General Information for Math 104
The Topology Seminar

Spring 1998

Instructor: My name is Thomas Hunter. My office is Dupont 185. You can reach me by phone at 328-8244 or by email at `thunter1@swarthmore.edu`.

Office Hours: Sundays from 7:00pm to 9:00pm are hours set aside particularly for this seminar. In addition, I will have regular office hours Monday and Wednesday from 2:00pm to 3:00pm and Friday from 9:00am to 10:00am. These slots are times you can be sure to find me in my office and willing to talk. Other times are fine, but to be sure that I am available, you should make an appointment with me. Of course you should feel free to stop by anytime and see whether I am available.

Text: We will use the text *Topology, A first course* by James Munkres. It is available at the bookstore.

General Game Plan: To cover chapters one through four and chapter eight of Munkres. Also to study the classification of surfaces and an introduction to simplicial homology. These last two topics will have reserve books and my notes as their primary sources.

Seminar Meetings: We will meet 6:00pm–11:00pm on Monday in Dupont 189. The two main activities during each meeting will be student presentation of several of the main points of the reading and student discussion of the problems.

Student Presentations: Each meeting will have several student presentations on topics I will select from the readings. These presentations will be fifteen to forty-five minutes long (depending on the number and scope) and will take the place that would be occupied by lectures were this not a seminar. Each student will be responsible for somewhere between three and ten presentations during the semester. From time to time I may assign two students to collaboratively prepare one presentation.

Student Conferences: At the beginning of the semester I will meet individually with each student each week to discuss how things are going. Later in the semester, I may meet less often with some or all of you, but those students giving presentations will always have to meet with me during the week before their presentations.

Homework: Each week, I will list some "common" problems. I expect each of you to work through all of these doing as many as you can. If it seems clear to me that the average student may not have enough time to do all of these, I will try to give some indication of the priority that I give them. In addition I each student to do one or more other problems, selected by that student from the text's problems, difficulties encountered in the reading, or any other source. The number of such additional problems will vary from student to student depending on their difficulty and the difficulty the student had that week with the common problems and the reading.

Notebooks: I ask that each of you keep an organized notebook containing your work for the course. In it, you should keep your notes on the reading and the seminar meetings, your problems and rewrites, your presentation outlines, and anything else pertinent to the course which you choose to include. I reserve the right to inspect your notebooks from time to time during the semester.

Give to me: I ask each student to give to me the following things for evaluation and comment.

1. By Tuesday at 2:00pm following each seminar give me a careful write up of one problem or one page of problems–which ever is more. Choose whatever problem or problems from the week before of which you are the most proud. The delay will give you time to make minor corrections based on your experience in seminar, but you should have most of the work done before the seminar meeting.

2. Before each seminar in which you are to give a presentation, give me a careful outline of the presentation you plan to give.

3. By 4:00pm on Mondays, give me a list of all the problems you have attempted–common and uncommon–together with a one or two word phrase describing your perception of your level of success. Don't list a problem unless you have work in your notebook documenting your work. I will use your lists to help me keep the flow of discussion going during our seminar meetings.

Exams: I expect all students in the course to take the written honors exam. I will grade the exams of the students who are not standing for honors.

Grades: For those students to whom I must assign grades, I will base my evaluation on the exam, the weekly handed-in problems, the presentations (including the handed-in outlines), and class participation (including the quality and veracity of your lists of completed problems).

Computer and Electronic resources: There is a web page associated to this course. A link to it is on my home page `http://www.swarthmore.edu/NatSci/thunter1/`.

Late work: Generally speaking late work will never be accepted and exams may never be taken late. In the case of irreconcilable conflicts you may schedule an exam earlier than the official time, but make up exams will not be given after the regularly scheduled exam except for the most extraordinary circumstances. (For example, global invasion by extraterrestials.)

Assignment one, for January 19

Our first meeting will be Monday, the first day of classes, at 7:00pm. I am asking you all to read a bunch of the text and do a bunch of problems. In our first meeting, there will be no student presentations, but I may give an informal overview of the material we will cover.

