r which is less than the rate of time preference, δ , of the debtor-country's government. Competition drives expected profits to zero.

2.1 Investment, production, and debtor preferences

Since trade is central to our story, we model the borrowing country as a small open economy that trades a perishable commodity export for a (numeraire) import good that is not produced locally. Debt is initially zero, and is accumulated in the first period ($t = 0$) to finance a major investment project that requires an indivisible input of one unit of the imported good. The investment project produces a stochastic output of a second, storable export good at time $t = 1$, where s is a discrete random variable with finite support whose distribution is common knowledge.

The country's budget constraint states that in period 0, gross external borrowing, B , must finance the current account deficit plus any accumulation of reserves:

$$B = 1 + c_0 - y + ((1 + r)^{-1} R_1 - R_0)$$

Here c_t and R_t denote consumption and start-of-period reserves in t , respectively; the commodity export accrues as an endowment at the rate of y units per annum, with an international terms of trade equal to 1.

The country's preferences are given by

$$U_0 = u(c_0) + \beta E v(W_1) \tag{1}$$

where $u(\cdot)$ and $v(\cdot)$ are twice-differentiable, concave functions and W_1 is an index of future consumption. The expectation in (1) is taken over the probability distribution of output from the investment project. Concavity

of $v(\cdot)$ implies that the country will wish to insure against variability of W_1 arising from the stochastic production technology.

Although a two-period structure is all we need to study the insurance role of reserves, we want debt service on the original loan to be determined by a potentially time-consuming bargaining process. We therefore treat W_1 as a measure of consumption over the indefinite future. To generate closed-form solutions, we specify W_t as the present value of consumption, so that for $t \geq 1$, the borrower maximizes

$$W_t = \sum_{i=0}^{\infty} [\beta(h)]^i c_{t+hi} \quad , \quad t \geq 1 \quad (2)$$

where $\beta(h) = (1 + \delta h)^{-1}$ is the country's discount factor and where h will coincide with the interval between alternate proposals during a debt renegotiation (we will suppress the dependence of β on h when this can be done without confusion). The country therefore maximizes utility over an infinite horizon, although at time 0 all that is relevant is the expected discounted value of future consumption. The import of (1) and (2) is that while the country is risk-averse with respect to the index it is risk-neutral with respect to the timing of consumption after period 0. These preferences are not stationary, but they are well-behaved.⁵ They imply that at time $t = 1$, any bargaining that may take place over the servicing of external debt will involve two risk-neutral players.

⁵In particular, this particular form of non-stationarity does not introduce a time-consistency problem: the marginal rate of substitution between consumption in any two future periods is the same regardless of the period from which it is viewed.

2.2 Trade finance

An adequate account of short-term trade finance must incorporate both the substitutability of reserves and trade credits as alternatives to international barter and their fundamental asymmetry in the event of a repayment crisis. To capture these features we follow earlier work in the monetary theory literature (particularly Kimbrough (1986) and Smitt-Grohe and Uribe (2004)) in modeling the time cost of international trade transactions as an increasing function of the volume of (balanced) trade, y , and a decreasing function of the total liquidity, L , available to the borrowing country. More formally,

(Transactions technology) The time cost of international trade transactions is $T(L/y)$, where T is nonnegative and twice continuously differentiable function which satisfies $\partial T/\partial(L/y) \leq 0$. Time costs display satiation for some finite value of liquidity, so that $\lim_{L \rightarrow \bar{L}} T(L/y) = 0$ for some $\bar{L} > 0$. Total liquidity is defined as the sum of gross reserves and undrawn credit lines. The latter are zero if the borrower is in arrears on long-term debt. Otherwise undrawn credits are always at least equal to \bar{L} , generating liquidity satiation regardless of the level of gross reserves.

Total time costs of transacting are equal to $T(L_t/y) \cdot y$. Time costs can arise either on the export side or on the import side, and the precise mix is unimportant for our analysis. The key is that in an equilibrium with balanced trade, the country's consumption of exportables net of transactions costs will

be

$$c_t = p(L_t/y) \cdot y$$

where $p(L_t/y) = 1 - T(L_t/y)$ represents the terms of trade net of transactions costs. In normal times, satiation prevails and we get $p_t = 1$ and $c_t = y$. But when the borrower is cut off from short-term trade finance, $L_t = R_t$ and the country's effective terms of trade become an increasing, concave function of the stock of international reserves. The cost of operating in financial autarky is the loss in real income per unit time due to the non-availability of trade credits, $T(R_t/y) \cdot y$.

The dependence of the terms of trade on liquidity gives lenders the ability to harass a recalcitrant borrower by interfering with its access to short-term trade credits during a debt rescheduling. Lenders have a strong incentive to do so, in order to increase the borrower's impatience to reach an agreement. We assume that they are able to cut off short-term trade finance completely until the relationship with current creditors is terminated, either through a negotiated agreement or through unilateral repudiation by either party. Access to trade credits is restored once this point is reached.

3 The Bargaining Game

Output arrives at period 1, at which point the country chooses unilaterally whether to repay its external debt in full or to renegotiate. Since all uncer-

tainties have been resolved, the payoffs to these two strategies are known. While the debt may be owed to multiple banks, we assume that once arrears have emerged a single lead bank acts on behalf of all lenders. In this section we analyze the bargaining game in order to determine the payoffs from renegotiation. Section 4 then takes up the repayment decision.

At time 1, the country's total resources consist of reserves, durable and perishable export goods. On paper, these assets are offset by debt service obligations where z is the promised interest rate on debt incurred in period 0. Its actual liability, however, only amounts to the minimum of what it owes and what it can be bargained into repaying. To analyze the bargaining game, we adopt the alternating offers framework of Rubinstein (1982), as outlined in Figure 2. The bank and country take turns at making proposals over how to divide the country's resources at time t , denoted by $\pi_t = R_t + Q$.

We use $q^*(t)$ to denote the share of the pie to be received by the country when the bank makes the proposal and $q(t)$ to denote this share when the country makes the proposal (throughout the paper, starred variables will refer to banks). Supposing that the bank has the first offer, the bargaining game is characterized by a sequence of alternating offers $q^*(t)$, $q(t+h)$, $q^*(t+2h)$, $q(t+3h)$, ..., that take place at intervals of length h .

After each proposal, the responding player either accepts or turns down the offer. In case of agreement, π_t is split according to the proposed terms. The agreement restores the country's creditworthiness and its access to trade

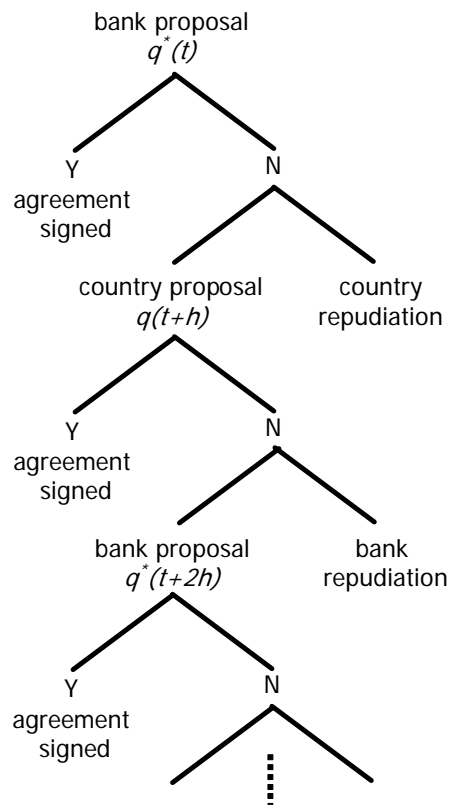


Figure 2: The bargaining game

credits, allowing the country to trade its perishable export at value $p = 1$, regardless of reserves. At this point the demand for foreign reserves will be zero, and the pressure of discounting will induce the country to consume its remaining assets and its claim on current export proceeds immediately. If the players disagree, the responder may terminate the negotiation unilaterally by walking away, or may wait to make a counter-offer.⁶

The repudiation option, if exercised by either player, terminates the good-faith negotiation and induces banks to seize what they can of the country's reserves and confiscate what they can of the country's storable export good. If immediate repudiation is efficient, then there is nothing to bargain over and the country's choice at $t = 1$ reduces to one of repayment or repudiation. But a hostile default is unlikely to be handled passively by creditor banks and governments. We therefore follow Bulow and Rogoff (1989a) in assuming that the confiscation of debtor-country output is costly. The debtor loses a fraction α of its output, but lenders only collect a fraction $\alpha(1 - \mu) < \alpha$ of it. The deadweight loss $\alpha\mu Q$ is an essential feature of our model: it gives the country and its creditors an incentive to engage in bargaining, with the attendant possibility of costly delay. We also allow lenders to attach a fraction $\gamma \geq 0$ of reserves, an option that in our analysis creates no additional deadweight loss.

⁶Sutton (1986) analyzes a game in which the responder has access to an outside option with a positive probability. The game here assumes that the probability is 1 and the outside option is unilateral termination of the negotiation.

There are three ways, then, that a negotiation can end: by agreement to the bank's proposal, by agreement to the country's proposal, or by unilateral repudiation by one of the players. The country's post-negotiation utility is given by

$$W_t = \begin{cases} q(t)\pi_t + \delta^{-1}y & \text{if agreeing to country's proposal} \\ q^*(t)\pi_t + \delta^{-1}y & \text{if agreeing to bank's proposal} \\ \lambda(t)\pi_t + \delta^{-1}y & \text{if repudiation occurs} \end{cases}$$

where $\lambda(t) = \pi_t^{-1} [(1 - \gamma) R_t + (1 - \alpha) Q]$ is the country's share of the pie under repudiation. $\delta^{-1}y$ represents the present value of imports financed by the future commodity endowment, assuming full access to trade credit.

