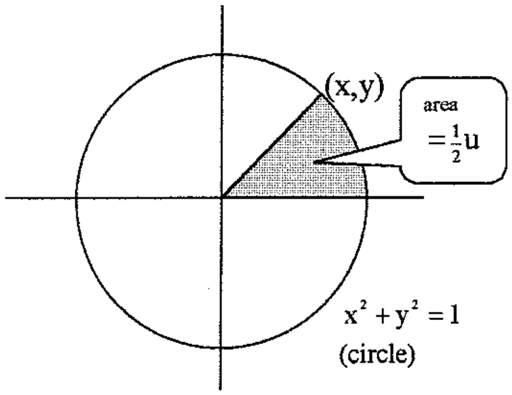
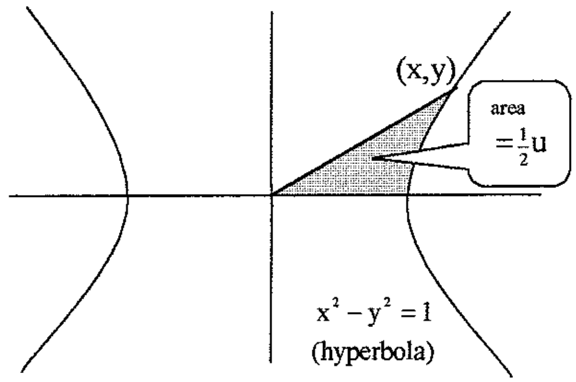
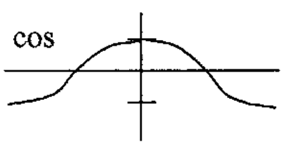
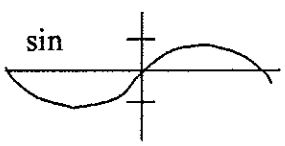
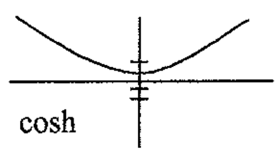
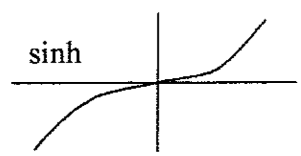
Review of Ordinary and Hyperbolic Sines and Cosines

Cosine (cos) Sine (sine)	Hyperbolic Cosine (cosh) Hyperbolic Sine (sinh)
	
Construct a sector as shown with a vertex at the point (x,y) and area $\frac{1}{2}u$. Then by definition: $\boxed{\cos u = x, \quad \sin u = y.}$ (Since u equals the radian measure of the central angle, we can think of sin and cos as functions of angles. Note that u also equals the length of the bounding arc.)	Construct a sector as shown with a vertex at the point (x,y) and area $\frac{1}{2}u$. Then by definition: $\boxed{\cosh u = x, \quad \sinh u = y.}$ (Note that u does not equal the arc length or the central angle.)
$\cos^2 u + \sin^2 u = 1$	$\cosh^2 u - \sinh^2 u = 1$
 	 
$\cos u$, $\sin u$ are bounded, and both are periodic with period 2π.	$\cosh u \geq 1$ for all u; min at u=0; unbounded above. sinh is a bijection from R to R, and is increasing.
cos is even: $\cos u = \cos(-u)$. sin is odd: $\sin u = \sin(-u)$.	cosh is even: $\cosh u = \cosh(-u)$. sinh is odd: $\sinh u = \sinh(-u)$.
$\cos u = \dfrac{e^{iu}+e^{-iu}}{2}$ $\quad \sin u = \dfrac{e^{iu}-e^{-iu}}{2}$	$\cosh u = \dfrac{e^{u}+e^{-u}}{2}$ $\quad \sinh u = \dfrac{e^{u}-e^{-u}}{2}$
$\dfrac{d}{du}\cos u = -\sin u$ $\quad \dfrac{d}{du}\sin u = +\cos u$	$\dfrac{d}{du}\cosh u = +\sinh u$ $\quad \dfrac{d}{du}\sinh u = +\cosh u$
The functions $z=\cos u$, $z=\sin u$ are a basis for the solutions of the differential equation $z'' = -z.$ (There are other bases.)	The functions $z=\cosh u$, $z=\sinh u$ are a basis for the solutions of the differential equation $z'' = +z.$ (There are other bases, such as e^{u}, e^{-u}.)

wrs 1/15/01