Topology Seminar Week 12 Homework

April 15, 2010

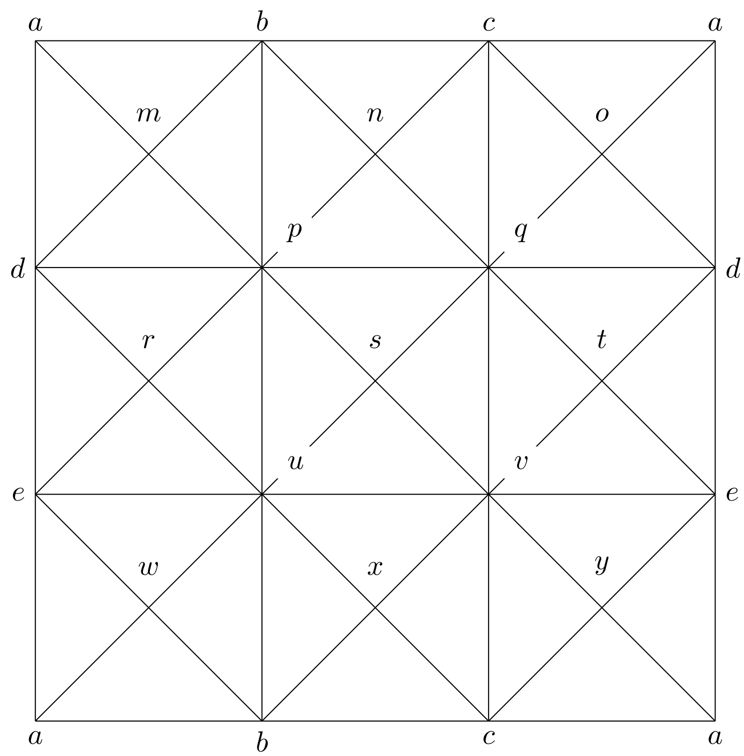
[A] Ch. 8 # 33, 34, 35
and these problems:

1. Let $n \in \mathbb{Z}$. Find triangulations K and L for S^1 and a simplicial map $f\colon |K| \to |L|$ so that

   - if α is the sum of the 1-simplices of K, then α generates $H_1(K)$,
   - if β is the sum of the 1-simplices of L, then β generates $H_1(L)$, and
   - the map $f_*\colon H_1(K) \to H_1(L)$ is such that $f_*(\alpha) = n\beta$.

   Your map f should, in some sense, be equivalent to a familiar map $g\colon S^1 \to S^1$. What is g?

2. Consider the complex T below. It is a triangulation for the torus.



Let σ be the sum of all the 2-simplices of T, oriented counter-clockwise. Then σ is a generating cycle for $H_2(T) \cong \mathbb{Z}$. Let $\alpha = [a, b] + [b, c] + [c, a]$ and $\beta = [a, d] + [d, e] + [e, a]$. (The orientation on $[a, b]$ is directed from a to b.) Then α and β are generators for $H_1(T) \cong \mathbb{Z} \oplus \mathbb{Z}$.

(a) Define simplicial maps f, g, h, and k from T to itself extending the following data:

$f\colon a \mapsto a$	$g\colon a \mapsto a$	$h\colon a \mapsto d$	$k\colon a \mapsto p$
$b \mapsto d$	$b \mapsto c$	$b \mapsto p$	$b \mapsto q$
$c \mapsto e$	$c \mapsto b$	$c \mapsto q$	$c \mapsto d$
$d \mapsto c$	$d \mapsto d$	$d \mapsto d$	$d \mapsto u$
$e \mapsto b$	$e \mapsto e$	$e \mapsto d$	$e \mapsto b$

(b) Compute the values of f_*, g_*, h_*, and k_* on σ, α, and β.

3. Let T be as above, and let S be the the boundary of a 3-simplex having vertices A, B, C, and D. (So $|S| \cong S^2$.) Let τ be the 2-cycle $\partial[A, B, C, D]$. Then τ generates $H_2(S) \cong \mathbb{Z}$.

   (a) Let $f\colon T \to S$ be the simplicial map with $f(m) = f(r) = A$, $f(p) = B$, $f(b) = f(u) = C$, and all other vertices are sent to D. Compute the induced homomorphism $f_*\colon H_2(T) \to H_2(S)$.

   (b) Let $g\colon T \to S$ be the simplicial map which agrees with f on all vertices except $g(r) = C$. Compute g_*.

   (c) Let $h\colon T \to S$ be the simplicial map which agrees with g on all vertices except $h(u) = A$. Compute h_*.

   (d) What are the induce maps f_*, g_*, and h_* on H_1?

4. Alter the schematic for T above by switching the labels for d and e on the right to get a triangulation K for the Klein bottle. Let α and β be as before.

   (a) Find a simplicial map from K to K so that the induced map on homology sends α to β.

   (b) Show that there is no simplicial map from K to K sending β to α.