Math 34 Review

April 28, 2011
Disclaimer

Our course follows Sue Colley’s *Vector Calculus* fairly closely. This presentation is basically an annotated table of contents of that book.
Preliminaries

- The basic language of functions
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- The basic language of functions
  - domain, codomain, graph, sections
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- The basic language of functions
  - domain, codomain, graph, sections
  - one-to-one, onto
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- The basic language of functions
  - domain, codomain, graph, sections
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- The basic language of linear algebra
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  - domain, codomain, graph, sections
  - one-to-one, onto
- The basic language of linear algebra
  - dot and cross products
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  - determinants
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  - domain, codomain, graph, sections
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- The basic language of linear algebra
  - dot and cross products
  - determinants
  - equations of lines, planes, etc.
Preliminaries

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  - domain, codomain, graph, sections
  - one-to-one, onto
- The basic language of linear algebra
  - dot and cross products
  - determinants
  - equations of lines, planes, etc.
- Spherical, Cylindrical, and Polar Coordinates
Preliminaries

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- The basic language of linear algebra
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- Spherical, Cylindrical, and Polar Coordinates
- The basic language of subsets of $\mathbb{R}^n$
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  - dot and cross products
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- Spherical, Cylindrical, and Polar Coordinates
- The basic language of subsets of $\mathbb{R}^n$
  - interior, exterior, boundary
Preliminaries

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- The basic language of linear algebra
  - dot and cross products
  - determinants
  - equations of lines, planes, etc.

- Spherical, Cylindrical, and Polar Coordinates

- The basic language of subsets of $\mathbb{R}^n$
  - interior, exterior, boundary
  - open, closed
Preliminaries

- The basic language of functions
  - domain, codomain, graph, sections
  - one-to-one, onto
- The basic language of linear algebra
  - dot and cross products
  - determinants
  - equations of lines, planes, etc.
- Spherical, Cylindrical, and Polar Coordinates
- The basic language of subsets of $\mathbb{R}^n$
  - interior, exterior, boundary
  - open, closed
- Continuity and limits of several variable functions
Differentiation

- The derivative as a good linear approximation (tangent plane to graph)
Differentiation

- The derivative as a good linear approximation (tangent plane to graph)
- The derivative as a matrix of partial derivatives
Differentiation

- The derivative as a good linear approximation (tangent plane to graph)
- The derivative as a matrix of partial derivatives
- The chain rule
Differentiation

- The derivative as a good linear approximation (tangent plane to graph)
- The derivative as a matrix of partial derivatives
- The chain rule
- Directional derivatives and the gradient
Differentiation

- The derivative as a good linear approximation (tangent plane to graph)
- The derivative as a matrix of partial derivatives
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- Directional derivatives and the gradient
- Partial derivatives as slopes of sections
Differentiation

- The derivative as a good linear approximation (tangent plane to graph)
- The derivative as a matrix of partial derivatives
- The chain rule
- Directional derivatives and the gradient
- Partial derivatives as slopes of sections
- Higher order partial derivatives
Differentiation

- The derivative as a good linear approximation (tangent plane to graph)
- The derivative as a matrix of partial derivatives
- The chain rule
- Directional derivatives and the gradient
- Partial derivatives as slopes of sections
- Higher order partial derivatives
- $C^n$ functions ($n$-th order partials are continuous.)
Differentiation

- The derivative as a good linear approximation (tangent plane to graph)
- The derivative as a matrix of partial derivatives
- The chain rule
- Directional derivatives and the gradient
- Partial derivatives as slopes of sections
- Higher order partial derivatives
- $C^n$ functions ($n$-th order partials are continuous.)
- Linearity and the product rule
Differentiation

- The derivative as a good linear approximation (tangent plane to graph)
- The derivative as a matrix of partial derivatives
- The chain rule
- Directional derivatives and the gradient
- Partial derivatives as slopes of sections
- Higher order partial derivatives
- $C^n$ functions ($n$-th order partials are continuous.)
- Linearity and the product rule
- (Omit Newton’s Method)
Sample problem

Problem

In this problem, let \( f(x, y, z) = x^3 - 2y^2 + z^2 \). Let \( S \) be the surface defined by \( f(x, y, z) = 25 \).
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a) Find the gradient, \( \nabla f \) of \( f \).
Sample problem

