This is a closed book, in-class exam. Calculators are not allowed. You may use a pre-prepared half-sheet (one-sided) of notes.

Write all of your work in the space provided. If you run out of space, use the backs of the pages. If you use the backs of the pages, label your work with the appropriate problem number.

Show your work. You should always give at least a brief justification for each answer; correct answers with no justification may receive little or no credit. No uncalled-for simplification is required.

Write your name below, and also on any of the pages that become detached from this cover-page as you work the exam.

Name: ________________________________

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1. (20 points) Let $D$ be the region in the first quadrant of the $xy$-plane bounded by the ellipse $x^2 + 4y^2 = 4$. Thus, $D$ is the quarter ellipse drawn below.

Find $\iint_D xy \, dx \, dy$ using the substitution $x = \sqrt{u}$, $y = \sqrt{v}$. 

![Diagram of the quarter ellipse]
2. (a) (10 points) Use Lagrange multipliers to find the maximum value taken by \( f(x, y) = y^2 - x^3 + x \) on the circle \( x^2 + y^2 = 1 \). (As often happens, this problem can be done without Lagrange multipliers, but you won’t get credit if you do it that way.)

(b) (10 points) Find and classify the critical points \( f \) has in the interior of the circle.
3. A wire of uniform density lies in the $xy$-plane along the graph of the function $y = \frac{2}{3} (x - 1)^{3/2}$ in the range $1 \leq x \leq 4$.

(a) (10 points) Find the length of the wire.

(b) (10 points) Find the $x$-coordinate of the center of mass of the wire.
4. Of the following two vector fields

\[ \vec{F} = (y + z, x + 2y + z, y) \]
\[ \vec{G} = (y, x + 2y + z, y) \]

one is conservative and the other is not.

(a) (10 points) Determine which is which.

(b) (10 points) For the conservative one, find a scalar potential.
5. (20 points) Use Green’s theorem to calculate the area within the circle \(x^2 + y^2 = 4\) bounded on the left by the line \(x = 1\). It is possible to do this problem without using Green's theorem, but that won't get you any credit. In your solution, clearly show how you use Green’s theorem.