I would hope that the first chapter of the text has some things which are review and others which are new. I would very much like us all to become familiar with order types in preparation for some examples and counterexamples in topology. Hence, our first assignment is to master all of chapter one. (Read the prefaratory material as well.) Ideally I would like all of you to do all the problems which you have to think to do in this chapter. Here is a prioritization with which to make this task more manageable:

- Highest Priority:
  - page 67: 4 through 8
  - page 33: number 11
  - page 28: 1, 3 through 5, 10, 12 through 15
- Medium-High Priority:
  - page 67: 1 through 3
  - page 33: 2,9, and 10.
- Medium Priority:
  - page 67: 9–12:
  - The rest of the problems in section 4
  - The rest of the problems in section 3
- Low priority:
  - The rest of the problems throughout chapter 1. (But don't do any which don't require you to think.)

Do as many of these problems as you can working your way down the list until you have spent what time you intend to spend for the first seminar meeting. Come with a positive agenda: Specific items which are problems you have solved of difficulties with the reading which you have resolved and are willing to talk about. Also come prepared with a negative agenda: Problems which you cannot solve, or parts of the text which have unresolved difficulties. Email me a sketch of both of your agendas by noon on Monday. I will be in charge of food at this first meeting.

As always, feel free to contact me about anything at all. I will be in my office from 7 to 10 each Sunday night preceding a seminar.

Assignment two, for January 26

Reading

Read all of Chapter two, including all of the problems.

Presentations

Ben 2.3 & 2.5

Dan 2.6

Naomi 2.7

Tim 2.8 & 2.4

Aaron 2.9 & 2.10

James 2.11

Higher priority problems

2.2 4,7.

2.5 4, 6–10.

2.6 1, 5, 7, 10–13, 16, 17, 19.

2.7 1, 4, 5, 9, 12–15.

1. Identify the resulting topological space when G is the group of real numbers, $\mathbb{R}$, under addition and H is the integers, $\mathbb{Z}$.
2. Identify the resulting topological space when G is $\mathbb{R} \oplus \mathbb{R}$ and H is $\mathbb{Z} \oplus \mathbb{Z}$.

Assignment three, for February 2

Reading

Finish reading chapter 2. Also read the first six sections of Chapter 3. You may choose to read the starred sections less carefully than the others–I won't assign any problems from them this time, but I may well do so in the future.

Higher Priority Problems

old 2-8 1–3, 6, 8.

2-9 2, 4, 5.

2-10 2, 4, 9.

2-11 2, 4, 5, 8.

Custom If a group G has a topology (of any sort) and H is any subgroup of G, then the epimorphism of sets $G \to G/H$ induces a quotient topology on G/H. Identify this topological space in a few example cases.

new 3-1 1–3, 7, 8, 10, 11.

3-2 1, 3, 4, 6, 7, 9, 13.

3-5 1–5, 8–10.

3-6 1–3, 6

Presentations

James 2-11: The quotient topology. (Maybe topological groups as well.)

Naomi 3-1 & 3-2: Connectedness.

Dan 3-5 & 3-6: Compactness.

Assignment four, for February 9.

Reading

We will have three "new" sections. In reading these carefully, you may well have to go back and complete some reading from earlier assignments. I have given some suggestions about things you may want to do as you read.

3-7 Make sure you can fill in all the details to example 1. Draw a picture of part 2 of the proof of Lemma 7.2. Go back to the arguments from real analysis having to do with uniform continuity and see if you can make any connections.

3-8 After reading this section, go back and see if you can prove all the statements without looking at Munkres' arguments.

4-2 We could get by without Chapter 4 for the rest of the seminar, but 4-2 has a wealth of examples and should force you to use most of what you have learned in Chapters 2 and 3. You shouldn't need anything from 4-1 to understand this section.

