

**LABORATORY 6: INTRODUCTION TO FOURIER SERIES  
AND SUPERPOSITION**

**Objectives**

1. To gain an intuitive understanding of Fourier series and to apply this technique to the analysis of a physical system.
2. To gain a greater appreciation for the superposition principle.

**Apparatus**

Computer with data acquisition system, function generator, resistor, capacitor, and oscilloscope.

**Background**

Fourier series are generally used to express complicated functions as a series of sine and cosine functions, which are relatively easy to manipulate mathematically. If an input can be expressed as a sum of sines and cosines (or equivalently, a sum of shifted sines *or* cosines); the response of the system to the input is the sum of the responses of the system to the sine and cosine inputs. In this lab, you will decompose a square wave input into a series of sine waves and construct the response of the system to the square wave as the sum of the responses to the series of sine waves.

In general, if  $f(t)$  is a bounded, periodic function with period  $T_0 = T$ , is continuous (except for a finite number of jump discontinuities), and has a finite number of maxima and minima,  $f(t)$  can be represented by a Fourier series given by:

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left[ a_n \cos\left(\frac{2n\pi t}{T}\right) + b_n \sin\left(\frac{2n\pi t}{T}\right) \right]. \quad (1)$$

Although this definition sounds quite restrictive, it encompasses almost all functions that are of practical interest in engineering.

As you can see from (1), the coefficients  $a_n$  are associated with cosines, which are even functions ( $f(t) = f(-t)$ ) while the  $b_n$  are associated with odd functions ( $f(t) = -f(-t)$ ). Therefore it is not surprising that if  $f(t)$  is an odd function, all the  $a_n$  (the coefficients for the even functions) are zero. Likewise, for an even function all the  $b_n$  are zero.

In this laboratory we will be dealing with the special case when  $f(t)$  is an odd square wave as shown below in Fig. 1.

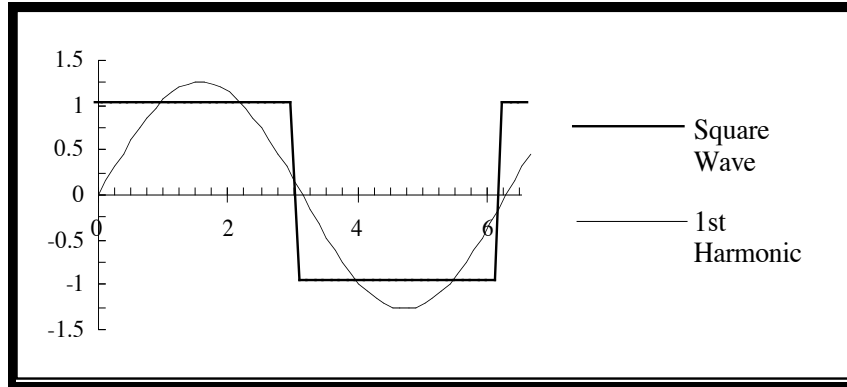


Figure 1: Square Wave Input and First Harmonic of the Square Wave

Since the function is odd, the  $a_n$  will be 0, and the  $b_n$  are given by

$$b_n = \frac{2}{T} \int_0^T f(t) \sin\left(\frac{2n\pi t}{T}\right) dt. \quad (2)$$

To prove this, you can multiply both sides of (1) by  $\sin(m\pi t/T)$  and integrate from  $-T$  to  $T$ . The function  $f(t)$  is then given by:

$$f(t) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{2n\pi t}{T}\right), \quad (3)$$

with

$$b_n = \begin{cases} 0; & n \text{ even} \\ \frac{4}{n\pi}; & n \text{ odd} \end{cases}, \quad (4)$$

where  $T$  is half the period of the square wave. In Fig. 1, the square wave is shown, along with its first harmonic, which has an amplitude corresponding to  $b_1$ . Figure 2 shows the third harmonic multiplied by  $b_3$ , and the sum of the first and the third harmonics. You can see that the sum of harmonics is beginning to approach the square wave.