⁷ If the negotiation ends amicably (as it does in equilibrium), there is no deadweight loss and creditors collect either $(1 - q^*) \pi$ or $(1 - q) \pi$, as relevant. In the repudiation case, lenders collect only $(1 - \lambda(t) - D_t) \pi_t$, where $D_t = \pi_t^{-1} \mu \alpha Q$ is the deadweight loss associated with confiscation of output.

3.1 The bargaining solution

We solve the model by exploiting recursive nature of the game. Consider first the case in which the bank places the offer at time t . Since delay is costly, the bank's optimal strategy is to offer the minimum acceptable share to the borrower. If the country is to accept this offer, however, the resulting

⁷Recall that any resolution of the negotiation (including repudiation) restores credit-worthiness. Note that with $\delta > r$, the country could consider selling its output stream to lenders, who attribute a higher value to it. Lucas (1979) underscores the incentive and enforceability problems associated with such a contract, and we are assuming that these are prohibitive.

utility must be at least equivalent to what the country could get by turning the offer down and either repudiating or (after a delay of length h) making the minimal acceptable counter-offer. The bank's offer is therefore pinned down by

$$q^*(t)\pi_t + \delta^{-1}y = \max [\lambda(t)\pi_t + \delta^{-1}y; \beta (q(t+h)\pi_{t+h} + \delta^{-1}y) + p(R_t/y)hy + rhR_t] \quad (3)$$

The second term inside the brackets measures the country's utility if it waits to make the minimum acceptable counter-offer.⁸ Note that the borrower in (3) consumes the proceeds from the sale of the perishable export and interest accruing on reserves; we show in section 3.3. that this constitutes an optimal reserve policy during renegotiation.⁹ Also, and more crucially for our story: although lenders cannot impose any penalties beyond the period of repudiation, their ability to cut off trade credits during the negotiation reduces the minimum offer they must make. The debtor country suffers a (deadweight) loss amounting to $T(R_t/y) \cdot y$ each period, by virtue of financing its trade using reserves rather than trade credit. If the stock of reserves is constant (as under an optimal reserve policy), this term induces a fixed bargaining cost of the type introduced by Rubinstein (1982).

A second relationship between bank and country offers can be ob-

⁸More precisely, within-period timing is as follows: I) the borrower consumes out of the reserve stock R ; II) interest payments and endowments accrue; III) trades take place (and reserves are replenished); IV) bargaining or repudiation.

⁹To maintain simplicity, we consider that the conversion of interest or export proceeds into reserves does not entail time costs. Although the inclusion of such cost would reduce the country's share, this simplification is otherwise without loss of generality.

tained by considering the country's counter-offer at time $t + h$. As before, the optimal offer leaves the responder – in this case, the bank – indifferent between accepting and refusing. The payoff from refusing, in turn, is the maximum of what the bank can get by either repudiating or waiting to make the next offer. We therefore have

$$1 - q(t + h) = \max \left[\begin{array}{l} 1 - \lambda(t + h) - D(t + h); \\ \beta^* \frac{\pi_{t+2h}}{\pi_{t+h}} (1 - q^*(t + 2h)) \end{array} \right] \quad (4)$$

Substituting (4) into (3) to eliminate $q(t + h)$, we obtain

$$q^*(t)\pi_t = \max \left[\begin{array}{l} \lambda(t)\pi_t; \beta \min \left[\begin{array}{l} (\lambda(t + h) + D(t + h)) \pi_{t+h}; \\ \pi_{t+h} - \beta^* \pi_{t+2h} (1 - q^*(t + 2h)) \end{array} \right] \\ - (\beta - p(R_t/y)) hy + rhR_t \end{array} \right] \quad (5)$$

the solution to which takes the form

$$q^*(t) = \max \left[\lambda(t); \min \left[\begin{array}{l} q_N^*(t); \beta \frac{\pi_{t+h}}{\pi_t} (\lambda(t + h) + D(t + h)) \\ - (\beta - p(R_t/y)) \frac{hy}{\pi_t} + \frac{rhR_t}{\pi_t} \end{array} \right] \right] \quad (6)$$

where $q_N^*(t)$ is the unique solution to the second-order difference equation in the bank's offer $q^*(t)$ that is imbedded in expression (5):

$$q_N^*(t) = \sum_{k=0}^{\infty} (\beta\beta^*)^k \left[\begin{array}{l} \beta \frac{\pi_{t+(2k+1)h}}{\pi_t} - \beta\beta^* \frac{\pi_{t+2(k+1)h}}{\pi_t} \\ - (\beta - p(R_{t+2kh}/y)) \frac{hy}{\pi_t} + \frac{rhR_{t+2kh}}{\pi_t} \end{array} \right] \quad (7)$$

Although we have been referring to q as the minimum share the country receives in a subgame perfect equilibrium, it is also the maximum share and therefore the equilibrium solution (Appendix A).¹⁰ The bank's

¹⁰Rubinstein (1982) studied the cases of discounting and (constant) bargaining costs separately. In the case of constant bargaining costs, the solution is discontinuous in the bargaining cost and possibly non-unique, with the player with lower cost receiving either the entire pie (if he moves first) or anything greater than or equal to the pie less his bargaining cost (the solution is not unique if the high-cost player moves first). We get uniqueness and continuity in the bargaining cost due to the simultaneous presence of discounting in our setup.

equilibrium strategy is to propose $q^*(t)$ given by (6) when it has the offer, and to refuse any offer below $1 - q(t)$, given by equation (4), after substituting from (6) for $q^*(t + h)$. Conversely, the country offers the amount given by equation (4) and refuses any offer below the quantity $q^*(t)$ defined by equation (6). The solution is immediate: the first offer will be implemented, so that deadweight losses due to delay or repudiation are avoided.

The unwieldy form of (7) is in part an artifact of the bank's arbitrary advantage as the first proposer. This advantage disappears if there are no barriers to the rapid exchange of offers and counter-offers. As the time between counter-offers gets arbitrarily small, the bargaining solution takes the simpler form

$$q^* = \max [\lambda; \min [q_N^*; \lambda + D]] \quad (8)$$

where $\lambda = \lambda(1)$ and where the equilibrium offer ignoring outside options is given by

$$q_N^* = \lim_{h \rightarrow 0} q^*(1) = \frac{r - \pi^{-1} (T(R/y) \cdot y - rR)}{r + \delta} \quad (9)$$

We interpret the expression for q_N^* below, after studying the country's optimal reserve policy during a renegotiation. Meanwhile the logic of equation (8) appears in Figure 3, where for a given value of π_1 we measure the country's share on the horizontal axis and the bank's share on the vertical axis. Potential bargaining solutions lie on the efficient sharing locus \overline{ab} . Since confiscation of output involves a deadweight loss, the repudiation payoffs $[\lambda, 1 - \lambda - D]$ lie strictly inside this locus.

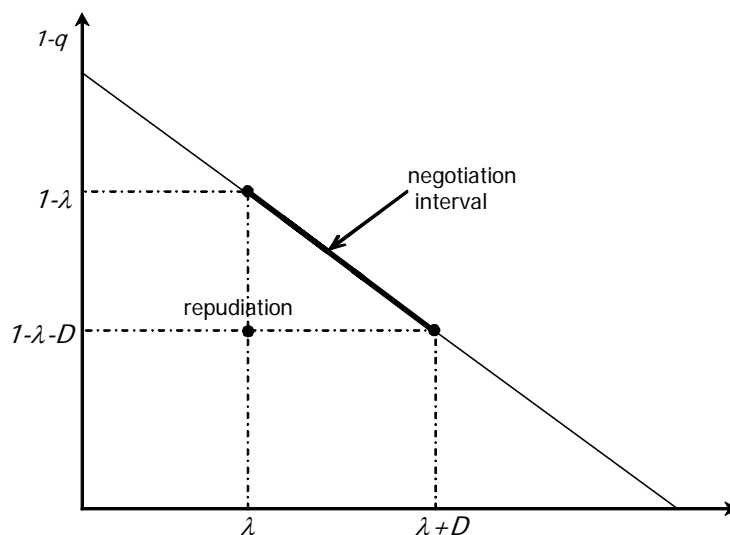


Figure 3: The contract curve

The bargaining outcome depends on the position of q_N^* relative to the negotiation interval $[\lambda, \lambda + D]$, the endpoints of which are determined by the outside option of repudiation. If q_N^* falls within this interval, bargaining is resolved as if there were no outside option. In this region, the players know that repudiation threats will not be carried out. Such non-credible threats are excluded by the requirement of subgame perfection. If q_N^* falls outside the negotiation interval, then one player can credibly threaten to repudiate, and this threat determines the split of the pie. If $q_N^* < \lambda$, for example, the country has no incentive to continue bargaining; understanding this, the bank ‘buys off’ the country and consumes what would otherwise have been a deadweight loss.

3.2 Optimal reserve policy during renegotiation

From the perspective of time 0, foreign reserves provide a form of insurance against investment-related risks. Once uncertainty has been resolved, however, reserve policy is motivated solely by the tradeoff between the consumption value of reserves and their value in shifting the bargaining outcome in the borrower's favor.

