(1) The reliability of polygraphs ("lie detectors") is a controversial topic. Courts usually do not allow evidence from polygraphs during trials, but many employers use polygraph screening as part of the hiring process in the hope that they can avoid hiring dishonest employees. According to some studies, a polygraph can detect 65% of liars, but incorrectly identifies 15% of truthful people as liars.

Suppose a company believes 95% of its applicants are truthful. The company gives all its applicants a polygraph test, asking, “Have you ever stolen anything from your place of work?” Of course, all the applicants answer “No”, but the polygraph identifies some of these people as liars. **What is the probability that an applicant identified as a liar is actually trustworthy and was wrongly accused?**

Hint: assume there are 10000 applicants and make a diagram like the one we made in class on Thursday, October 2. Out of 10000 applicants, on average, how many are telling the truth and how many are lying? Of the honest applicants and the liars, how many of each group will be identified as liars? Of those so accused, how many are actually telling the truth? The correct answer should be .81.

Trivia question (for fun; do not hand in): One of the inventors of the polygraph was also the creator of what comic book character? The answer makes sense if you think about it.

(2) Calculate the following areas under the Normal distribution, and sketch an appropriate picture. In a Normal distribution…

(a) What percent of the area under the curve lies to the left of 1.6 SDs below the mean?
(b) What percent lies to the right of 0.7 SDs above the mean?
(c) What percent lies between 1.6 SDs below the mean and 0.7 SDs above the mean?
(d) What percent lies farther than ±2.57 SDs from the mean?
(e) What percent lies within ±0.5 SDs of the mean?
(f) 79% of the area lies to the right of the point Z. How many SDs above or below the mean is Z?

(3) Well-designed opinion polls usually report a margin of error along with their estimates (e.g., “27% approve of the job President Bush is doing, with ±3% margin of error”).

(a) Why is a margin of error necessary? That is, why do the polls not simply report that 27% approve of the job President Bush is doing? What causes the uncertainty that makes such a margin of error necessary? Be specific.
(b) Less than one week before the 2000 presidential election between Bush and Gore, pollsters were estimating that as many as 25% of potential voters were undecided. This uncertainty makes it harder to predict the outcome of the election. Is this uncertainty accounted for in the ±3% margin of error?

(4) Suppose a university that is better known for its basketball program than for its academic strength claims that historically, 80% of its basketball players get degrees. An investigation examines the outcomes of 40 randomly selected players who entered the program over a period of years. Of these 40 players, 65% graduated with a degree.

(a) If the university’s claim is true, the proportion of players graduating in a sample of 40 will have a sampling distribution that is approximately Normal. What is the mean of this sampling distribution? What is its standard deviation?
(b) If the university’s claim is true, calculate the probability that, in a randomly selected sample of 40 players, 65% or less will graduate. Also, sketch a picture that corresponds to the probability that you are calculating.

(c) Given your answer to part (b), do you think the university’s claim is plausible? Explain briefly.

(5) In a recent Gallup Poll (October 21, 2008), 513 Latvian adults were asked, “Do you think who is elected president of the United States makes a difference to your country or not?” (Yes, Gallup conducts polls in Latvia, too.) The poll found that 34% of Latvians believe that the upcoming US election makes a difference to Latvia.

(a) Identify the population and the sample.

(b) Identify the parameter and the statistic.

(c) Calculate the margin of error of this estimate. (Don’t just assume it’s ±3%; do the calculation.)

(d) Give a 95% confidence interval for the true proportion of Latvian adults who believe that the US election makes a difference to Latvia.

(6) A research firm conducts polls to estimate the proportion of people who would vote for presidential candidates. Just before an election, the firm increases the size of its sample from the usual 1200 people to 2500 people.

(a) Does the larger sample lessen the bias of the poll result? Explain briefly.

(b) Does the larger sample improve the precision (i.e., decrease the margin or error) of the result? Explain briefly.

(7) Suppose that the price of a technology company stock starts out at $100 per share. Each day the stock price can go up exactly one dollar or down exactly one dollar, with probability .52 and .48, respectively. Assume that whether the stock price goes up or down on any one day does not affect whether it goes up or down on any other day. What is the probability that after 250 business days (approximately one year) the stock will have increased in price?

(Hint: Let \( p = .52 \). Consider the 250 business days to be a random sample of all possible days on which this stock could be traded. In order for the stock to increase in price under this scenario, what must be true about \( \hat{p} \), the proportion of days on which it increased in value? What are the chances of this happening?)

(8) In a recent poll, a random sample of 537 adults was asked this question: “If you could choose between the following two approaches, which do you think is the better penalty for murder, the death penalty or life imprisonment, with absolutely no possibility of parole?” Of those polled, 52% chose the death penalty. Find a 95% confidence interval for the proportion of adults who favor the death penalty.