

DIARY
AND
CORRESPONDENCE
OF
SAMUEL PEPYS, ESQ., F.R.S.

FROM HIS MS. CYPHER IN THE PEPYSIAN LIBRARY,

WITH A LIFE AND NOTES BY
RICHARD LORD BRAYBROOKE.

DECIPHERED, WITH ADDITIONAL NOTES, BY
REV. MYNORS BRIGHT, M.A.,
PRESIDENT AND SENIOR FELLOW OF MAGDALENE COLLEGE, CAMBRIDGE.

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the world with wondrous contempt, when I consider for what we keep such a mighty bustle. *O fortunate* Mr Pepys! who knows, possesses, and enjoys all that's worth the seeking after. Let me live among your inclinations, and I shall be happy.

J. EVELYN.

C.]

S. Pepys to Isaac Newton.

November 22, 1693.

Sir — However this comes accompanied to you with a little trouble, yet I cannot but say, that the occasion is welcome to me, in that it gives me an opportunity of telling you that I continue sensible of my obligations to you, most desirous of rendering you service in whatever you shall think me able, and no less afflicted when I hear of your being in town, without knowing how to wait on you till it be too late for me to do it. This said, and with great truth and respect, I go on to tell you that the bearer, Mr. Smith, is one I bear great goodwill to, no less for what I personally know of his general ingenuity, industry, and virtue, than for the general reputation he has in this town, inferior to none, but superior to most, for his mastery in the two points of his profession; namely, fair writing, and arithmetic, so far, principally, as is subservient to account-antship. Now, so it is, that the late project, of which you cannot but have heard, of Mr. Neale, the Groom-Porter's lottery, has almost extinguished for some time, at all places of public conversation in this town, especially among men of numbers, every other talk but what relates to the doctrine of determining between the true proportion of the hazards incident to this or that given chance or lot. On this occasion, it has fallen out that this gentleman is become concerned, more than in jest, to compass a solution that may be relied upon beyond what his modesty will suffer him to think his own alone, or any less than Mr. Newton's, to be, to a question which he takes a journey on purpose to attend you with, and prayed my giving him this introduction to you to that purpose, which, not in common friendship only, but as due to his so

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earnest application after truth, though in a matter of speculation alone, I cannot deny him; and therefore trust you will forgive me in it, and the trouble I desire you to bear, at my instance, of giving him your decision upon it, and the process of your coming at it: wherein I shall esteem myself on his behalf greatly owing to you, and remain,

Honoured Sir, your most humble,
And most affectionate and faithful Servant,

S. P.

C. orig.]

Isaac Newton to S. Pepys.

Cambridge, Novbr 26, 1693.

Sr—I was very glad to hear of your good health by Mr Smith, and to have any opportunity given me of showing how ready I should be to serve you or your friends upon any occasion, and wish that something of greater moment would give me a new opportunity of doing it, so as to become more useful to you than in solving only a mathematical question. In reading the question, it seemed to me at first to be ill stated; and in examining Mr Smith about the meaning of some phrases in it, he put the case of the question the same as if A played with six dice till he threw a six; and then B threw as often with twelve, and C with eighteen, the one for twice as many, the other for thrice as many, sixes. To examine who had the advantage, I took the case of A throwing with one dice, and B with two—the former till he threw a six, the latter as often for two sixes; and found that A had the advantage. But whether A will have the advantage when he throws with six, and B with twelve dice, I cannot tell; for the number of dice may alter the proportion of the chances considerably, and I did not compute it in this case, the problem being a very hard one. And, indeed, upon reading the question anew, I found that these cases do not come within the question; for here an advantage is given to A by his throwing first till he throws a six: whereas, the question requires, that they throw upon equal luck, and by consequence that no advantage be given to any one by throwing

first. The question is this: A has six dice in a box, with which he is to fling a six; B has in another box twelve dice, with which he is to fling two sixes; C has in another box eighteen dice, with which he is to fling three sixes. Q^y, whether B and C have not as easy a task as A at even luck? If this last question must be understood according to the plainest sense of the words, I think that sense must be this:

1st. Because A, B, and C, are to throw upon even luck, there must be no advantage of luck given to any of them by throwing first or last, by making anything depend upon the throw of any one, which does not equally depend on the throws of the other two: and, therefore, to bar all inequality of luck on these accounts, I would understand the question as if A, B, and C, were to throw all at the same time.

2^{dly}. I take the most proper and obvious meaning of the words of the question to be, that when A flings more sixes than one, he flings a six, as well as when he flings but a single six, and so gains his expectation: and so, when B flings more sixes than two, and C more than three, they gain their expectations. But if B throw under two sixes, and C under three, they miss their expectations; because, in the question, 'tis expressed that B is to throw two, and C three sixes.

3^{dly}. Because each man has his dice in a box, ready to throw, and the question is put upon the chances of that throw, without naming any more throws than that. I take the question to be the same as if it had been put thus upon single throws.

What is the expectation or hope of A to throw every time one six, at least, with six dice?

What is the expectation or hope of B to throw every time two sixes, at least, with twelve dice?

What is the expectation or hope of C to throw every time three sixes, or more than three, with eighteen dice?

And whether has not B and C as great an expectation or hope to hit every time what they throw for, as A hath to hit what he throws for?

