Topology Seminar Week 8 Homework

March 18, 2010

[M] §67 # 4
[M] §68 # 2, 4
[M] §69 # 1, 3, 4
[M] §70 # 1, 3
[M] §71 # 1, 2
[M] §72 # 1
[M] §73 # 1, 2
and four problems more:

1. Let $X \subset \mathbb{R}^3$ be the union of n lines through the origin. Compute $\pi_1(\mathbb{R}^3 \setminus X)$.

2. Let X be the space obtained by taking two copies of a torus $S^1 \times S^1$ and glueing a circle $S^1 \times \{x_0\}$ in one torus with the same circle in the other torus. Compute $\pi_1(X)$.

   Let X_1 and X_2 be surfaces. The **connected sum** of X_1 and X_2 is the surface $X_1 \# X_2$ obtained by deleting the interior of a closed discs $D_1 \subset X_1$ and $D_2 \subset X_2$ and identifying the resulting boundary circles ∂D_1 and ∂D_2 via some homeomprphism between them. (For example, if X is a surface and $T = S^1 \times S^1$, then $X \# T$ is X with a handle attached, $X \# \mathbb{R}P^2$ is X with a cross-cap attached, and $X \# S^2 = X$.)

3. Compute $\pi_1(M_g)$ where M_g is the genus g orientable surface in two ways:
   (a) First by considering $M_g = M_{g-1} \# T$, where $T = S^1 \times S^1$.
   (b) Second by considering M_g as the quotient of a regular $4g$-gon by identifying pairs of edges in some appropriate way.

4. Compute $\pi_1(K)$ where K is the Klein bottle in four ways:
   (a) First by considering K to be the quotient of $S^1 \times I$ obtained by identifying $(z, 0)$ with $(-z, 1)$ for all $z \in S^1$.
   (b) Second by considering $K = T \# \mathbb{R}P^2$.
   (c) Third by considering $K = \mathbb{R}P^2 \# \mathbb{R}P^2$.
   (d) Fourth by considering K as the quotient of a square with edges identified in the usual way.