Appendix B: DETAILS ABOUT THE SIMULATION MODEL

The simulation model is carried out on one spreadsheet and has five modules, four of which are contained in lookup tables that are all calculated on an auxiliary spreadsheet.

1. Population and Labor Force Module

The lookup table contains the three L-T population projections (1994) for selected years between 2000 and 2050 for four age groups: 1 - 19, 20 - 64, over 64 and over 85. For the years between these points, I assumed a constant growth rate of each population group - simplifications to make interpretation of the results less problematic. From these data I roughly estimated the population within two special age groups (20 - 24 and 65 - 69) by multiplying the relevant L-P age groupings by the fixed ratios representing what these were in 1998. So that the working age can start at 25 and the retirement age can be changed from 65 to 70. Such demographic approximations have little impact on the final results.

The labor supply was calculated separately for the 25 - 64 and the 65 - 69 age groups. For the former, I merely multiplied the population in that age group by a participation ratio which could be fixed or changed at a constant percentage increments over time. Workers in the 65 - 69 age group were estimated in two steps. If the retirement age remained at 65, the number of workers in this cohort was zero; if the retirement age was chosen to be 70 in 2050, I multiplied the population in the 65 - 69 age group by a fraction rising from 0 to 1 in regular increments and then by the same labor force participation ratio used for the 25 - 64 age group.

2. Saving Rate Module

This calculation centered around a lookup table of the consumption level of workers that is maintained throughout their working lifetime so that they accumulate enough savings to finance consumption during retirement at some fraction of their former consumption level (consumption-replacement ratio). The consumption level depends on the growth of the workers' annual income, the interest rate, their year of retirement, and their year of death. This consumption level was determined iteratively on the auxiliary spreadsheet and the solution required the worker to have exhausted all saving at the time of death.

From the data on the pattern of consumption and income of a single worker over a working lifetime, I calculated the ratio of the chosen consumption level to the average aggregate total income over the working life. Assuming the same number of workers in each age group within the cohort of workers, this was used as the saving rate for all workers in each year.

A problem arose because I assumed that the life expectancy between 2000 and 2050 was increasing from 80 to 85 and that, at least in some simulations, the age of retirement was rising from 65 to 70 in the same period. To take these changes into account, I calculated two sets of optimal savings rates, one assuming a life expectancy of 80 years and a retirement age of 65, which is used to define the initial saving rate; and another assuming a life expectancy of 85 years, and a retirement age of either 65 or 70. These set the endpoint saving rates, with the actual rate in the other years rising at even increments between these two values.

3. Income Module

The net income of workers on which they base their saving decisions was set equal to their work income (which increases at a constant annual rate) plus their interest income (or minus their interest payments if they are in debt). As noted in the text, the interest income on saving or interest payments on loans came from a source outside the model.

When calculating the saving rate, I also calculated the ratio of total aggregate income over the working lifetime to total work income over the same period. This varied according to the interest rate, growth rate, income replacement rate, age of retirement, and age of death; much of the relevant information was contained in a lookup table.

4. Module of Consumption (Dissaving) Rates by Retired Workers

This calculation centered around a lookup table providing rates of consumption of retired workers with a given life expectancy. This level was merely the level of consumption maintained through the working lifetime times the consumption-replacement ratio and was part of the calculations used to determine the optimum consumption level during a worker's lifetime.

This dissaving needed, however, to be related to the current level of income of active workers, which was easily determined by calculating the ratio of consumption of a retired worker to the average income of current workers. For simplicity, I made three calculations: the ratio of the consumption of a worker who just retired to the average income of workers 1 to 5 years in the past; to 6 to 20 years in the past, and 21 to 30 years in the past. Using these ratios I could then calculate from the current income of a worker the consumption level of retired workers in these three different age brackets. Total consumption of for these three groups of retired workers could be calculated by simply multiplying the average income of workers in the current year times these ratios times the number of retired workers in the age group corresponding to the ratio.

Because life expectancy and, for some simulations, the age of retirement were increasing, I followed the same procedure as for saving, determining the initial consumption ratios using one set of assumptions about age and retirement and the end consumption ratios using another set of assumptions and then creating a weighted average that increased in regular increments of the 50-year period.