The basic outline of an optimal reserve policy can be understood by considering the autonomous reserve policy the country would run following a debt repudiation, if repudiation (counterfactually, in our analysis) were accompanied by a permanent cutoff from trade credits. Since the country is risk-neutral, the optimal policy would involve moving as rapidly as possible to the level of reserves that equates the marginal return to immediate consumption with the marginal increase in the discounted value of liquidity services. As shown in Appendix B, the reserve target \hat{R} satisfies $p'(\hat{R}/y) = \delta - r$. Given convexity of the transaction cost function, this target is approached monotonically over time, in a pattern reminiscent of the target-adjustment models in the reserve-demand literature (e.g., Frenkel 1983). Note also that if $r = \delta$, the borrower would accumulate reserves up to the satiation level $\hat{R} = \bar{L}$. Adjustment is immediate if reserves initially exceed the target level, because in this case the excess can be consumed immediately. If reserves are initially below the target, export proceeds $p(R/y) \cdot y$ are added to reserves in their entirety as they accrue, until the target is reached, after which point all

export proceeds are immediately consumed. Consumption therefore satisfies

$$c_{t+kh} = \begin{cases} (1 + hr) R_{t+kh} + hy - \widehat{R} & \text{if } R_{t+kh} \geq \widehat{R} \\ 0 & \text{if } R_{t+kh} < \widehat{R} \end{cases} \quad (10)$$

The rationale underlying this policy extends to the case of optimal reserve management during a debt renegotiation, given that the country is free to choose its level of consumption out of reserves and export proceeds in between offers. Ignoring repudiation, the optimal policy is again characterized by an interior reserve target, this time with the property that the marginal return to immediate consumption is equal to the marginal deterioration in the value of the bargaining game. The country follows a policy similar to (10), so as to approach the reserve target as rapidly as possible. The cost of delay would adjust endogenously over time, reaching a constant level as soon as reserves reach the target.¹¹

In what follows we assume that the expected value of Q is sufficiently large, so that R_1 exceeds the interior reserve target \widehat{R} .¹² In this case the country attains the target immediately at $t = 1$, and reserves remain constant over the course of the negotiation. The country's equilibrium payoff is then given by equation (9), which we reproduce here after substituting for the net terms of trade, p :

$$q_N^* \pi = \frac{r}{r + \delta} Q + \left(1 - \frac{\delta - r}{r + \delta} \right) \widehat{R} - \frac{\delta}{r + \delta} \frac{T(\widehat{R}/y) \cdot y}{\delta} \quad (11)$$

¹¹Because the country discounts at a rate higher than the return on reserves, the demand for reserves is zero conditional on the outside option.

¹²A sufficient condition for this is that $E[Q(s)] \geq \delta^{-1} [2r\bar{L} + (\delta + r)(1 + r)(1 - R_0) + (1 - \delta h)y]$.

The logic of this solution is straightforward. Consider first the division of Q : if discount rates are equal, the familiar symmetric Nash bargaining solution of a half-and-half split emerges. If (realistically) the country has a higher discount rate than the bank ($\delta > r$), the country's greater impatience reduces its share of output relative to this symmetric benchmark. Next, consider the split of reserves, captured by the second term. Since the country consumes the interest on reserves during negotiation, the country is impatient with respect to this portion of the pie only to the degree that the yield on reserves is below the country's discount rate. This cost is absent when $\delta = r$, implying that in this case the country can credibly demand the entire stock of reserves. When $\delta > r$ the country receives less than the full stock of reserves. Finally, the last term in (11) reflects the impact of the trade credit cutoff. As long as the negotiation continues, the country suffers increased transaction costs in converting its perishable export good into imports. The present value of these costs – assuming agreement is never reached – comes to $T(\widehat{R}/y) \cdot y/\delta$. The country must hand over a share $\delta/(r + \delta)$ of these costs.

Equations (10) and (11) characterize optimal reserve policy and its implications when neither player can credibly threaten to repudiate. The repudiation option brings in two potential complications. The first is that $R = \widehat{R}$ may produce a sufficiently large payoff for the country that the bank prefers repudiation. If this is the case, the country's marginal return to reserves at $R = \widehat{R}$ is no longer 1, but rather $1 - \gamma$. The country therefore gains by

consuming reserves down to the level that makes the bank just indifferent between repudiating and renegotiating; at this point any further reduction imposes a marginal cost above 1, so it is locally suboptimal to reduce reserves further. The second complication applies to any strictly positive reserve target and is potentially relevant if $R = 0$ produces a low enough payoff to the country that its own threat to repudiate becomes credible. In this case the overall bargaining solution is no longer concave at low levels of reserves, as we will see below. The locally optimal reserve policy must therefore be compared directly with the payoff from consuming the entire stock of reserves immediately and collecting the resulting repudiation payoff $(1 - \alpha)Q$. The latter strategy is less likely to be optimal the larger α . In what follows we restrict attention to the case in which reserves are retained.

For any given level of net indebtedness $B(1 + z) - R$, higher gross debt rewards the country in two ways, conditional on rescheduling (and provided the outside options are not binding). First, ignoring the transactions costs of trade, it raises consumption nearly dollar for dollar, because the country retains a fraction $2r/(r + \delta)$ of its stock of gross reserves. Second, it reduces the transactions cost burden that lenders could otherwise impose by virtue of their ability to withhold trade credits.

3.3 Lender haircuts in the bargaining region

The foregoing analysis generates a precise set of predictions about the ratio of bank payoffs to the face value of debt. We summarize these in terms of the “haircut” or percentage loss suffered by creditors if the solution lies in the negotiation region.

Proposition 1: In the negotiation region, the haircut is an increasing function of the stock of debt and the discount rate of lending banks, and a decreasing function of borrowing-country exports. It is a decreasing function of the borrower’s discount rate if the time transactions cost of international trade are sufficiently large relative to the value of the recipient’s resources at the outset of the bargaining game.

Proof. The bank’s payoff is $(1 - q_N^*(t)) \pi_t$, so in the bargaining region the proportional haircut H is given by

$$H = \frac{B - (1 - q_N^*(t)) \pi_t}{B} = 1 - \frac{1}{B} \left[\frac{\delta}{r + \delta} (\widehat{R} + Q) + \frac{T(\widehat{R}/y) \cdot y - r\widehat{R}}{r + \delta} \right]$$

It follows that $\frac{\partial H}{\partial B} > 0$, $\frac{\partial H}{\partial Q} < 0$ and $\frac{\partial H}{\partial r} > 0$. Moreover

$$\frac{\partial H}{\partial \delta} = -\frac{1}{B} \left[\frac{rQ - T(\widehat{R}/y) \cdot y}{(r + \delta)^2} \right]$$

which is negative as long as $r^{-1}T(\widehat{R}/y) \cdot y > \pi - \widehat{R}$, i.e., as long as the discounted value of transaction costs the lender can impose (if negotiation were to go on forever) exceeds the value of the borrower’s exportable goods.

QED. ■

Note that because the level of international reserves converge to their optimal level, their effect on the haircut at the time of the renegotiation is zero: exactly so if reserves are initially above \widehat{R} , because adjustment to \widehat{R} is immediate; approximately so if reserves are initially below and must be accumulated during the negotiation.¹³ Our analysis in section 5 below suggests that borrowers will generally accumulate reserves in excess of \widehat{R} during period 0, so we do not attempt a solution in the latter (non-stationary) case. The logic of the analysis suggests, however, that if it were in fact optimal for the borrower to accumulate reserves *during* the negotiation, the effect of low initial reserves on the borrower's payoff would be moderated, to some degree, by a greater impatience on the part of the lender to conclude the negotiation quickly and therefore suffer a smaller haircut.

What is clear in either case is that the value of gross reserves during a renegotiation undermines the appeal of a debt buyback from the borrower's perspective. At the margin, a buyback financed by international reserves would reduce the borrower's welfare. This may help explain the typical reluctance of debtor countries to engage in debt buyback operations.

¹³

$$\frac{\partial H}{\partial R} = -\frac{1}{B(r+\delta)} [\delta - r + T'(R/y)] = 0$$

where the latter equality follows from the derivation in Appendix B.

3.4 Extension: fixed costs to lenders

The framework allows us to analyze the outcome in the presence of banking regulations that may act to increase the bank's impatience and thereby reduce their bargaining power. Suppose that the lender faces a fixed cost K if the negotiation is still unresolved at time $T + 1 > 1$. The deadline at $T + 1$ can be thought of as coming from regulations that require a loan in arrears for T periods to be declared as non-performing. Such action calls for provisions which can lower bank equity values. In Appendix C, we derive the following bargaining solution, for the case of constant reserves:¹⁴

$$q^*(t) = \max \left[\lambda(t); \min \left[q_N^* + \frac{K}{2\pi} e^{-\frac{r+\delta}{2}(T-t)}; \lambda(t) + D(t) \right] \right] \quad (12)$$

where q_N^* is defined as in equation (9). This expression is intuitively appealing. The fixed cost is irrelevant only if, in its absence, lenders would already have been able to issue a credible threat of default. In all other circumstances, a rise in K shifts bargaining power towards the country, raising its share $q^*(t)$. The country's share is non-decreasing in the proximity of the deadline T , implying that the bank would increase its offer to the country if the deadline were closer at hand. As before, the country's share in the negotiation region is capped by what it would receive if the bank could credibly

¹⁴The one-time cost K renders the problem nonstationary up to time T . After T , however, the stationary solution of equation (7) holds. Note that the solution at $t \leq T$ hinges on who has the last proposal before time T . To avoid the problems associated with taking the limit as $h \rightarrow 0$, we follow the approach of Binmore (1980) to remove the first mover advantage, assuming that the proposer is decided by the flip of a coin in each period.

threaten to abandon negotiations.

4 The Repayment Decision

In this section we examine the effect of the country's assets on its choice to repay debts in full or reschedule and, in case the latter option is chosen, on the terms of the rescheduling agreement.