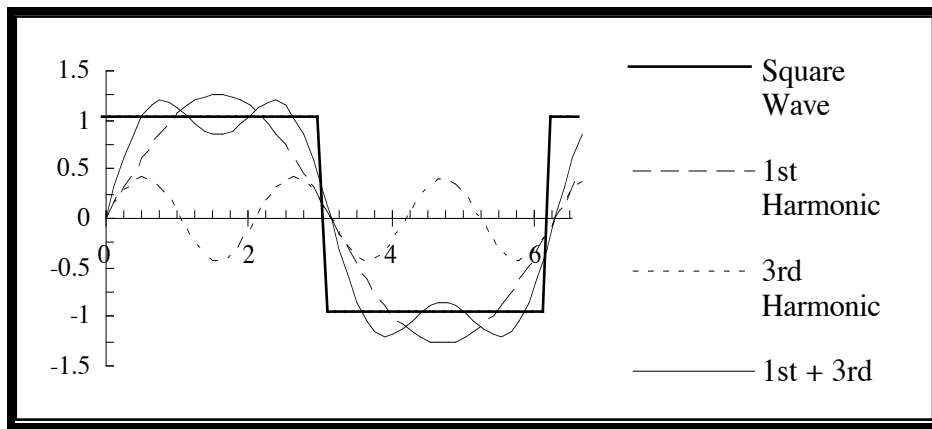


Figure 2: Square Wave Input, First and Third Harmonics, and their Sum

The approximation is even better as we add more and more harmonics, up to the 11<sup>th</sup> harmonic in Fig. 3.

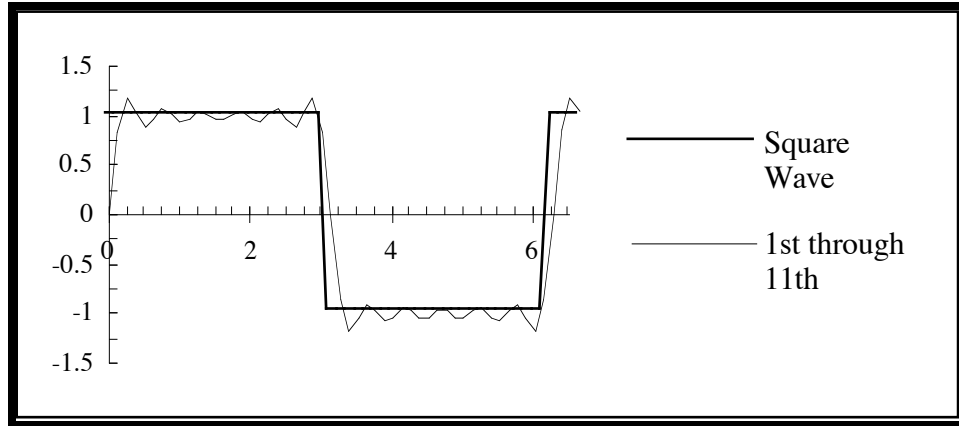


Figure 3: Square Wave Input and the Sum of the First and Third Harmonics of the Square Wave

If the function  $f(t)$ , as given above, is the input to a linear system then the output is

$$g(t) = \sum_{n=1}^{\infty} b_n r_n(t), \quad (5)$$

where  $r_n(t)$  is the response of the system to an input of  $\sin(2n\pi t/T)$ . Your task in this lab is to apply several sine functions with frequency  $n\pi/T$  to an RC circuit and to collect the input and output waveforms. From these you will construct an approximation of the square wave input from (3) and the response to a square wave input from (5).

### Procedure

1. Set up an RC circuit with a time constant of 0.01 sec. Use a low pass configuration.
2. Using a 10 Hz square wave (+1 to -1 V) from a signal generator as the input to the circuit, display both the input and output signals on the oscilloscope. Show one full period of the square wave. Use the Agilent IntuiLink data grabber to collect the input and output waveforms. Save this data.
3. Decompose your square wave into its Fourier components using the trigonometric form of the Fourier Series. Repeat step 2 for the first several harmonics (go at least to 90 Hz). Carefully check the frequency and amplitude of each input signal, and make sure that  $t = 0$  corresponds to a positively-sloped zero-crossing of each sinusoid. Later you will need to add all of the Fourier components to recreate the input and output waveforms. Please make sure that your data make sense before leaving the lab.

### Report

1. Please follow the format of a formal lab report.
2. Derive the Fourier coefficients of equation (4) using equation (2).
3. Include a schematic of the circuit.

4. Include graphs of the input and output waveforms for the square wave input. Compare to a theoretical graph that you construct using the equation you obtain from analysis of the circuit with a square wave input. (You can use a step input and infer the response during the second half of the period.) Did the graphs look like you expected them to look? Why or why not?
5. For each of the Fourier components include the input and output waveforms. Also provide a “running sum” of it and the other components with lower frequencies. Did the graphs look like you expected them to look? Why or why not?
6. Create a theoretical prediction of the sums of all Fourier components for the input and output waveforms. Compare to your experimental results.
7. Note the overshoot (known as Gibbs' phenomenon) at the edges of the wave formed by the sum of the sine waves. What percent of the initial amplitude is the overshoot as a function of the number of sine waves included in the sum?
8. Discuss any errors or discrepancies you note in the above. What are the sources of these errors? How do they affect your results? Can you think of ways to reduce the effects of the errors?