5. The Main Simulation

Since most of the calculations are carried out in the saving, income, and dissaving modules, the calculations containing the aggregate results were simple and consisted of the calculation for income, average consumption of workers, aggregate saving of active workers, aggregate dissaving of retired workers, and, finally, net saving.

Average income increased at a constant annual rate. Average consumption was determined by multiplying the income by the saving rate determined in the saving module. Total saving was calculated by multiplying the average saving times the number of active workers. Total dissaving was determined by multiplying the number of retired workers times the average current consumption of workers times the consumption-replacement rate times the ratio of consumption of retired to active workers that was calculated in the dissaving module. Net saving is simply the sum of aggregate saving of active workers and aggregate dissaving of retired workers.

Appendix C: AN ALGEBRAIC DEMONSTRATION OF SOME SIMULATION RESULTS

The discussion in the text is based on intuitive arguments and numbers derived from the simulations. Nevertheless, if we assume a world without an interest rate, the various results can be derived from a simple algebraic model that provide more rigor to the argument that net saving falls between 2000 and 2050.

Let S_t = net saving, the sum of the saving of active workers and the dissaving of the retired workers. The saving of active workers in time period t is SA_t.

Equation C1: $SA_t = .\ (ó_t\ Y_t)\ (a_t\ A_t)$,

where ó = the saving rate, Y = average income, a_t = percentage of adult population who are active workers, and A = adult population. The expression in the first parentheses is the saving of one worker and the expression in the second parentheses is the number of active workers. Dissaving by retired workers = consumption by retired workers is SA_t .

Equation C2: $SA_t = (ñ\ (1 - ó_t)\ Y_t\ z_t)\ ((1-a_t)\ A_t)$,

where ñ = consumption-replacement ratio, and z = ratio of income on which saving decisions of retired workers were based to current income of active workers. The expression in the first brackets is the dissaving of a single worker and the expression in the second brackets is the number of retired workers. Thus, in any given year,

Equation C3-a: $S_t = (ó_t\ Y_t)\ (a_t\ A_t) - (ñ\ (1 - ó_t)\ Y_t\ z_t)\ ((1-a_t)\ A_t)$

Since we are interested in the net saving ratio, that is, the ratio of net saving over total income, this expression can be arranged for easier analysis:

Equation C3-b: $(S_t / a_t\ A_t\ Y_t) = ó_t - (ñ\ (1 - ó_t)\ z_t)((1-a_t)/a_t)$.

Given the assumed growth of 1.8% a year of income and assuming that the population in each year cohort between retirement and death is the same, z = .60. Since ñ = 1, the expression reduces to:

Equation C3-c: $(S_t / a_t A_t Y_t) = ó_t - (.6 (1 - ó_t)) ((1-a_t)/a_t)$.

Since $ó_t$ depends on the age of retirement and the life expectancy, two variables which may change over time, it is necessary to know how ó will change when these two variables change. Since lifetime saving (total average annual savings times number of work years) = total dissaving in retirement (annual consumption times number of retirement years). Letting k = percentage of adult years spent working and K = total adult years, then:

Equation C4-a: $ó\ Y\ k\ K = (ñ (1 - ó_t) Y)((1-k) K)$.

Rearranging and simplifying:

Equation C4-b: $ó = ñ (1-k)/ (k - ñk + ñ)$.

Given the assumptions of the model, this can be simplified to:

Equation C4-c: $ó = (1-k)$.

Since life expectancy rises from 80 to 85 and the retirement age from 65 to 70, k changes from 0.727 to 0.750, and ó falls from 0.273 to 0.250 (in the simulations). This should be obvious since the number of retirement years is the same, but workers have more years to accumulate the necessary savings.

Given the assumptions of the model, the question is how the aggregate saving rate changes is simple to derive. From the L-T data we determine that a falls from 0.820 to 0.766 and, as a result, (1-a)/a rises from 0.219 to 0.305 (in the simulations it falls somewhat less because I am defining the adult labor force as 25 to retirement, rather than 20 to retirement). Because the expressions in both of the brackets in equation 1a are increasing and because $ó_t$ is falling, the saving rate falls.

Although the numerical results are sensitive to the population estimates, they do not affect the qualitative results. With the SSA estimates, the same fall in (1-a)/a occurs, but with the Census estimates there is a slight increase. Nevertheless, in the latter case the result of multiplying the two bracketed expressions in equation 1a still shows an increase.