Since the country may always settle the claims by repaying outstanding debts at face value, its payoff in period 1 will be given by

$$W_1 = \max[V^p, V^r]$$

where V^p and V^r are the values of repaying in full and rescheduling, respectively.

4.1 The value of rescheduling

The value of rescheduling, in turn, can be expressed as

$$V^r = (R_1 - \widehat{R}) + \max \left[V; \min \left[V_N(\widehat{R}); V^* \right] \right] , \quad (13)$$

where $V = \lambda\pi_1$, $V_N = q_N\pi_1$, and $V^* = (\lambda + D)\pi_1$. To streamline terminology, we will define the *bank region*, *country region* and *negotiation region*, as the range of reserve levels for which bank's threat to repudiate is credible (i.e. $V^r = V^*$), the country's threat is credible ($V^r = V$), and neither is credible

($V^r = V_N$), respectively. While the exact configuration of V^r will depend on all the parameters, one can see from (8) that V^r is a differentiable function of R_1 except at a finite number of switch points where the equilibrium moves from one region to another. Since $\lambda\pi_1$, $q_N\pi_1$, and $(\lambda + D)\pi_1$ are all nondecreasing in R_1 , a rise in the level of reserves cannot decrease the value of rescheduling. Put alternatively,

Lemma 1: The return to gross reserves is strictly positive conditional on debt renegotiation.

Proof. By equations (8) and (9), the value of rescheduling is

$$V^r(R_1, Q) = \left(R_1 - \widehat{R} \right) + \max[(1 - \gamma) \widehat{R} + (1 - \alpha) Q + \frac{y}{\delta}; \\ \min\left[\frac{r(\widehat{R} + Q) - T(\widehat{R}/y) \cdot y + r\widehat{R}}{r + \delta} + \frac{y}{\delta}; \quad (1 - \gamma) \widehat{R} + (1 - \alpha + \mu\alpha) Q + \frac{y}{\delta} \right]]$$

It follows that the return on reserves is strictly positive. QED. ■

The return to gross reserves has two distinct components in a world with debt renegotiations. Under default, a portion $R_1 - \gamma\widehat{R}$ of gross reserves constitutes *net wealth*; this is nonnegative given that $R_1 \geq \widehat{R}$ and $\gamma \leq 1$. In the renegotiation region, reserves also have a liquidity role. They substitute for trade credit, making the borrower appear more patient; this puts the borrower in a position to demand a greater share of the surplus. Figures 4 and 5 illustrate these two roles of gross reserves. We do this by tracing out the country's rescheduling payoff (the maximum expression in (13)) as a

function of reserves held at the outset of a negotiation. In Figure 4¹⁵ we set $T(R_t/y) \cdot y \equiv 0$, so that the value of reserves stems entirely from their imperfect appropriability by lenders. In Figure 4 we set $\gamma = 0$ so that reserves cannot be attached in default; but we allow $p(R_1/y) > 0$ so that the borrower gains from the liquidity services provided during a negotiation. Note that the relationship between the level of reserves and the operative bargaining region depends on the parameters: in Figure 4, for example, higher reserves eventually shift the surplus in favor of the bank, because the marginal return to reserves in the negotiation region is below that in the country and bank regions. The opposite would be true if the ranking were reversed, as it is in Figure 5.¹⁶

Note that there is no case in which the value of rescheduling depends on the stock of debt. In the country and bank regions, this is because the default penalty consists of a given fraction of output and/or reserves, and is independent of the depth of default. In the negotiation region, it is because

¹⁵To ensure that all three regions are non-empty the restrictions that $V_N(R_1 = 0) < V(R_1 = 0)$ and $\frac{r}{r+\delta} < 1 - \gamma$ are imposed, so that in the absence of reserves the country would prefer negotiation to repudiation, whereas the bank prefers repudiation.

The V and V^* schedules differ by the amount of the deadweight loss, DQ , having a common slope of $1 - \gamma$ - (the fraction of non-attachable reserves). The shape of the V_N schedule hinges on whether creditors are able to interfere with trade finance during the negotiation. Fig. 4 is drawn assuming that creditors cannot affect terms of trade (that is the case if $R \rightarrow \infty$). In this case, the slope of V^N is determined by the relative impatience rates $\frac{r}{r+\delta}$.

¹⁶More complicated configurations are clearly possible when reserves offer liquidity services as in Fig. 5. With $\gamma > 0$, for example, the V and V^* schedules are upward-sloping. In the case of strictly concave V^N , a negotiation region of the type shown in Fig. 5 would be followed, at higher reserve levels, by another negotiation region with a transition back to the country region.

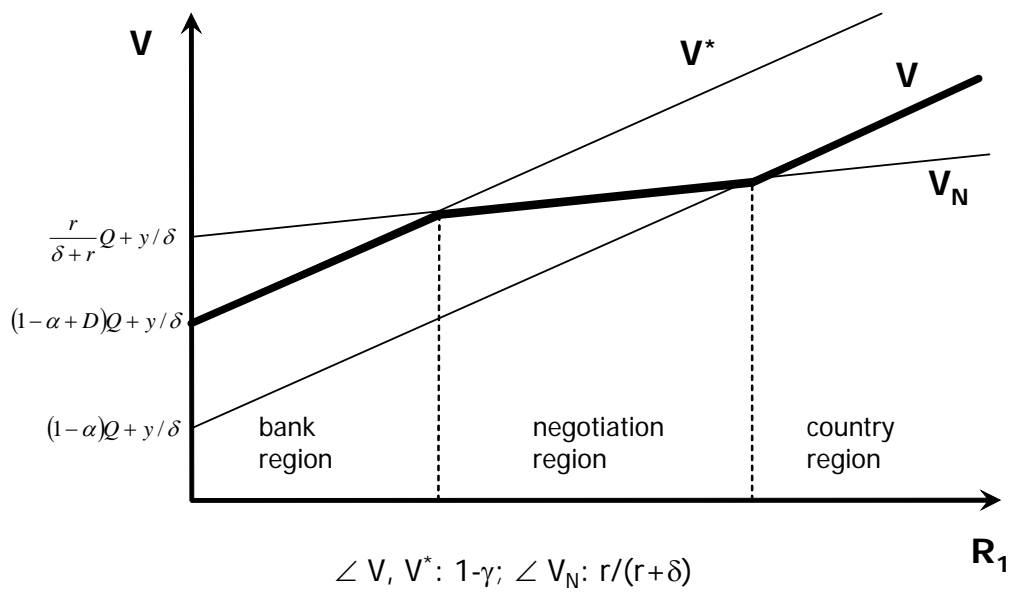


Figure 4: Reserves as net wealth

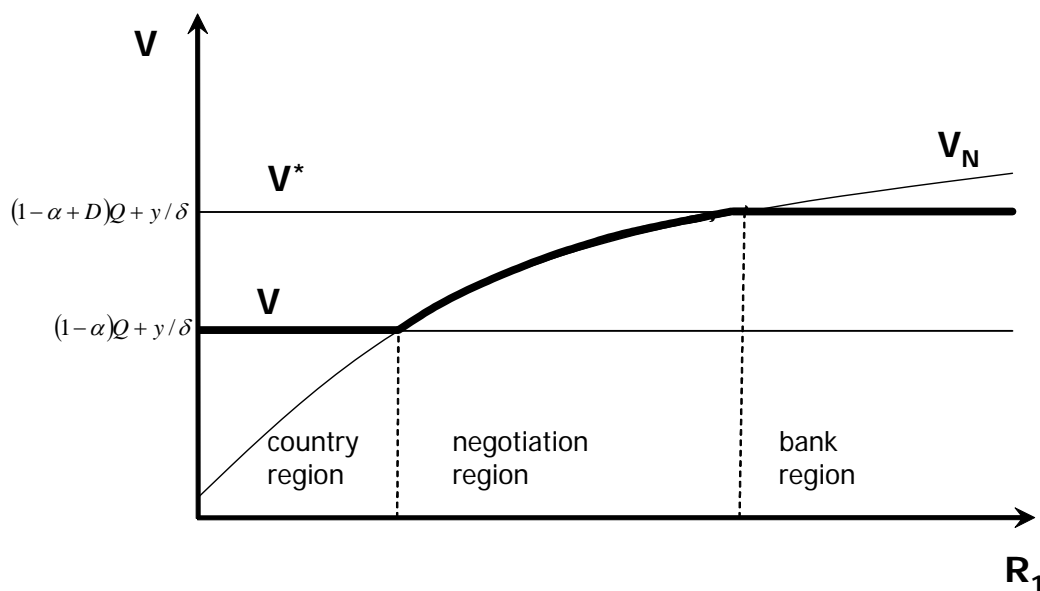


Figure 5: Reserves as liquidity

repayment is limited to what the country can be bargained into repaying. In either case, it follows that *net* reserves, $R_1 - D$, are irrelevant to the rescheduling decision, given the level of gross reserves.

Figure 5 depicts the case in which reserves are fully confiscated in the event of a repudiation – implying that V and V^* are flat – and creditors can impose a terms of trade loss on the country by interfering with trade finance during negotiation.¹⁷

One can see from the diagrams that the *ex post* marginal gross return

¹⁷We assumed that liquidity services are substantial enough to ensure that $V_N(R_1 = 0) < V(R_1 = 0)$ and that Q exceeds the level that would make repudiation by either part optimal.

of reserves conditional on rescheduling can only exceed 1 in case of a trade credit cutoff with the agreement falling in the negotiation region. There is a strong sense, therefore, in which the liquidity role is more central than the net wealth role in explaining the demand for reserves. In the model presented here, liquidity services are a necessary condition for reserves to be held past the first negotiation period if the country is following an optimal reserve policy. If trade credit were always readily available, the demand for borrowed reserves would be zero.

