Theorem. If $G(V, E)$ is a connected graph with even degree everywhere, then Algorithm IterativeEuler outputs an Euler cycle for $G$.

**Input** $G(V, E), v$  
[Graph and start vertex]

**Output** $C$  
[Euler cycle as sequence of edges]

**Algorithm** IterativeEuler

Pathgrow($E, v; C$)  
[Same Pathgrow as in DAM3]

$e \leftarrow$ first edge of $C$

repeat until $e$ is last edge of $C$

$v \leftarrow$ other end of $e$

if some edge of $E - C$ is incident to $v$

then Pathgrow($E - C, v; C'$)

splice $C'$ into $C$ after $e$

[C is now the augmented cycle]

else $e \leftarrow$ next edge of $C$

endif

endrepeat

Proof of the Theorem. By loop invariant. The invariant is the following statement $S$: $C$ is a cycle that uses each edge of $G$ at most once, and no edges in $E - C$ are incident to $v$ or any earlier vertex in $C$.

We show that $S$ is a loop invariant by showing that it meets the three conditions of Section 2.5.

1) $S$ is true just before entering the repeat-loop for the first time because at that point $C$ is the result of the single call $\text{Pathgrow}(E, v; C)$, and we know (from DAM3) that $\text{Pathgrow}$ returns a cycle (that begins and ends at $v$ and doesn’t reuse edges) when $G(V, E)$ has even degree everywhere, and furthermore that every edge incident to $v$ is in this cycle. Therefore, there are no edges of $E - C$ incident to $v$, and there are no earlier vertices in $C$ to worry about.

2) If $S$ is true just before some entrance to the repeat-loop, then it is true just before the next entrance: The additional pass through the loop either just checks that the next vertex $v$ in $C$ is not incident to $E - C$, in which case the invariant certainly remains true; or else it finds an edge of $E - C$ incident to this $v$. In the latter case, since $E - C$ has even degree at every vertex, $\text{Pathgrow}(E - C, v; C')$ returns a cycle that doesn’t reuse edges but does use every edge incident to $v$ that is not in $C$. Therefore, splicing $C'$ into $C$, and renaming the whole thing $C$ means that $C$ is still a cycle that uses each edge at most once, and no edges of the now smaller $E - C$ are incident at the current $v$ either. This completes the pass through the repeat-loop, so the invariant is again true just before we reenter the loop.

3) $S$ is true at termination, because termination occurs at an entrance to the loop.

Finally at termination $C$ is an Euler cycle because $E - C$ is empty. Why? By the loop invariant, at termination $E - C$ is not incident to any vertex on $C$, so if $E - C$ were nonempty, $G$ would be disconnected.
Moreover, termination must occur, because each pass through the repeat-loop either reduces $|E - C|$ (but never below 0) or moves us one vertex further along $C$ (which never grows to more than $|E|$ edges).