Problem
In this problem, let \( f(x, y, z) = x^3 - 2y^2 + z^2 \). Let \( S \) be the surface defined by \( f(x, y, z) = 25 \).

a) Find the gradient, \( \vec{\nabla} f \) of \( f \).

b) Find a vector perpendicular to the surface, \( S \), at the point \( (3, 1, 0) \).
Problem

In this problem, let $f(x, y, z) = x^3 - 2y^2 + z^2$. Let $S$ be the surface defined by $f(x, y, z) = 25$.

a) Find the gradient, $\vec{\nabla} f$ of $f$.

b) Find a vector perpendicular to the surface, $S$, at the point $(3, 1, 0)$.

c) Find a conditional equation for the plane tangent to $S$ at $(3, 1, 0)$. (That is to say find an equation in $x$, $y$, and $z$ which is satisfied if and only if $(x, y, z)$ is on the specified tangent plane.)
Sample Problem

Problem

a) State the general form of the chain rule.

b) Let \( w = f(u, v) \) where \( u = x^2 + y^2 \) and \( v = x^2 - y^2 \).

Show how the chain rule as you stated it above allows you to calculate \( \frac{\partial w}{\partial x} \) and \( \frac{\partial w}{\partial y} \) in terms of \( x, y, f_u, \) and \( f_v \). (I.e. do this calculation using the chain rule as you stated it.)
Sample Problem

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Vector valued functions

► Parametrized Curves
Vector valued functions

- Parametrized Curves
  - functions $\mathbb{R} \rightarrow \mathbb{R}^n$ as parametrized paths
Vector valued functions

- Parametrized Curves
  - functions $\mathbb{R} \to \mathbb{R}^n$ as parametrized paths
  - velocity and acceleration
Vector valued functions

- Parametrized Curves
  - functions $\mathbb{R} \rightarrow \mathbb{R}^n$ as parametrized paths
  - velocity and acceleration
  - tangent line
Vector valued functions

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  - functions $\mathbb{R} \rightarrow \mathbb{R}^n$ as parametrized paths
  - velocity and acceleration
  - tangent line
  - product rules (for $\cdot$ and $\times$)
Vector valued functions

- Parametrized Curves
  - functions $\mathbb{R} \to \mathbb{R}^n$ as parametrized paths
  - velocity and acceleration
  - tangent line
  - product rules (for $\cdot$ and $\times$)
  - speed and arclength
Vector valued functions

- Parametrized Curves
  - functions $\mathbb{R} \rightarrow \mathbb{R}^n$ as parametrized paths
  - velocity and acceleration
  - tangent line
  - product rules (for $\cdot$ and $\times$)
  - speed and arclength
  - the moving frame: tangent, normal and binormal
Vector valued functions

- **Parametrized Curves**
  - functions $\mathbb{R} \to \mathbb{R}^n$ as parametrized paths
  - velocity and acceleration
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  - product rules (for $\cdot$ and $\times$)
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- **Vector Fields**
Vector valued functions

- Parametrized Curves
  - functions $\mathbb{R} \rightarrow \mathbb{R}^n$ as parametrized paths
  - velocity and acceleration
  - tangent line
  - product rules (for $\cdot$ and $\times$)
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- Vector Fields
  - read and write pictures
Vector valued functions

- **Parametrized Curves**
  - functions $\mathbb{R} \to \mathbb{R}^n$ as parametrized paths
  - velocity and acceleration
  - tangent line
  - product rules (for $\cdot$ and $\times$)
  - speed and arclength
  - the moving frame: tangent, normal and binormal

- **Vector Fields**
  - read and write pictures
  - flowlines
Vector valued functions

- **Parametrized Curves**
  - functions $\mathbb{R} \rightarrow \mathbb{R}^n$ as parametrized paths
  - velocity and acceleration
  - tangent line
  - product rules (for $\cdot$ and $\times$)
  - speed and arclength
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- **Vector Fields**
  - read and write pictures
  - flowlines
  - **Divergence, Gradient, and Curl**
Sample problem

Problem
Let \( \vec{x}(t) = (t \cos t, t \sin t, t) \), and suppose that \( \vec{x} \) represents the position of a particle at time \( t \).
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a) Find the velocity of the particle as a function of \( t \).
Sample problem