4.2 The value of repayment vs. rescheduling

Lemma 2: V^p is a non-increasing function of R_1 and a strictly increasing function of R_0 .

Proof. Recall that the country borrowed for consumption, accumulation of reserves and one unit for the investment project. If z is the promised interest rate on debt incurred in period 0, the borrowers repayment value is

$$\begin{aligned} V^p &= Q + R_1 + \frac{y}{\delta} - (1 + c_0 - y + (1 + r)^{-1} R_1 - R_0) (1 + z) = \\ &= Q + y \frac{1 + \delta}{\delta} - \frac{z - r}{1 + r} R_1 - (1 + c_0 - R_0) (1 + z) \end{aligned} \quad (14)$$

QED. ■

The above proposition makes two important points. First, gross reserves will be dominated in rate of return – and will therefore not be held at all at

$t = 1$ – unless the borrower reschedules its debt in some states of the world. This is because reserves carry a strictly positive opportunity cost of $z > 0$ in states of the world in which the borrower repays. Second, while we have just noted that net reserves do not affect the payoff to rescheduling, they do affect the value of repaying, and in the opposite direction to gross reserves. Given the level of gross reserves, an increase in net reserves implies a reduction in debt and therefore an *increase* in the probability of repayment.

The country repays if $V^p \geq V^r$ and reschedules otherwise. Since the value of rescheduling is non-decreasing in reserves and the value of repayment is strictly decreasing in reserves, the impact of reserves on the rescheduling decision is straightforward:

Proposition 2: For given values of Q , R_o and z , either the country reschedules for all values of R_1 , or there is a unique level of reserves, $R^(Q, R_o, z)$, above which the country reschedules and below which the country repays. This cutoff level of reserves is continuous and piecewise differentiable in its arguments, with $\frac{\partial R^*(.)}{\partial Q} > 0$, $\frac{\partial R^*(.)}{\partial R_o} \geq 0$ and $\frac{\partial R^*(.)}{\partial z} < 0$.*

Proof. Follows from Lemmas 1 and 2, together with equations (??) and (14). ■

In Fig. 6 we plot the cutoff level of reserves for selected values of Q , holding R_0 and z constant. We assume that reserves are not fully attachable ($\gamma < 1$) and that reserves deliver liquidity services ($T'(R_t/y) < 0$). Kinks

in the schedule may occur where the bargaining solution switches between regions. For R_1 sufficiently large, the outcome will fall in the country or bank regions as Figures 4 and 5 make clear, and the R^* schedule will approach a horizontal asymptote. Given z , the cutoff value rises with output because the bank is not a residual claimant of the storable export good under repayment; this means that for the country, the value of repaying rises by more than that of rescheduling as output rises.

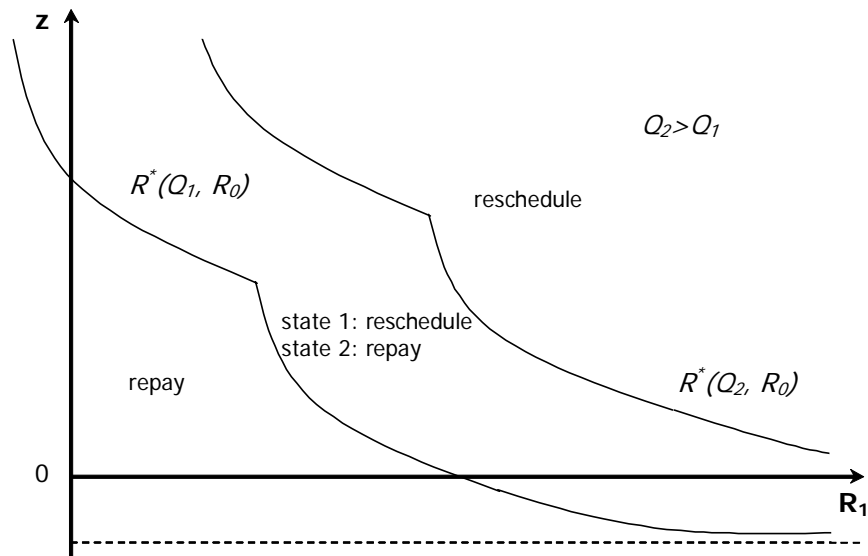


Figure 6: The rescheduling decision

The R^* schedule partitions the (z, R) plane into areas in which the pattern of rescheduling and repayment is clearly defined. If a country chooses to repay (reschedule) for a given level of output, it will always choose to repay (reschedule) for any

higher (lower) output level. More generally, the comparative statics of the repayment decision satisfy:

Corollary: The country repays (reschedules) when output is above (below) a critical level $Q^(R_0, R_1, z)$, where $\frac{\partial Q^*}{\partial R_0} < 0$, $\frac{\partial Q^*}{\partial R_1} < 0$ and $\frac{\partial Q^*}{\partial z} < 0$. Given z , the probability of repayment is a non-decreasing function of R_1 and a non-decreasing function of R_0 .*

5 The Supply and Demand of Borrowed Reserves

In the previous section we concluded that gross reserves may increase the value of rescheduling, and at the same time reduce the value of repayment. In this section we show that rational banks will lend reserves to the country - in spite of the fact that they increase the bargaining power of the country - as long as penalties on output are large enough so that it can credibly claim a share of the borrower's resources. As we assume that banks are perfectly competitive ex ante, this amounts to showing that reserve lending in the first period satisfies the zero-profit condition.

We study the case in which there are two possible states for the economy, s_1 and s_2 , associated with output realizations $Q_2 > Q_1$. The arbitrage condition requires that $E(z(s_i)) = r$, where the expectation is taken given all information available at $t = 0$, which includes the specification of the bar-

gaining problem that players will face in period $t = 1$. Below the $R^*(Q_1, R_0)$ schedule in Fig. 7, repayment occurs in both states so that lending is risk-free (i.e. $z(s_1) = z(s_2) = z$). Competition among banks drives the promised rate z down to r . Notice that the existence of the horizontal segment \overline{ab} in the zero-profit locus on Fig. 7 requires that the condition $R^*(Q_1, r) > -R_0$ is met. The range of borrowed reserves in which lending is risk-free increases with Q_1 , α and $T(\cdot)$.

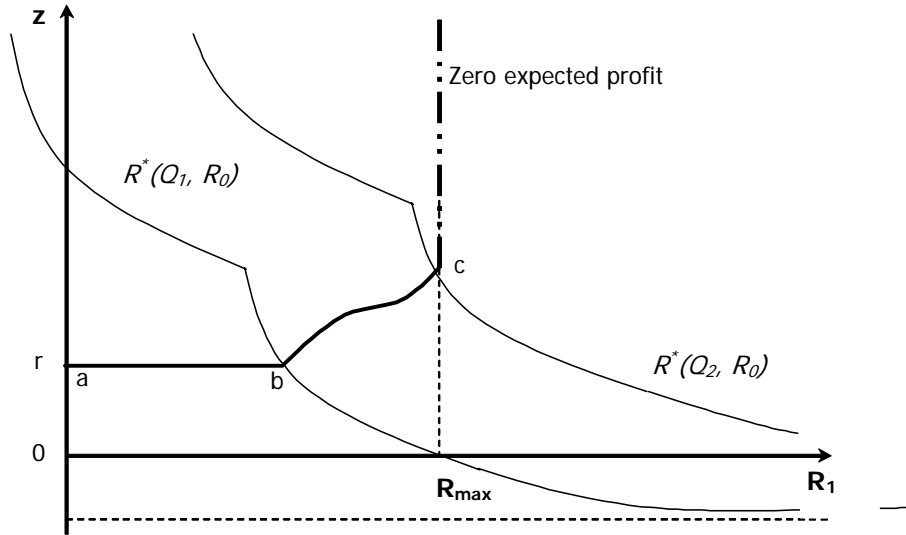


Figure 7: The supply of borrowed reserves

Between the $R^*(Q_1, r)$ and the $R^*(Q_2, r)$ schedules, the country repays only in the high-output state. Notice that the return in the low output state falls with R_1 , so that the promised return (which is paid only in the high

output state) must rise with R_1 in this interval. This gives the segment \overline{bc} in the zero-profit locus, which must be above r . There is no discontinuity at b because the rescheduling process is efficient and involves no deadweight loss.

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At point c the country reschedules in the low output state and is indifferent between rescheduling and repaying in the high output state. Hence, any further rise in the promised interest rate z is irrelevant, as both players anticipate that it will never be honored. Since the return conditioned on rescheduling can never exceed r , the zero profit locus becomes vertical at c . We denote the maximum amount of borrowed reserves at time 1 by R_{\max} , so that the country's overall long-term credit ceiling at time 0 is $1 + (1 + r)^{-1} R_{\max}$. The supply schedule is given by \overline{abc} .¹⁹

Credit ceilings are a well known characteristic of the sovereign debt literature at least since Eaton and Gersovitz (1981). Here the credit ceiling can be derived by equating the risk-free return on borrowed reserves with the bank's expected yield assuming rescheduling in both states. Defining $q(s)$ as the share of resources received by the borrower in a rescheduling agreement in state s , and assuming that reserves earn the risk-free rate from $t = 0$ to 1, R_{\max} satisfies

¹⁸If the rescheduling process involves a deadweight loss, there would be a discontinuity at b and the possibility of two equilibrium promised interest rates over some interval of reserves.

