ENGINEERING 12 SPRING 2008
LINEAR PHYSICAL SYSTEMS ANALYSIS

LABORATORY 3: CONVOLUTION AND THE IMPULSE RESPONSE

Objectives

The goal of this lab is to try to answer the question:
"Can the temperature inside a passive structure (a building made from straw bales and stucco, see pictures on the course webpage) be predicted by the outside temperature if the house is modeled by a first order system?"

A situation like this might come up if you were working for a group that built such structures and they wanted to know the temperature variations in a building where the external temperature are typically 60 degrees during the day and 30 at night, or that the longest cold spell is 3 days at 40 degrees, and you want to be able to predict the interior temperatures.

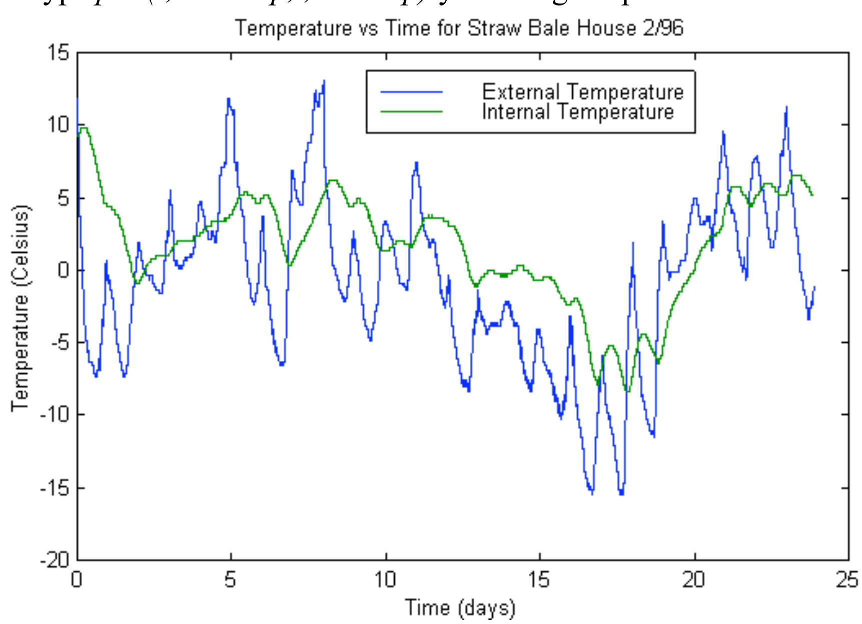
Apparatus

For this lab you will need a computer running Matlab.

Procedure

1. Load the file temps.mat from the course webpage onto your computer (where Matlab can find it). Type *load temps* at the Matlab prompt and three variables will be loaded into the Matlab workspace.
   1) The variable *t* holds time values (spaced at 192 second intervals).
   2) The variable *ExtTemp* holds values for the outside temperature for most of the month of February 1996.
   3) The variable *IntTemp* holds values for the internal temperature of the straw bale house over the same time period.

If you type *plot(t,ExtTemp,t,IntTemp)* you will get a plot like the one shown below.



The rest of this lab includes some things to think about. You do not need to do all of them, and you may add some other ideas of your own.

2. If we assume that the internal temperature can be predicted by modeling the straw bale house as a simple first order system, with the output of the system being the internal temperature, and the input to the system being the external temperature, this yields the first order differential equation:

$$\tau \cdot \dot{\theta}_i(t) + \theta_i(t) = \theta_e(t)$$

where θ_i is the internal temperature, θ_e is the external temperature, and τ is the time constant of the system.

3. Even thought there was no heater in the building, the average internal temperature was higher than the external temperature (Why?). There is no simple way for our first order model to account for this difference, so before beginning the analysis of the data, you should give the two data sets the same average value. You can set the average of the External Temperature to zero by subtracting off the average. You can do this with Matlab with the command *ExtTemp=ExtTemp–mean(ExtTemp);* You should also do this for the internal temperature.

4. What is the impulse response of the system based on the differential equation?

5. The Matlab *conv* (convolution) command does the following operation (assuming *y* is the output, *f* is the input, and *h* is the impulse response):

$$y_i = \sum_j f_j \cdot h_{i-j}$$

Matlab does not do an integration, and the summation does compensate for the time step chosen. Also, Matlab does the calculation such that length(*y*)=length(*f*)+length(*h*)-1. You will probably want to truncate *y* to set its length the same as that of *f.*

6. Recall that we originally defined convolution from the sum

$$y(t) = \sum_j (f(j \cdot \Delta T) \cdot \Delta T) \cdot (h(t - j \cdot \Delta T))$$

which we formed into an integral as we let $\Delta T \to 0$. In our case we only have values at discrete time intervals, t_i such that $t_i = i \cdot \Delta T$. So we can rewrite the summation as

$$y(i \cdot \Delta T) = \sum_j (f(j \cdot \Delta T) \cdot \Delta T) \cdot (h(i \cdot \Delta T - j \cdot \Delta T))$$

or

$$y_i = \Delta T \cdot \sum_j f_i \cdot h_{i-j}$$

which is just like the convolution sum done by Matlab multiplied by ΔT.

7. Matlab's *conv* function is probably the easiest way for you to find the internal temperature, given the external temperature. All you need is two arrays holding the input data (external temperature), and the impulse response (which depends on the time constant), to find the internal temperature.

8. What value of τ best fits the data? How do you know? Finding the best value of the time constant is not trivial, and it depends on how you define "best". A common definition of "best" is the least squared error criterion.

$$sum\ squared\ error = sse = \sum_{data\ po\mathrm{int}\,s} (data(t_i) - fitted(t_i))^2$$

What does a plot of the sum squared error, sse, look like as a function of the time constant τ? You can find a best value for τ by trial and error, or by writing a Matlab program.

9. After you get your best fit to the data, plot the difference between the measured internal temperature and the temperature predicted by your model, called the residuals or the error. If the model is accurate, this difference should be random. Are there any regular features in the error? If there are any regular features, can you explain them in terms of factors that were not considered in the model? (Factors that would cause the internal temperature to be higher than the external temperature?)

10. If you use Matlab's *conv* function, it just calculates the zero-state response. Because of this, your calculations will be off near the beginning, until the startup transients die out.

1) Are these errors important?
2) How can you add ther zero-input response and thus negate these startup errors?
3) Is It really necessary to get rid of these startup errors to decide if the model is a good one?

Report

1. Your report should answer the question "Can the temperature inside the straw bale house be predicted by the outside temperature if the house is modeled by a first order system?". Your report should be written so it could be read and understood by someone who is at your level, but knows nothing about convolution or the particular measurements that were made.
2. No formal lab report is required, but think about the organization for this report, and make your presentation clear.
3. Also include your Matlab code as an appendix in your lab report .
4. Be sure to include at least one graph that shows External Temperature, Internal Temperature, and your "best" fit to the Internal Temperature. Make sure you clearly indicate your calculated time constant.