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Let $\vec{x}(t) = (t \cos t, t \sin t, t)$, and suppose that $\vec{x}$ represents the position of a particle at time $t$.

a) Find the velocity of the particle as a function of $t$.

b) Find the speed of the particle as a function of $t$. 
Problem
Let \( \vec{x}(t) = (t \cos t, t \sin t, t) \), and suppose that \( \vec{x} \) represents the position of a particle at time \( t \).

a) Find the velocity of the particle as a function of \( t \).

b) Find the speed of the particle as a function of \( t \).

c) Write down but do not evaluate an integral which gives the total distance traveled by the particle between time \( t = a \) and time \( t = b \).
Problem
Let $\vec{x}(t) = (t \cos t, t \sin t, t)$, and suppose that $\vec{x}$ represents the position of a particle at time $t$.

a) Find the velocity of the particle as a function of $t$.

b) Find the speed of the particle as a function of $t$.

c) Write down but do not evaluate an integral which gives the total distance traveled by the particle between time $t = a$ and time $t = b$.

d) Describe the trajectory traced out by the particle.
Sample Problem

Problem
For each of the following descriptions, either give a specific example of the thing described or a brief explanation of why no such thing exists.

a) A scalar function $f(x, y, z)$ which is not constant, but for which $\vec{\nabla}f = \vec{0}$.

b) A vector field $\vec{F}(x, y, z)$ none of whose partial derivatives are zero, but for which $\vec{\nabla} \times \vec{F} = \vec{0}$.

c) A vector field $\vec{F}(x, y, z)$ none of whose partial derivatives are zero, but for which $\vec{\nabla} \cdot \vec{F} = 0$.

d) Are there vector fields $\vec{F}(x, y, z)$ none of whose partial derivatives are zero, but for which both $\vec{\nabla} \times \vec{F} = \vec{0}$ and $\vec{\nabla} \cdot \vec{F} = 0$? What would the geometry of such a field look like? Can you relate such fields to solutions to Laplace's equation, $\nabla^2 f = 0$ for an appropriately chosen scalar field, $f$?
Sample Problem

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a) A scalar function $f(x, y, z)$ which is not constant, but for which $\nabla f = \vec{0}$. 
Sample Problem

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b) A vector field $\vec{F}(x, y, z)$ none of whose partial derivatives are zero, but for which $\nabla \times \vec{F} = \vec{0}$. 
Sample Problem

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For each of the following descriptions, either give a specific example of the thing described or a brief explanation of why no such thing exists.

a) A scalar function \( f(x, y, z) \) which is not constant, but for which \( \vec{\nabla} f = \vec{0} \).

b) A vector field \( \vec{F}(x, y, z) \) none of whose partial derivatives are zero, but for which \( \vec{\nabla} \times \vec{F} = \vec{0} \).

c) A vector field \( \vec{F}(x, y, z) \) none of whose partial derivatives are zero, but for which \( \vec{\nabla} \cdot \vec{F} = 0 \).
Sample Problem

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Taylor’s Theorem and Finding Maxima and Minima

- Taylor approximations for scalar functions of several variables
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- Taylor approximations for scalar functions of several variables
  - first order
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- Taylor approximations for scalar functions of several variables
  - first order
  - second order
Taylor’s Theorem and Finding Maxima and Minima

- Taylor approximations for scalar functions of several variables
  - first order
  - second order
  - remainder formulas
Taylor’s Theorem and Finding Maxima and Minima

- Taylor approximations for scalar functions of several variables
  - first order
  - second order
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  - higher order
Taylor’s Theorem and Finding Maxima and Minima

- Taylor approximations for scalar functions of several variables
  - first order
  - second order
  - remainder formulas
  - higher order

- Finding extrema
Taylor’s Theorem and Finding Maxima and Minima

- Taylor approximations for scalar functions of several variables
  - first order
  - second order
  - remainder formulas
  - higher order

- Finding extrema
  - The multivariable second derivative test
Taylor’s Theorem and Finding Maxima and Minima