¹⁹We are implicitly assuming that the reserve generating debt instruments are issued sequentially and contain a seniority clause, so that rational competitive lenders will never be willing to hold such instruments beyond the credit ceiling.

$$E \left[(1 - q(s)) \left(\widehat{R} + Q(s) + hy \right) \right] = \left(\frac{R_{\max}}{1 + r} + (1 - R_0) \right) (1 + r) \quad (15)$$

If $R_{\max} \leq -(1 + r)$, the country is excluded from long-term credit markets, and its investment can only be self-financed via accumulation of current account surpluses. The credit ceiling on borrowed reserves is a non-decreasing function of the penalties the lender can impose in case of repudiation, with comparative statics depending on the bargaining region that is operative in each output state at the credit limit.

Proposition 3: The amount of borrowed reserves is decreasing in international interest rates and increasing in expected export revenues.

Proof. The cap on borrowed reserves is obtained by rearranging the investor's arbitrage condition (15):

$$R_{\max} = (\delta + r)^{-1} \left(\delta hy + (\delta - r) \widehat{R} + \delta EQ(s) + T(\widehat{R}/y) \cdot y \right) - (1 + r) (1 - R_0)$$

If we use the optimal reserve choice condition (i.e. $\frac{\partial T}{\partial \widehat{R}} = \delta - r$), this gives

$$\frac{\partial R_{\max}}{\partial r} = - \frac{2\delta \widehat{R} + \delta EQ(s) + T(\widehat{R}/y) \cdot y}{(r + \delta)^2} - 1 < 0$$

■

As lenders are competitive *ex ante*, the country obtains the entire surplus from the relationship with lenders. It can choose the equilibrium level of

reserves taking the bank's zero expected profit locus as given. Hence, equilibrium occurs at the point on the zero expected profit locus that maximizes the country's utility. If reserves are remunerated at the risk-free rate until $t = 1$, the country augments its consumption by $S = E(Q) - (1 + r)$, regardless of the level of reserves it holds. In this case, reserves serve a pure insurance role, redirecting consumption from high output states to low output states without changing its expected value.²⁰ The two state case is summarized in the proposition below:

Proposition 4: If the country is risk-neutral ($u'' = 0$), it is indifferent to the amount of borrowed reserves held, including zero. If the country is risk-averse ($u'' < 0$), the country borrows up to its credit ceiling and holds the maximum amount of borrowed reserves. Borrowed reserves provide partial insurance.

Fig. 8 shows the consumption allocation across the two states of nature that can be achieved by various contracts. Taken at face value, a debt contract has the borrower bearing all the risk, with consumption on a point like E . As Hellwig (1986) and others have pointed out, this makes the use of standard international debt instruments somewhat puzzling, given that lenders are probably less risk-averse than borrowers. It would seem efficient to have payments contingent on output, thus shifting some of the risk to the

²⁰In the case where reserves earn less than the risk-free rate, it is Pareto inefficient for the country to hold reserves if its debt is positive. The country still gets the entire surplus of the relationship at $t = 0$, but the surplus is a declining function of borrowed reserves.

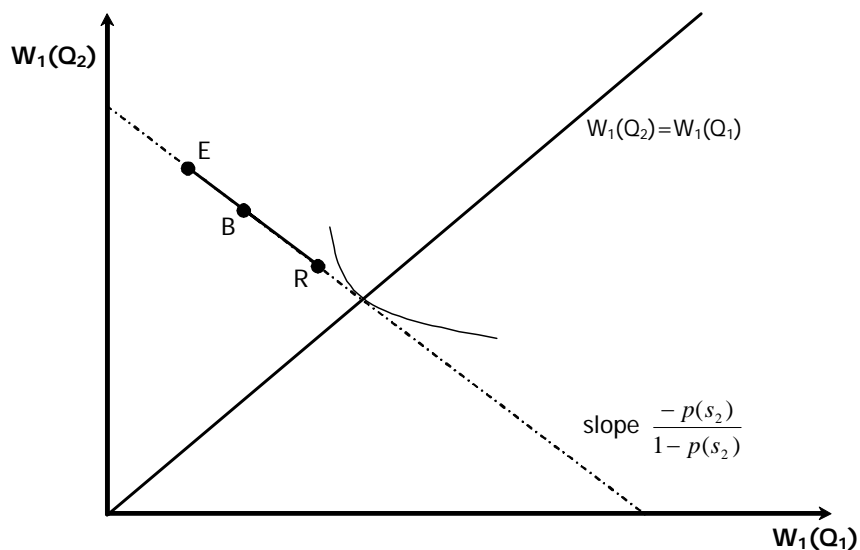


Figure 8: The insurance role of borrowed reserves

lender.²¹

Point E however represents only *enforceable* debt contracts, and in equilibrium, the promised rate on debt contains a premium above the risk-free rate to compensate the lender for losses in case of a rescheduling (e.g., Grossman and van Huyck (1988)). The actual, ex post return paid by the borrower is below the risk-free return in the states in which debt is rescheduled. Sovereign lending, in the absence of borrowed reserves, moves the equilibrium to B , i.e., the possibility of rescheduling provides some of the missing insurance.

Borrowed reserves expand the range of achievable consumption alloca-

²¹Atkeson (1991) argues that the optimal contract does not provide full insurance because of moral hazard.

tions further. As borrowed reserves move from 0 to R_{\max} , the consumption allocation moves from B to R . A risk averse country will clearly choose the maximal amount of insurance given that lenders are competitive. This involves borrowing up to the credit ceiling and holding the excess over investment needs as reserves. It is easy to see that the insurance that is made available through the resort to borrowed reserves is only partial: full insurance would require the transfer from the borrower to the lender to rise one-for-one with output. Since at its credit limit the country reschedules in the low output state and is indifferent between rescheduling and repayment in the high output state, the difference in payments in the two states is just the difference between the rescheduling payments. As long as $\alpha < 1$, these payments differ by less than output.

Propositions 3 and 4 suggest that risk averse borrowers are able to accumulate larger stocks of international reserves during times in which interest rates are lower or when increases in export revenues are anticipated. Hence, the recent buildup of reserves by emerging market economies carrying substantial net international debt is fully consistent with optimal behavior by banks in a situation of credit rationing - given the historically low interest rates.

6 Testable Implications

The theory delivers testable implications for the magnitude of haircuts during debt renegotiations (Proposition 1). Haircuts should be larger for larger debt stocks and lender's discount rates, whereas higher exports should affect haircuts negatively. The prediction for the borrower's discount rate is less clear-cut as it hinges on the unobservable transaction time cost. In order to test the implications we use sizes of haircuts for foreign currency bonds estimated by Sturzenegger and Zettelmeyer (2005). Summary statistics and the results for the 246 rescheduled bonds are shown in Tables 1 and 2. Regression results support our predictions: a 1% increase in the debt/GNP ratio increases the size of the haircut by 2 to 2.5% according to the random and fixed effects estimates. Shrinking exports and low international interest rates favor lenders: a 10 b.p. rise in the 5 year T-Bill rate increases the haircut by about 2.5%. Furthermore, the estimates suggest that the degree of impatience of the borrower (proxied by the domestic money market rate) may increase investor losses.²²

²²In principle, one could also test the results concerning the credit ceiling of borrowed reserves. However this would involve identifying credit constrained countries in a first stage. We leave this for future research.

7 Concluding Remarks

Since the landmark paper of Eaton and Gersovitz (1981) many studies of the sovereign debt market have been presented and many more debt reschedulings have taken place. Yet some key aspects within the sovereign debt literature remain puzzling. This paper has tried to shed light on selected aspects, leaving others for future research. Empirical evidence points to the importance of diminished trade flows during debt arrears and suggests a potentially important modification in how the literature has viewed the punishment strategies available to unlucky creditors. In our analysis, debt renegotiation does not imply a halt to export production, but – realistically – export seizing ‘gun-boats’ are not deployed. What creditors do instead is simply to stop rolling over short-term trade finance during the negotiation process. This cut-off from trade finance has the effect of increasing the impatience of the borrower to seek an agreement in order to maximize the proceeds that accrue from its exports. In this sense, creditors are less active than in Bulow and Rogoff (1989a) and are likely to incur smaller costs, attenuating the free-rider problem.

The side effect of the assumed punishment strategy is to highlight a new rationale for reserve holdings: borrowing countries may accumulate reserves to guarantee their liquidity in anticipation of a bargaining game. This is certainly not always the main reason for reserve accumulation and many borrowers go considerable lengths in reducing their reserve holdings to avoid

falling into arrears. Conditional on renegotiation, however, greater liquidity plays into the hands of the borrower. The relative impatience of players – which ultimately defines the outcome – is the endogenous result of the liquidity position of the borrower. For this reason, the distinction between gross and net international reserves is predicted to be central to the outcome of the bargaining process. Borrowers with higher gross reserves find themselves in a position to reach a better deal during a debt renegotiation. Hence, the model may explain why some borrowers may not risk to exhaust their reserves to meet repayments, defaulting with positive reserve holdings. It may also explain the notable reluctance of borrowers in arrears to engage in immediate debt buyback operations.

Further extensions of the model could incorporate the IFIs as a third player into the bargaining game. The recent involvement of multilateral organizations, as the IFC and the IADB, in trade financing and the policy of export credit agencies of lending or not into arrears is likely to affect the degree of impatience of creditors and borrowers and consequently shift bargaining power. Also, the set up of contingent credit lines as the one arranged between Mexico and private financial institutions in 1997 or more recently by a group of South-East Asian economies may provide a cheaper alternative to borrowed reserves. In the case of Mexico however, some financial institutions objected to the exercise of the line in 1998, although contingencies were reasonably broad (Sidaoui (2000)). Future research might focus on the

conditions under which such arrangements may be substitutes for borrowed reserves.

