Another approach to finding Moment of Inertia

Let I_z be the moment of inertia around the z-axis. This is what we have computed so far, but in this problem we will also compute I_y and I_x. Of course, for a ball or sphere with center at the origin they will all be the same by symmetry. This observation makes this method computationally simpler than the original method.

Let a point dM of mass be at position (x, y, z). Then what we have written as

$$dI_z = r^2\, dM$$

could just as well be written

$$dI_z = (x^2 + y^2)\, dM,$$

since the distance r of the point-mass from the z-axis is $r = \sqrt{x^2 + y^2}$.

Similarly, you should write down expressions using (x, y, z) coordinates for I_y and I_x.

Now, let $a = \sqrt{x^2 + y^2 + z^2}$. Thus a is the distance of the point mass from the origin. We would normally call this distance r or ρ or R, but all these letters are already taken!

Anyway, if we had to integrate $a^2\, dM$, that would be a lot easier (for either a ball or a sphere) than integrating $r^2\, dM$. Why?

Last piece of the puzzle: What is a simplified integral formula for $I_x + I_y + I_z$, and what is the relationship between $I_x + I_y + I_z$ and I_z ?

April 21, 1999