- Taylor approximations for scalar functions of several variables
  - first order
  - second order
  - remainder formulas
  - higher order
- Finding extrema
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  - the hessian
Taylor’s Theorem and Finding Maxima and Minima

- Taylor approximations for scalar functions of several variables
  - first order
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- Finding extrema
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  - the hessian
  - positive and negative definite
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- Lagrange multipliers
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  - applying the method
Taylor’s Theorem and Finding Maxima and Minima

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- The multivariable second derivative test
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- Lagrange multipliers
  - applying the method
  - understanding the geometry
Taylor’s Theorem and Finding Maxima and Minima

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- Applications
Taylor’s Theorem and Finding Maxima and Minima

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  - applying the method
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- Applications
  - least squares
Taylor’s Theorem and Finding Maxima and Minima

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- The multivariable second derivative test
  - the hessian
  - positive and negative definite
  - max, min, and saddle points
- Lagrange multipliers
  - applying the method
  - understanding the geometry
- Applications
  - least squares
  - Newtonian motion
Sample Problem

Problem
In this problem, let $f(x, y) = 6x - 8y - x^2 - y^2$. 

a) Find the gradient and the Hessian of $f$.

b) Find and classify the critical points of $f$.

c) Use the method of Lagrange Multipliers to find the maximum and minimum values that $f$ takes on the circle $x^2 + y^2 = 100$.

d) Use the previous parts of this problem to find the maximum and minimum values that $f$ takes on the disk $D_{10} = \{(x, y) | x^2 + y^2 \leq 100\}$.
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Multiple Integrals

- Integrals as Riemann sums
Multiple Integrals

- Integrals as Riemann sums
- Fubini’s Theorem
Multiple Integrals

- Integrals as Riemann sums
- Fubini’s Theorem
- Iterated integrals
Multiple Integrals

- Integrals as Riemann sums
- Fubini’s Theorem
- Iterated integrals
- Type I, II, ... regions
Multiple Integrals

- Integrals as Riemann sums
- Fubini’s Theorem
- Iterated integrals
- Type I, II, … regions
- Changing the order of integration
Multiple Integrals

- Integrals as Riemann sums
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- Changing the order of integration
- Change of variables
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- Change of variables
- Applications
Multiple Integrals

- Integrals as Riemann sums
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- Changing the order of integration
- Change of variables
- Applications
  - area and volume
Multiple Integrals

- Integrals as Riemann sums
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  - area and volume
  - average values
Multiple Integrals

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  - area and volume
  - average values
  - total stuff
Multiple Integrals

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  - area and volume
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  - stuff-weighted averages
Multiple Integrals

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- Changing the order of integration
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  - area and volume
  - average values
  - total stuff
  - stuff-weighted averages
    - center of mass and centroid
Multiple Integrals

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  - area and volume
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  - stuff-weighted averages
    - center of mass and centroid
    - moment of inertia
Sample Problem

Problem

Let $D_R = \{(x, y) \mid x^2 + y^2 \leq R^2\}$ and let $S_R = [-R, R] \times [-R, R]$. 

a) Calculate $\int\int_{D_R} e^{-x^2 - y^2} \, dA$.

b) Express $\int\int_{S_R} e^{-x^2 - y^2} \, dA$ in terms of the quantity $\int_{-R}^{R} e^{-x^2} \, dx$.

c) Justify the following inequalities:

$$\int\int_{D_R} e^{-x^2 - y^2} \, dA < \int\int_{S_R} e^{-x^2 - y^2} \, dA < \int\int_{D_R \sqrt{2}} e^{-x^2 - y^2} \, dA.$$

d) Use the previous parts of this problem to deduce the value of $\int_{-\infty}^{\infty} e^{-x^2} \, dx$. 
Sample Problem

Problem
Let $D_R = \{(x, y) \mid x^2 + y^2 \leq R^2\}$ and let $S_R = [-R, R] \times [-R, R]$.

a) Calculate $\int \int_{D_R} e^{-(x^2+y^2)} \, dA$. 
Sample Problem

Problem
Let \( D_R = \{(x, y) \mid x^2 + y^2 \leq R^2 \} \) and let \( S_R = [-R, R] \times [-R, R] \).

a) Calculate \( \int \int_{D_R} e^{-(x^2+y^2)} \, dA \).

