EXERCISES FOR SECTION 1.2

1. Bob, Glen, and Paul are once again sitting around enjoying their nice, cold glasses of iced cappucino when one of their students asks them to come up with solutions to the differential equation

$$\frac{dy}{dt} = \frac{y+1}{t+1}.$$

After much discussion, Bob says $y(t) = t$, Glen says $y(t) = 2t + 1$, and Paul says $y(t) = t^2 - 2$.

(a) Who is right?

(b) What solution should they have seen right away?

2. Make up a differential equation of the form

$$\frac{dy}{dt} = 2y - t + g(y)$$

that has the function $y(t) = e^{2t}$ as a solution.

3. Make up a differential equation of the form

$$\frac{dy}{dt} = f(t, y)$$

that has $y(t) = e^{t^3}$ as a solution. (Try to come up with one whose right-hand side $f(t, y)$ depends explicitly on both t and y.)

4. In Section 1.1, we guessed solutions to the exponential growth model

$$\frac{dP}{dt} = kP,$$

where k is a constant (see page 6). Using the fact that this equation is separable, derive these solutions by separating variables.

In Exercises 5–22, find the general solution of the differential equation specified. (You may not be able to reach the ideal answer of an equation with only the dependent variable on the left and only the independent variable on the right, but get as far as you can.)

5. $\dfrac{dy}{dt} = ty$

6. $\dfrac{dy}{dt} = t^4 y$

7. $\dfrac{dy}{dt} = 2y + 1$

8. $\dfrac{dy}{dt} = 2 - y$

9. $\dfrac{dy}{dt} = e^{-y}$

10. $\dfrac{dx}{dt} = 1 + x^2$

11. $\dfrac{dy}{dt} = \dfrac{t}{t^2 y + y}$

12. $\dfrac{dy}{dt} = t\sqrt[3]{y}$

13. $\dfrac{dy}{dt} = \dfrac{1}{2y + 1}$

14. $\dfrac{dy}{dt} = 3y^2 - 4ty^2$

15. $\dfrac{dy}{dt} = y(1-y)$

16. $\dfrac{dy}{dt} = \dfrac{t}{y}$

17. $\dfrac{dv}{dt} = t^2v - 2 - 2v + t^2$

18. $\dfrac{dy}{dt} = \dfrac{1}{ty + t + y + 1}$

19. $\dfrac{dy}{dt} = \dfrac{e^t y}{1 + y^2}$

20. $\dfrac{dy}{dt} = y^2 - 4$

21. $\dfrac{dw}{dt} = \dfrac{w}{t}$

22. $\dfrac{dy}{dt} = 1 + \dfrac{1}{y^2}$

In Exercises 23–32, solve the given initial-value problem.

23. $\dfrac{dy}{dt} = 2y + 1, \quad y(0) = 3$

24. $\dfrac{dy}{dt} = ty^2 + 2y^2, \quad y(0) = 1$

25. $\dfrac{dy}{dt} = -y^2, \quad y(0) = 1/2$

26. $\dfrac{dy}{dt} = t^2y^3, \quad y(0) = -1$

27. $\dfrac{dy}{dt} = -y^2, \quad y(0) = 0$

28. $\dfrac{dy}{dt} = \dfrac{t}{y - t^2y}, \quad y(0) = 4$

29. $\dfrac{dx}{dt} = \dfrac{t^2}{x + t^3x}, \quad x(0) = -2$

30. $\dfrac{dy}{dt} = \dfrac{1 - y^2}{y}, \quad y(0) = -2$

31. $\dfrac{dy}{dt} = (y^2 + 1)t, \quad y(0) = 1$

32. $\dfrac{dy}{dt} = \dfrac{1}{2y + 3}, \quad y(0) = 1$

33. A 5-gallon bucket is full of pure water. Suppose we begin dumping salt into the bucket at a rate of 1/4 pounds per minute. Also, we open the spigot so that 1/2 gallons per minute leaves the bucket, and we add pure water to keep the bucket full. If the salt water solution is always well mixed, what is the amount of salt in the bucket after

(a) 1 minute? **(b)** 10 minutes? **(c)** 60 minutes?
(d) 1000 minutes? **(e)** a very, very long time?

34. Consider the following very simple model of blood cholesterol levels based on the fact that cholesterol is manufactured by the body for use in the construction of cell walls and is absorbed from foods containing cholesterol: Let $C(t)$ be the amount (in milligrams per deciliter) of cholesterol in the blood of a particular person at time t (in days). Then

$$\frac{dC}{dt} = k_1(C_0 - C) + k_2E,$$

where

C_0 = the person's natural cholesterol level,
k_1 = production parameter,
E = daily rate at which cholesterol is eaten, and
k_2 = absorption parameter.