Problems

I am going to assign only 12 problems. I will want every one of you to try every one of them and either get in some interesting state of stuckness, or solve it.

3-5 9.

3-7 2, 4, 6, 7.

3-8 1, 6, 8,9.

4-2 1, 5, 7.

Presentations

Ben 3-7

Aaron 3-8

Tim 4-2

Assignment five, for February 16.

Reading

Read the first four sections of chapter 8. Check all the omitted details.

Problems

I have assigned "twelve" problems. I will want every one of you to try every one of them and either get in some interesting state of stuckness, or solve it. If you finish or are stuck on all of these, certainly do some more. (Others may help relieve your stuckness.)

8–1 2, 4, 5.

8–2 (1), (2 or 5), (3 and 4). Here, I am implying that 2 and five teach much the same thing and giving you the hint that three and four together amount to less proof than the typical single problem, provided you do them the right way.

8–3 (1 and 3), 5, 7.

8–4 (1, 6 and 8), (4 and 9), 5, .

Presentations

James 8–1, 8–2.

Naomi 8–3.

Dan 8–4.

Assignment six, for February 23.

Reading

Finish reading section 8.4. Also read section 8.5

Problems

I will want every one of you to try every one of them and either get in some interesting state of stuckness, or solve it. If you finish or are stuck on all of these, certainly do some more. (Others may help relieve your stuckness.)

"old" 8–4: (1, 6 and 8), (4 and 9), 5.

8–4 : 2, 3, 7, 8, 10.

8–5 1, 2, 5, 6.

For 8.4.10 it is useful to think of the "action" of the fundamental group of the base on the fiber. Let E, B, p and so forth be as given in the problem. Let x be an element of $p^{-1}b_0$ and α be an element of $\pi_1(B, b_0)$. Define $x \cdot \alpha$ to be equal to $g(1)$ where g is a lift starting at x of a loop representing α. Verify that this action is well defined, that $x \cdot e = x$ and that $x \cdot (\alpha\beta) = (x \cdot \alpha) \cdot \beta$. If you have done all of this, you have done 8.4.10. Why?

For each of the coverings we discussed in the last seminar, find this action. (You may assume that the fundamental group of the figure eight is generated by the two loops given by the two lobes.)

Find some more covers. Try to find all of the three sheeted and all of the four sheeted covers of the figure eight. Can you figure out how to describe when you have them all?

Presentations

8–4 Dan

8–5 Ben

Assignment seven, for March 2.

Reading

tjh Read the notes on free products and actions and such (available on the web or from me.)

8–6 Note in particular, that theorem 6.1 is a special case of the Seifert Van Kampen theorem as stated in my notes.

8–7 Much of what is in this section has been hinted at already. Here it is explicitly with some good problems.

8–8 Here we continue to apply what we know about the fundamental group.

Problems

I will want every one of you to try every one of them and either get in some interesting state of stuckness, or solve it. If you finish or are stuck on all of these, certainly do some more. (Others may help relieve your stuckness.)

tjh Do all the problems (3, 4, 5, 8, 16, 18, 20). Some of these should be fairly straightforward checking of details. When confused, don't hesitate to ask me–maybe I've made a typo.

8–6 1, 2, 3.

8–7 1, 2, 3, 4. (However, read 5–9 as well, even if you don't work on them. This is another way that actions get into the picture. Can you find a relation with the actions I have written about? Also, problem 9 gives us the interesting fact that every cyclic group is the fundamental group of some three manifold.)

8–8 1, 2, 3, 4. (However, again the remaining problems are very interesting.)

Presentations

Tim 8-6

Aaron 8-7

Naomi 8-8

Assignment eight, for March 16.

Reading

8–14 and 8–9, 10, 11. (I list them this way because section 14 is both the most central part of the assignment and the lion's share of the work.) Here are some comments

8–9 Those of you who took the complex analysis seminar should review the proof of this theorem that you saw there. Those of you who did not should introspect on whether or not you have been implicitly assuming this theorem all of your life, and if so, whether you had any reason to believe it before now.