b) Express \( \int \int_{S_R} e^{-(x^2+y^2)} \, dA \) in terms of the quantity \( \int_{-R}^{R} e^{-x^2} \, dx \).
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c) Justify the following inequalities:

\[
\int \int_{D_R} e^{-(x^2+y^2)} \, dA < \int \int_{S_R} e^{-(x^2+y^2)} \, dA < \int \int_{D_{\sqrt{2}R}} e^{-(x^2+y^2)} \, dA.
\]
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Let $D_R = \{(x, y) \mid x^2 + y^2 \leq R^2\}$ and let $S_R = [-R, R] \times [-R, R]$.

a) Calculate $\int \int_{D_R} e^{-(x^2+y^2)} \, dA$.

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$$\int \int_{D_R} e^{-(x^2+y^2)} \, dA < \int \int_{S_R} e^{-(x^2+y^2)} \, dA < \int \int_{D_{\sqrt{2}R}} e^{-(x^2+y^2)} \, dA.$$

d) Use the previous parts of this problem to deduce the value of $\int_{-\infty}^{\infty} e^{-x^2} \, dx$. 
Sample Problem

Problem

In this problem, let
\( H = \{ (x, y, z) \mid x^2 + y^2 + z^2 \leq 25 \text{ and } x \geq 0 \} \), be the hemisphere of radius 5 centered at the origin and having positive x coordinate. Let \( f : H \rightarrow \mathbb{R} \) be an unknown scalar function.
Sample Problem

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In this problem, let
\[ H = \{(x, y, z) \mid x^2 + y^2 + z^2 \leq 25 \text{ and } x \geq 0\}, \]
be the hemisphere of radius 5 centered at the origin and having positive \(x\) coordinate. Let \( f : H \to \mathbb{R} \) be an unknown scalar function.

(a) Express \( \iiint_H f \, dV \) as an iterated integral in rectangular \((x, y, z)\) coordinates.
Sample Problem

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In this problem, let
\( H = \{(x, y, z) \mid x^2 + y^2 + z^2 \leq 25 \text{ and } x \geq 0\} \), be the hemisphere of radius 5 centered at the origin and having positive \( x \) coordinate. Let \( f : H \rightarrow \mathbb{R} \) be an unknown scalar function.

a) Express \( \iiint_H f \, dV \) as an iterated integral in rectangular \((x, y, z)\) coordinates.

b) Express \( \iiint_H f \, dV \) as an iterated integral in cylindrical \((r, \theta, z)\) coordinates.
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c) Express \( \iiint_H f \, dV \) as an iterated integral in spherical \((\rho, \phi, \theta)\) coordinates.
Line Integrals

- Scalar line integrals
Line Integrals

- Scalar line integrals
- Vector line integrals
Line Integrals

- Scalar line integrals
- Vector line integrals
- Work and circulation
Line Integrals

- Scalar line integrals
- Vector line integrals
- Work and circulation
- Reparameterization
Line Integrals

- Scalar line integrals
- Vector line integrals
- Work and circulation
- Reparameterization
- Orientation of curves
Line Integrals

- Scalar line integrals
- Vector line integrals
- Work and circulation
- Reparameterization
- Orientation of curves
- Green’s Theorem
Line Integrals

- Scalar line integrals
- Vector line integrals
- Work and circulation
- Reparameterization
- Orientation of curves
- Green’s Theorem
  - statement
Line Integrals

- Scalar line integrals
- Vector line integrals
- Work and circulation
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- Orientation of curves
- Green’s Theorem
  - statement
  - application
Line Integrals

- Scalar line integrals
- Vector line integrals
- Work and circulation
- Reparameterization
- Orientation of curves
- Green’s Theorem
  - statement
  - application
  - as special case of divergence theorem
Line Integrals

- Scalar line integrals
- Vector line integrals
- Work and circulation
- Reparameterization
- Orientation of curves
- Green's Theorem
  - statement
  - application
  - as special case of divergence theorem
  - as special case of curl theorem
Line Integrals

- Scalar line integrals
- Vector line integrals
- Work and circulation
- Reparameterization
- Orientation of curves
- Green’s Theorem
  - statement
  - application
  - as special case of divergence theorem
  - as special case of curl theorem
- Path independence, conservation and anti-gradients
Line Integrals

- Scalar line integrals
- Vector line integrals
- Work and circulation
- Reparameterization
- Orientation of curves
- Green’s Theorem
  - statement
  - application
  - as special case of divergence theorem
  - as special case of curl theorem
- Path independence, conservation and anti-gradients
- Fundamental theorem of calculus for curves
Sample Problem

Problem
Let $C$ be a simple closed curve going counter-clockwise around a region $D$ in the plane.