If the question be thus stated, it appears, by an easy com-

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putation, that the expectation of A is greater than that of B or C; that is, the task of A is the easiest: and the reason is, because A has all the chances on sixes on his dice for his expectation, but B and C have not all the chances upon theirs; for, when B throws a single six, or C but one or two sixes, they miss of their expectations. This Mr Smith understands, and therefore allows that, if the question be understood as I have stated it, then B and C have not so easy a task as A; but he seems of opinion, that the question should be so stated, that B and C, as well as A, may have all the chances of sixes on their dice within their expectations. I do not see that the words of the question, as 'tis set down in your letter, will admit it; but this being no mathematical question, but a question what is the true mathematical question, it belongs not to me to determine it. I have contented myself, therefore, to set down how in my opinion, the question, according to the most obvious and proper meaning of the words, is to be understood; and that, if this be the true state of the question, then B and C have not so easy a task as A: but, whether I have hit the true meaning of the question, I must submit to the better judgment of yourself and others. If you desire the computation, I will send it you.

I am, Sir,

Y^r most humble and obedient Servant,

IS. NEWTON.

Isaac Newton to S. Pepys.

Cambridge, Dec^r 16, 1693.

Sir—In stating the case of the wager, you seem to have exactly the same notion of it with me; and to the question, Which of the three chances should Peter chuse, were he to have but one throw for his life? I answer, that if I were Peter, I would chuse the first. To give you the computation upon which this answer is grounded, I would state the question thus:—

A hath six dice in a box with which he is to fling at least one six, for a wager laid with R.

B hath twelve dice in another box, with which he is to fling at least two sixes, for a wager laid with S.

C hath eighteen dice in another box, with which he is to fling at least three sixes, for a wager laid with T.

The stakes of R, S, and T, are equal; what ought A, B, and C, to stake, that the parties may play upon equal advantage?

To compute this, I set down the following progressions of numbers:—

Progr. 1.	1	2	3	4	5	6	the number of the dice.
Progr. 2.	0	1	3	6	10	15	
Progr. 3.	6	36	216	1296	7776	46656	{ the number of all the chances upon them.
Progr. 4.	5	25	125	625	3125	15625	{ the number of chances without sixes.
Progr. 5.	1	5	25	125	625	3125	
Progr. 6.	1	10	75	500	3125	18750	{ chances for one six and no more.
Progr. 7.	1	5	25	125	625		
Progr. 8.	1	15	150	1250	9375		{ chances for two sixes and no more.

The progressions in this table are thus found: the first progression, which expresses the number of the dice, is an arithmetical one; viz., 1, 2, 3, 4, 5, &c.; the second is found, by adding to every term, the term of the progression above it; viz., $0+1=1$, $1+2=3$, $3+3=6$, $6+4=10$, $10+5=15$, &c.; the third progression, which expresses the number of all the chances upon the dice, is found by multiplying the number 6 into itself continually; and the fourth, fifth, and seventh, are found by multiplying the number 5 into itself continually; the sixth is found by multiplying the terms of the first and fifth; viz., $1 \times 1=1$, $2 \times 5=10$, $3 \times 25=75$, $4 \times 125=500$, &c.; and the eighth is found by multiplying the terms of the second and seventh; viz., $1 \times 1=1$, $3 \times 5=15$, $6 \times 25=150$, $10 \times 125=1250$, &c.; and by these rules the progressions may be continued on to as many dice as you please.

Now, since A plays with six dice, to know what he and R ought to stake, I consult the numbers in the column under six,

and the dice, ex number and the with on stake of to 1; fo the num if you v dice, pr and the sions; number the nu 217678: therefo 211 to the pro the sur gressio ence b numbe their s that, v pound A mus that c The q fewer 1000/. upon upon value to se where chanc or ex

and there, from 46656, the number of all the chances upon those dice, expressed in the third progression, I subduct 15625, the number of all the chances without a six, expressed in the fourth: and the remainder, 31031, is the number of all the chances, with one six or above: therefore the stake of A must be the stake of R, upon equal advantage, as 31031 to 15625, or $\frac{31031}{15625}$ to 1; for their stakes must be as their expectations, that is, as the number of chances which make for them. In like manner, if you would know what B and S ought to stake upon twelve dice, produce the progressions to the column of twelve dice, and the sum of the numbers in the fourth and sixth progressions; viz., $244140625 + 585937500 = 830078125$, will be the number of chances for S; and this number, subducted from the number of all the chances in the third progression, viz., 2176782336, will leave 1346704211, the number of chances for B: therefore the stake of B would be to the stake of S, as 1346704211 to 830078125, or $\frac{1346704211}{830078125}$ to 1. And so, by producing the progressions to the number of eighteen dice, and taking the sum of the numbers in the fourth, sixth, and eighth progressions for the number of the chances for T, and the difference between this number and that in the third column for the number of the chances for C, you will have the proportion of their stakes upon equal advantage. And thence it will appear that, when the stakes of R, S, and T, are units, suppose one pound or one guinea, and by consequence equal, the stake of A must be greater than that of B, and that of B greater than that of C; and, therefore, A has the greatest expectation. The question might have been thus stated, and answered in fewer words: if Peter is to have but one throw for a stake of 1000*l.*, and has his choice of throwing either one six at least upon six dice, or two at least upon twelve, or three at least upon eighteen, which throw ought he to chuse; and of what value is his chance or expectation upon every throw, were he to sell it? Answer: Upon six dice there are 46656 chances, whereof 31031 are for him; upon twelve, there are 2176782336 chances, whereof 1346704211 are for him: therefore, his chance or expectation is worth the $\frac{31031}{46656}$ th part of 1000*l.* in the first

case, and the $\frac{1346704211}{2170782336}$ th part of 1000*l.* in the second; that is, 66*l.* 0*s.* 2*d.* in the first case, and 618*l.* 13*s.* 4*d.* in the second. In the third case, the value will be found still less. This, I think, Sir, is what you desired me to give you an account of; and if there be any thing further, you may command

Your most humble and most obedient Servant,

IS. NEWTON.

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