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Appendix A - Unicity

Let the proposer in period t be determined by the flip of a coin and $hc_i(t)$ represent the cost of delay of h in reaching an agreement for player i . Also, let $\pi_i(t)$ represent i 's expected continuation value in a perfect game before the proposer is determined and $v_i(t)$ and $v_i'(t)$ represent the continuation value conditioned on being the proposer at time t or not respectively. Further, $M_i(t) = \sup_{\Omega} \pi_i(t)$ and $m_i(t) = \inf_{\Omega} \pi_i(t)$ where Ω represents the set of subgame perfect equilibria and $\bar{\beta} = \max[\beta; \beta^*]$.

Lemma: If there exists $D(t) < \infty$ such that $M_i(t) - m_i(t) \leq D(t)$, then $M_i(t-h) - m_i(t-h) \leq \bar{\beta}D(t)$.

Proof: Suppose the country proposes the split (x, y) at $t-h$. The bank will surely reject if

$$y < \beta^* m^*(t) - c^*(t-h)$$

and accept if

$$y > \beta^* M^*(t) - c^*(t-h) \tag{16}$$

In case the bank rejects, the country will have to wait a period and will receive at least $m(t)$ in period t . The country will offer at most the value on the RHS of expression (16), since at this value the bank would already accept the offer for sure. Since the country has the offer, it will do no worse than receiving the better of this two payoffs:

$$v(t-h) \geq \max[\beta m(t) - c(t-h); x + y - \beta^* M^*(t) + c^*(t-h)] \tag{17}$$

$v(t-h)$ is also limited from above by the highest equilibrium payoff offered by the bank after a rejection by the country, $M(t)$, and the value given by least offer that is accepted by the bank. Hence, we also have

$$v(t-h) \leq \max [\beta M(t) - c(t-h); x + y - \beta^* m^*(t) + c^*(t-h)] \quad (18)$$

Similarly, if the bank makes the offer at $t-h$, the country rejects if

$$x < \beta m(t) - c(t-h)$$

and accepts if

$$x > \beta M(t) - c(t-h)$$

$$\beta m(t) - c(t-h) < v'(t-h) < \beta M(t) - c(t-h) \quad (19)$$

Substituting $M_i(t) \leq D(t) + m_i(t)$ in expressions (17), (18) and (19) we get

$$\begin{aligned} & \max [\beta m(t) - c(t-h); x + y - \beta^* m^*(t) - \beta^* D(t) + c^*(t-h)] \\ \leq & v(t-h) \leq \max [\beta m(t) + \beta D(t) - c(t-h); x + y - \beta^* m^*(t) + c^*(t-h)] \end{aligned}$$

and

$$\beta m(t) - c(t-h) < v'(t-h) < \beta m(t) - c(t-h) + \beta D(t)$$

Since $\pi_i(t) = E[v_i(t)]$, it follows that the bounds on $\pi(t-h)$ will be

$$\begin{aligned} & \max \left[\beta m(t) - c(t-h); \frac{1}{2} [x + y - \beta^* m^*(t) - \beta^* D(t) + c^*(t-h) + \beta m(t) - c(t-h)] \right] \\ \leq & \pi(t-h) \leq \max \left[\begin{array}{c} \beta m(t) + \beta D(t) - c(t-h); \\ \frac{1}{2} [x + y - \beta^* m^*(t) + c^*(t-h) + \beta m(t) + \beta D(t) - c(t-h)] \end{array} \right] \quad (20) \end{aligned}$$

A similar expression holds for $\pi^*(t-h)$.

Since $M(t-h)$ and $m(t-h)$ are defined as bounds to the equilibrium payoff, the difference $M(t-h) - m(t-h)$ must be bounded by the outer quantities in equation (20). Hence, the inequalities above imply

$$M(t-h) - m(t-h) \leq \beta D(t) + \frac{1}{2} (\max [0; \omega - \beta D(t)] - \max [0; \omega - \beta^* D(t)]) \quad (21)$$

, where $\omega = x + y - (\beta m(t) - c(t-h)) - (\beta^* m^*(t) - c^*(t-h))$. It is easy to see that for all values ω this implies

$$M(t-h) - m(t-h) \leq \max \left[\beta D(t); \frac{\beta^* + \beta}{2} D(t) \right] \leq \bar{\beta} D(t) \quad (22)$$

Similarly, one can also show that

$$M^*(t-h) - m^*(t-h) \leq \max \left[\beta^* D(t); \frac{\beta^* + \beta}{2} D(t) \right] \leq \bar{\beta} D(t) \quad (23)$$

QED.

Let $D(t) = R_t + Q$. From (22) and (23), as $t \rightarrow \infty$, $M_i(\tau) - m_i(\tau) = 0 \forall \tau$, i.e., each player has a unique equilibrium expected payoff for any finite time period.

Appendix B - Optimal Reserve Policy

Under financial autarky, the optimal reserve policy is given by the solution to

$$\max_{R_{t+(i+1)h}} \sum_{i=0}^{\infty} \frac{C_{t+ih}}{(1 + \delta h)^i}$$

s.t.

$$\begin{aligned} c_{t+ih} + R_{t+(i+1)h} &= (1 + rh) R_{t+ih} + p(R_{t+ih}/y)hy \\ c_{t+ih} &\geq 0 \quad \text{and} \quad R_{t+ih} \geq 0 \end{aligned}$$

The Euler equation that characterizes the optimal policy is

$$(1 + \theta_i (1 + \delta h)) \left(1 + rh + p'(R_{t+h}/y) \frac{1}{y} hy \right) + \lambda_i = (1 + \theta_{i-1}) (1 + \delta h)$$

where λ_i and θ_i are the shadow prices on the last two constraints, respectively. An interior solution is obtained when $\lambda_i = \theta_i = \theta_{i-1} = 0$. In this case, the condition for the interior optimum is:

$$p'(R_{t+h}/y) = \delta - r$$

Appendix C - The Solution with a Fixed Cost to Lenders

Assume that in each period players put their proposal in an envelope and the relevant offer is decided by the flip of a coin. Moreover, let V_b and V_c denote the country's payoff if the bank or the country gets to make the offer in a period t , respectively. We have

$$V(t) = \frac{E[V_c(t) + V_b(t)]}{2}$$

The optimal strategy for each player will be to make the minimum acceptable offer, i.e., to offer the amount that leaves the responder indifferent between accepting and turning the offer down. Hence, we get

$$V(t) = \frac{[1 - (\beta^* V^*(t+h) - hc^*(t))] + [\beta V(t+h) - hc(t)]}{2} \quad (24)$$

where $c(t)$ and $c^*(t)$ represent the cost of delay in reaching an agreement for the country and the bank respectively. But perfect information implies $V^*(t) = 1 - V(t)$ for all t , so that we can rewrite (24) as

$$V(t) = \frac{1 - \beta^* + h(c^*(t) - c(t)) + (\beta^* + \beta)V(t+h)}{2} \quad (25)$$

Starting at $T+h$, bargaining costs are constant at $c(t) = c$ and $c^*(t) = 0$. The subgames starting at T and $T+h$ (before the coin toss) are identical, rendering the solution

$$V(T+kh) = \frac{1 - \beta^* - hc}{2 - \beta^* - \beta} \quad \forall k \geq 1 \quad (26)$$

Now consider that the bank incurs a one time cost of K if the offer at time T is refused. We can obtain $V(T)$ by substituting equation (26) in (25) at time T :

$$V(T) = \min \left[\frac{1 - \beta^* - hc}{2 - \beta^* - \beta} + \frac{K}{2}; 1 \right]$$

where we ensured that the country share does not exceed 1.

Consider that the time between offers is given by $h = \frac{T}{n}$ with $n \in \mathbb{N}$. Iterating (25) and defining ϕ as the arithmetic average of β and β^* leads us to

$$V(t) = \min \left[\frac{1 - \beta^* - hc}{2 - \beta^* - \beta} + \frac{K}{2} \sum_{i=0}^{n-1} \phi^i hc^*(t+ih) + \phi^n V(t+nh); 1 \right]$$

If the interval h goes to zero (i.e. $n \rightarrow \infty$), the last term vanishes and we obtain

$$V(t) = \begin{cases} \min \left[\frac{r-c}{r+\delta} + \frac{K}{2} e^{-\frac{r+\delta}{2}(T-t)}; 1 \right] & \text{if } t \leq T \\ \frac{r-c}{r+\delta} & \text{if } t > T \end{cases}$$

Table 1**Summary Statistics**

	Obs	Mean	Std. Dev.	Min	Max
Haircut (%)	246	49.6	24.4	0.1	93.6
Debt Stock / GDP (%)	246	68.4	24.7	22.2	111.8
12-month export growth rate (%)	246	4.63	8.26	-18.8	24.1
r (5 year T-Bill)	246	4.10	0.76	2.78	6.68
r (10 year T-Bill)	246	4.74	0.57	3.81	6.52
Delta (dom. money market rate)	246	28.3	21.5	2.2	81.3
Log (outstanding amount in USD)	158	5.60	1.66	0.35	10.01