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b) Find $M$ so that $\oint_C M \, dx$ gives the $x$-coordinate of the centroid of $D$. (You may assume that the area, $A$, of $D$ is known.)
Surface Integrals

- Parameterization of Surfaces
Surface Integrals

- Parameterization of Surfaces
  - tangent space and normal vector from parameterization
Surface Integrals

- Parameterization of Surfaces
  - tangent space and normal vector from parameterization
  - smooth parameterizations
Surface Integrals

- Parameterization of Surfaces
  - tangent space and normal vector from parameterization
  - smooth parameterizations
  - orientation from parameterization
Surface Integrals

- Parameterization of Surfaces
  - tangent space and normal vector from parameterization
  - smooth parameterizations
  - orientation from parameterization
- Scalar surface integrals
Surface Integrals

- Parameterization of Surfaces
  - tangent space and normal vector from parameterization
  - smooth parameterizations
  - orientation from parameterization

- Scalar surface integrals

- Vector surface integrals
Surface Integrals

- Parameterization of Surfaces
  - tangent space and normal vector from parameterization
  - smooth parameterizations
  - orientation from parameterization

- Scalar surface integrals
- Vector surface integrals
- Flux and flow
Surface Integrals

- Parameterization of Surfaces
  - tangent space and normal vector from parameterization
  - smooth parameterizations
  - orientation from parameterization

- Scalar surface integrals
- Vector surface integrals
- Flux and flow
- Reparametrization
Surface Integrals

- Parameterization of Surfaces
  - tangent space and normal vector from parameterization
  - smooth parameterizations
  - orientation from parameterization

- Scalar surface integrals
- Vector surface integrals
- Flux and flow
- Reparametrization
- Orientability
Surface Integrals, ctd.

- Stokes’ theorem
Surface Integrals, ctd.

- Stokes’ theorem
  - the right hand rule for surfaces
Surface Integrals, ctd.

- Stokes’ theorem
  - the right hand rule for surfaces
  - the statement of Stokes’ theorem
Surface Integrals, ctd.

- Stokes’ theorem
  - the right hand rule for surfaces
  - the statement of Stokes’ theorem
  - applications and corollaries
Surface Integrals, ctd.

- Stokes’ theorem
  - the right hand rule for surfaces
  - the statement of Stokes’ theorem
  - applications and corollaries
  - anti-gradients and curl
Surface Integrals, ctd.

- Stokes’ theorem
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- Gauss’ Theorem
Surface Integrals, ctd.

- Stokes’ theorem
  - the right hand rule for surfaces
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- Gauss’ Theorem
  - Infinitesimal integrals explain the geometry of derivatives
Surface Integrals, ctd.

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- Gauss’ Theorem
  - Infinitesimal integrals explain the geometry of derivatives

- Gauss, Green, and Stokes are all fundamental too
Sample Problem

Problem
Let $D$ be the solid region, $D := \{(x, y, z) \mid x^2 + y^2 + 1 \leq z \leq 5\}$. Let $\vec{F}$ be the vector field defined by $\vec{F}(x, y, z) := (x, y^2, z)$.
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a) Use Gauss’ theorem (also known as the divergence theorem) to convert $\iint_{\partial D} \vec{F} \cdot d\vec{S}$ into a volume integral.
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b) Compute the value of the volume integral you gave as the answer to the previous part.
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c) Parametrize the boundary, $\partial D$, of $D$. (Split $\partial D$ into more than one piece, if necessary.)
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c) Parametrize the boundary, $\partial D$, of $D$. (Split $\partial D$ into more than one piece, if necessary.)

d) Express $\iint_{\partial D} \vec{F} \cdot d\vec{S}$ in terms of one or more explicit iterated integrals with respect to the parameters. Do not evaluate these integrals.