8–10 This is just the tip of the iceberg of vector fields. You may find a generalization of theorem 10.4 on page 190 of Massey. (However to understand the proof, we will have to wait until we have some tools of homology.) A more general problem is to find the maximum number of continuous vector fields on S^n which are nowhere linearly dependent. The somewhat surprising answer was completely determined by Adams in his fairly famous 1962 paper in the Annals of Mathematics. In order to prove this theorem, Adams used a form of generalized homology theory called K-theory.

8–11 Here we develop the language of homotopy equivalence, which is what algebraic topologists often really mean when they say that two things are the same as far as a topologist is concerned.

8–14 The main theorem of this section also appears in section 10 of chapter 5 of Massey (on reserve) and in section 8 of chapter 3 of Bredon. The curious reader may want to compare the three treatments.

Problems

I will want every one of you to try every one of them and either get in some interesting state of stuckness, or solve it. If you finish or are stuck on all of these, certainly do some more. (Others may help relieve your stuckness.)

8–9 2.

8–10 1, 3, 5, 6, 8.

8–11 2, 3, 5, 6, 7.

8–14 1, 2, 4, 6.

Presentations

§9 & §10 Dan

§11 Ben

§14 James

Assignment nine, for March 23.

Reading

Read the notes on the classification theorem and appropriate material from our reserve books. Most of this reading will be in Massey, although Armstrong also proves the main theorem.

Problems

I will want every one of you to try every one of them and either get in some interesting state of stuckness, or solve it. If you finish or are stuck on all of these, certainly do some more. (Others may help relieve your stuckness.)

tjh
1. Do the problems from the notes above. (I guess that means just 25 and 28.)
2. Use the library to find a proof of the triangulation theorem. (Armstrong and Massey both give pointers.) Take a look at it. Is it in language you can understand? Bring a Xerox of at least the first page or two to seminar. (If you already know a classmate has found a certain version, look for a different one.) Can you find out where the Jordan curve theorem is used?
3. Use the Seifert Van-Kampen theorem and Massey's treatment of the classification theorem to give a description of the groups which arise as fundamental groups of compact surfaces.

Massey Do I.5.1, I.7.1 through I.7.6.

Armstrong 7.1.2.

Old There was a fair portion of last week which we didn't get to. Try to finish your work on those problems. Add any difficulties or proudnesses to your agenda for this week

Presentations

Dan Munkres: Chapter 8: §9 & §10.

Tim The proof of classification theorem.

Aaron Either some examples for the classification theorem, or some comments on the triangulation theorem–you pick.

Assignment ten, for March 30.

Reading

My notes should introduce you to the material, but you will probably want to seek out the sources for a more complete treatment.

Armstrong The first three sections of chapter 6. These introduce simplicial complexes and the simplicial approximation theorem.

Big Munkres Parts of sections 23 and 24. These introduce homological algebra. For now you may stop reading section 23 halfway down page 131 and stop reading section 24 one-third of the way down page 140. See also Massey X.2.

Problems

I will want every one of you to try every one of them and either get in some interesting state of stuckness, or solve it. If you finish or are stuck on all of these, certainly do some more. (Others may help relieve your stuckness.)

tjh All of the embedded problems.

Armstrong Chapter 6 2, 3, 5, 7–14, 18.

Munkres 23.1.

Presentations

James Definitions and examples of triangulation.

Naomi Definitions and examples of the first ideas in homological algebra.

Ben The proof of the simplicial approximation theorem.

Assignment eleven, for April 6.

Reading

Read Armstrong chapter eight. My notes have a little added commentary, slightly changed organization, and a bit of alternate notation. However, you would loose very little by considering only Armstrong. I have corrected the homological notes from last week and brought the notation there into better accord with the notation for this week. Try to make sure that you believe that what I have stated is, in its sum, no stronger or weaker than what Armstrong states.