Table 2**Dependent variable: Average Market Haircut (%)**

	5 yr T-Bill			10 yr T-Bill		
	L.S.	R.E.	F.E.	L.S.	R.E.	F.E.
Debt Stock / GDP (%)	0.545***	1.946***	2.471***	0.568***	1.926***	2.438***
	5.54	22.20	30.30	5.87	22.1	29.40
12-month export growth rate (%)	-0.914**	-1.065**	-3.251***	-0.605	-0.851*	-3.087***
	-2.01	-2.4	-7.52	-1.33	-1.89	-6.98
r	6.24***	25.99***	22.66***	12.74***	34.00***	28.00***
	2.90	8.90	7.97	4.30	9.27	7.97
delta	-0.216	1.49***	1.269***	-0.141	1.343***	1.140***
	1.61	11.13	11.08	1.07	10.37	10.12
Log (amount issuance USD)	-8.18	-276.94***	-238.17***	-48.01***	-322.05***	-272.86***
	0.63	14.70	18.38	2.66	14.36	16.03
Constant	0.690***	0.607***	0.631***	0.670***	0.608***	0.631***
	3.83	6.73	9.23	3.79	6.89	9.23
Observations	246	246	246	246	246	246
R-squared	0.200	0.111	0.056	0.230	0.150	0.068
within		0.783	0.875		0.796	0.875
between		0.214	0.188		0.249	0.180

t statistics in parentheses. * significant at 10%; ** significant at 5%; *** significant at 1%

8 Detail of the Derivations

$$q^*(t)\pi_t + \frac{y}{\delta} = \max \left[\lambda(t)\pi_t + \frac{y}{\delta}; \beta \left(q(t+h)\pi_{t+h} + \frac{y}{\delta} \right) + p(R_t/y)hy + rhR_t \right] \quad (27)$$

$$1 - q(t+h) = \max \left[1 - \lambda(t+h) - D(t+h); \beta^* \frac{\pi_{t+2h}}{\pi_{t+h}} (1 - q^*(t+2h)) \right] \quad (28)$$

$$q^*(t)\pi_t = \max \left[\lambda(t)\pi_t; \beta q(t+h)\pi_{t+h} + p(R_t/y)hy + rhR_t + (\beta - 1) \frac{y}{\delta} \right]$$

$$q^*(t) = \max \left[\lambda(t); \beta \frac{\pi_{t+h}}{\pi_t} q(t+h) - (\beta - p(R_t/y)) \frac{hy}{\pi_t} + \frac{rhR_t}{\pi_t} \right]$$

$$q(t+h) = \min \left[\lambda(t+h) + D(t+h); 1 - \beta^* \frac{\pi_{t+2h}}{\pi_{t+h}} (1 - q^*(t+2h)) \right]$$

$$q_N^*(t) = \beta \frac{\pi_{t+h}}{\pi_t} \left(1 - \beta^* \frac{\pi_{t+2h}}{\pi_{t+h}} (1 - q^*(t+2h)) \right) - (\beta - p(R_t/y)) \frac{hy}{\pi_t} + \frac{rhR_t}{\pi_t}$$

$$q_N^*(t) = \beta \frac{\pi_{t+h}}{\pi_t} - \beta \beta^* \frac{\pi_{t+2h}}{\pi_t} + \beta \beta^* \frac{\pi_{t+2h}}{\pi_t} q^*(t+2h) - (\beta - p(R_t/y)) \frac{hy}{\pi_t} + \frac{rhR_t}{\pi_t}$$

$$q_N^*(t+2h) = \beta \frac{\pi_{t+3h}}{\pi_{t+2h}} - \beta \beta^* \frac{\pi_{t+4h}}{\pi_{t+2h}} + \beta \beta^* \frac{\pi_{t+4h}}{\pi_{t+2h}} q^*(t+4h) - (\beta - p(R_{t+2h}/y)) \frac{hy}{\pi_{t+2h}} + \frac{rhR_{t+2h}}{\pi_{t+2h}}$$

$$\begin{aligned} q_N^*(t) &= \beta \frac{\pi_{t+h}}{\pi_t} - \beta \beta^* \frac{\pi_{t+2h}}{\pi_t} - (\beta - p(R_t/y)) \frac{hy}{\pi_t} + \frac{rhR_t}{\pi_t} + \\ &\quad \beta \beta^* \left[\beta \frac{\pi_{t+3h}}{\pi_t} - \beta \beta^* \frac{\pi_{t+4h}}{\pi_t} - (\beta - p(R_{t+2h}/y)) \frac{hy}{\pi_t} + \frac{rhR_{t+2h}}{\pi_t} \right] + \\ &\quad \beta \beta^{*2} \frac{\pi_{t+4h}}{\pi_t} q^*(t+4h) \end{aligned}$$

$$q_N^*(t) = \sum_{k=0}^{\infty} (\beta \beta^*)^k \left[\beta \frac{\pi_{t+(2k+1)h}}{\pi_t} - \beta \beta^* \frac{\pi_{t+2(k+1)h}}{\pi_t} - (\beta - p(R_{t+2kh}/y)) \frac{hy}{\pi_t} + \frac{rhR_{t+2kh}}{\pi_t} \right]$$

when $R_t = R$

$$q_N^*(t) = \sum_{k=0}^{\infty} (\beta \beta^*)^k \left[\beta - \beta \beta^* - (\beta - p(R/y)) \frac{hy}{\pi_t} + \frac{rhR}{\pi_t} \right]$$

$$\begin{aligned}
q_N^*(t) &= \sum_{k=0}^{\infty} \left(\frac{1}{1+\delta h} \frac{1}{1+rh} \right)^k \left[\beta \frac{rh}{1+rh} - (\beta - p(R/y)) \frac{hy}{\pi_t} + \frac{rhR}{\pi_t} \right] \\
q_N^*(t) &= \frac{\beta \frac{rh}{1+rh} - (\beta - p(R/y)) \frac{hy}{\pi_t} + \frac{rhR}{\pi_t}}{1 - \frac{1}{1+\delta h} \frac{1}{1+rh}} \\
q_N^*(t) &= \frac{rh + \left[\frac{rhR}{\pi_t} - (\beta - p(R/y)) \frac{hy}{\pi_t} \right] (1 + \delta h)(1 + rh)}{(\delta + r)h + \delta rh^2} \\
q_N^*(t) &= \frac{r + \left[\frac{rR}{\pi_t} - (\beta - p(R/y)) \frac{y}{\pi_t} \right] (1 + \delta h)(1 + rh)}{\delta + r + \delta rh}
\end{aligned}$$

if $h \rightarrow 0$ (or alternatively, if δ , r and h small), we have

$$q_N^*(t) = \frac{r + \frac{rR}{\pi_t} - (1 - p(R/y)) \frac{y}{\pi_t}}{\delta + r}$$

or

$$q_N^*(t) = \frac{r + \frac{rR - T(R/y) \cdot y}{\pi_t}}{\delta + r}$$

Conditions for Figure 5

Ruling out attachment:

I) Borrower

$$\begin{aligned}
\frac{r(\widehat{R} + Q) - T(\widehat{R}/y)y + r\widehat{R}}{r + \delta} + \frac{y}{\delta} &\geq \widehat{R} + (1 - \alpha)Q + \frac{y}{\delta} \\
r\widehat{R} + rQ - T(\widehat{R}/y)y + r\widehat{R} &\geq (r + \delta)\widehat{R} + (r + \delta)(1 - \alpha)Q \\
(r - (r + \delta)(1 - \alpha))Q &\geq (\delta - r)\widehat{R} + T(\widehat{R}/y)y = \frac{(\delta - r)\widehat{R} + T(\widehat{R}/y)y}{\alpha r - \delta(1 - \alpha)} \\
[\alpha r - \delta(1 - \alpha)]Q &\geq (\delta - r)\widehat{R} + T(\widehat{R}/y)y
\end{aligned}$$

II) Bank

$$\widehat{R} + Q - \frac{r(\widehat{R} + Q) - T(\widehat{R}/y)y + r\widehat{R}}{r + \delta} \geq \gamma\widehat{R} + \mu\alpha Q$$

$$\frac{1}{r + \delta} [(\delta - r)\widehat{R} + \delta Q + T(\widehat{R}/y)y] \geq \gamma\widehat{R} + \mu\alpha Q$$

$$(\delta - (r + \delta)\mu\alpha)Q \geq [r(1 + \gamma) - \delta(1 - \gamma)]\widehat{R} - T(\widehat{R}/y)y$$

Footnote 12: Sufficient condition for $R_{\max} > \widehat{R}$

$$\delta h y + E [(\delta - r)\widehat{R} + \delta Q(s) + T(\widehat{R}/y) \cdot y] - (1 + r)(1 - R_0)(\delta + r) = (\delta + r)R_{\max}$$

$$\delta h y + E [(\delta - r)\widehat{R} + \delta Q(s) + T(\widehat{R}/y) \cdot y] - (1 + r)(1 - R_0)(\delta + r) - (\delta + r)\widehat{R} \geq 0$$

$$E[Q(s)] \geq \delta^{-1} [(\delta + r)\widehat{R} - E [(\delta - r)\widehat{R} + T(\widehat{R}/y) \cdot y] + (1 + r)(1 - R_0)(\delta + r) - \delta h y]$$

$$E[Q(s)] \geq \delta^{-1} [2r\widehat{R} + T(\widehat{R}/y) \cdot y + (\delta + r)(1 + r)(1 - R_0) - \delta h y]$$

Since increases in otherwise prohibitive transaction costs are irrelevant, we can restrict the attention to the range in which $T(\widehat{R}/y) < 1$. Moreover, the internal solution for optimal reserve holdings implies that $\widehat{R} \leq \bar{L}$. Hence, the above condition will certainly be satisfied if

$$E[Q(s)] \geq \delta^{-1} [2r\bar{L} + (\delta + r)(1 + r)(1 - R_0) + (1 - \delta h)y]$$