Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. Show your work. Correct answers with no justification may receive little or no credit. This is a closed book exam. No calculators are allowed. No uncalled-for simplification is required. Write your name below, and also on any of the pages that become detached from this cover-page as you work the exam.

Name: ________________________________

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1. In this problem, let $\vec{F}$ be the vector field given by $\vec{F}(x, y, z) = (xz, xy, \sin(xz))$ and $g$ be the scalar function defined by $g(x, y, z) = x^2 z + xy^2$. For each of the expressions below, either calculate the specified expression, or explain why it makes no sense.

(a) (4 points) $\nabla \times \vec{F}$.

(b) (4 points) $D_{\vec{u}}g(\vec{a})$ where $\vec{u} = \frac{(1,1,1)}{\sqrt{3}}$ and $\vec{a} = (-1,0,1)$.

(c) (4 points) $\nabla \times (\nabla \cdot \vec{F})$.

(d) (4 points) $\nabla \cdot (\nabla g)$.

(e) (4 points) $\nabla \cdot (\nabla \times \vec{F})$. 
2. Suppose a particle in $\mathbb{R}^2$ has position $\vec{x}(t) = (t \sin t + \cos t, t \cos t - \sin t)$ at time $t$.

(a) (10 points) Find the position and velocity of the particle at times $t = 0$, $t = \pi/4$, and $t = \pi/2$. Make a rough sketch including this data and a trajectory consistent with it. Explain how the trajectory you drew is consistent with the data in one or two brief sentences.

(b) (10 points) Where will the particle be when it has moved $2\pi^2$ units (in the sense of arc-length) from the point where it started at $t = 0$? Give the coordinates of the particle when it has traveled that far, as well as the distance (along a straight line) it is from its starting point.
3. Suppose that \( x = e^u \sin v \) and \( y = e^u \cos v \). Let \( f(x, y) \) be an otherwise unspecified differentiable scalar function of \( x \) and \( y \).

(a) (10 points) Show how the chain rule allows you to express \( \frac{\partial f}{\partial u} \) and \( \frac{\partial f}{\partial v} \) in terms of \( u \), \( v \), \( \frac{\partial f}{\partial x} \), and \( \frac{\partial f}{\partial y} \).

(b) (10 points) Now use basic linear algebra to find \( \frac{\partial f}{\partial x} \) and \( \frac{\partial f}{\partial y} \) in terms of \( u \), \( v \), \( \frac{\partial f}{\partial u} \), and \( \frac{\partial f}{\partial v} \).
4. In this problem we consider the surface described by the equation $x^2 + y^2 = z^2 - z^3$.  
(Note that this is a level surface of the function $F(x, y, z) = x^2 + y^2 - z^2 + z^3$.)

(a) (5 points) Find the tangent plane to the surface at the point $(1, 1, -1)$

(b) (5 points) Find any points on the surface where the tangent plane is parallel to the plane $z = 0$.

(c) (5 points) Find any points on the surface where the tangent plane is perpendicular to the plane $z = 0$.

(d) (5 points) What can you say about the tangent plane at $(0, 0, 0)$?
5. In the figure below I have presented a vector field which is the gradient of a certain function, \( f \). At the point marked \( A \) in the picture, the value of \( f \) is 5. The vectors drawn to represent the vector field have been scaled so that the one marked with an ellipse around it represents a vector of length three. In this problem, I ask you to deduce some more information about \( f \).

(a) (10 points) Draw on the picture a curve representing the level curve for \( f \) at height 5. In the space below provide a very brief (one sentence should do) explanation of why you drew what you did.

(b) (10 points) Estimate the value of \( f \) at the point marked \( B \). Again, be sure to include a brief explanation of how you arrived at your estimate.