Problems

I will want every one of you to try every one of them and either get in some interesting state of stuckness, or solve it. If you finish or are stuck on all of these, certainly do some more. (Others may help relieve your stuckness.)

tjh The embedded problems.

Armstrong All problems in Chapter eight. These problems break into two types: those which are examples and those which are steps remaining to be filled in in the proofs from the text. Be sure to spend equal time on both kinds. There are more problems than usual here, but many should be fairly straightforward.

tjh Here is one further algebraic problem as a warm up for some of the algebra we will have to do. The result is called the five lemma and you may look it up in several of our reserve texts if you like. The comments in Massey are particularly apt. Suppose that we have a commutative diagram

$$\begin{array}{ccccccccc} A_1 & \longrightarrow & A_2 & \longrightarrow & A_3 & \longrightarrow & A_4 & \longrightarrow & A_5 \\ \cong\downarrow f_1 & & \cong\downarrow f_2 & & \downarrow f_3 & & \cong\downarrow f_4 & & \cong\downarrow f_5 \\ B_1 & \longrightarrow & B_2 & \longrightarrow & B_3 & \longrightarrow & B_4 & \longrightarrow & B_5 \end{array}$$

in which the rows are exact and the marked vertical arrows are isomorphisms. Show that it follows that the middle arrow is also an isomorphism. Show that the hypotheses can be weakened somewhat–some of the vertical arrows need not be isomorphisms. Can you give the minimum hypothesis that allow your argument?

Presentations

Ben Consequences and examples (post 8.3) of the main theorem.

Naomi Examples (Armstrong section 8.3).

Aaron Proof of the main theorem.

Assignment twelve, for April 13.

Reading

Read Chapter 9 of Armstrong. There is a great number of great theorems there all of which follow from the work we have done to this point to set up homology. Also read my page and a half of notes on what I call the fundamental lemma of homological algebra. Proofs of this lemma are given or commanded in both Massey and big Munkres. You may want to look at those texts as additional sources.

Problems

I would like you to carry out steps three through nine of the proof stated in my notes. I would also like you to work as many problems as you can from chapter 9 of Armstrong.

For presentations, I am going to assign one section of Armstrong to each of you (except Naomi who gets the five lemma). Ben, you may not have wanted to present–in that case, you may regard this as fair warning that I will look to you for solutions to problems in that section.

I will want every one of you to try every problem in your section and either get in some interesting state of stuckness, or solve it. If you finish or are stuck on all of these, certainly do some more. (Others may help relieve your stuckness.)

In this way I hope that although it may be the case that no one covers the entire chapter in the usual depth, we will together form an open covering of the chapter.

Presentations

Naomi The five lemma. (Also comment on the fundamental lemma, if you like.)

Ben 9.1: Maps of spheres.

James 9.2: Euler-Poincare.

Aaron 9.3: Borsuk-Ulam.

Tim 9.4: Lefshetz fixed point formula.

Dan 9.5: Dimension.

Assignment thirteen, for April 20.

Reading

Read my notes. You may want to look at the treatment of the topics as presented in big Munkres and in Massey. In the case of today's notes, my notes are pretty much my own, although I have used Munkres' notation when I felt I had a choice.

Problems

Do the embedded problems in the notes. I have tried hard to make them both doable and thought-provoking. If I have failed in either way, be sure to talk to me in time to do something and to be provoked. If you finish or are stuck on all of these, do some from the relevant sections in Munkres or Massey.

Presentations

Ben Mayer-Vietoris

Tim Relative homology

Assignment fourteen, for April 27.

Reading

Read my notes. Again, you may want to look at the axiomatic descriptions in various other sources. I have a history book if anyone is interested in learning more about the history of the axioms. (Tim has it right now.)

You will find out that exact sequences of pairs play a big role.

Problems

Again there are problems embedded in the text. We didn't discuss the pair problems much last week. Prepare to discuss them this week together with the new material.

Presentations

Dan Issues old and new with exact sequences of pairs.

James Excision.

Aaron The axioms.