Rate of Convergence

Note: We proved in class that

$$\Big(1+\frac{a}{n}\Big)^n = e^a\Big(1-\frac{a^2}{2}(\frac{1}{n})+(\frac{a^3}{3}+\frac{a^4}{8})(\frac{1}{n})^2+\text{h.o.t}\Big). \tag{1}$$

1. You tell your roommate that you have been learning how to do complicated interest computations in your head. Oh yeah, she asks, what's $(1.02)^{50}$? Figure it out. Explain your secret method briefly. Hint: you already know that $e = 2.718$ and that $(1+\frac{a}{n})^n$ is very close to $e^a(1-\frac{a^2}{2n})$.

2. In your head, figure out what $\Big(1-\frac{1}{100}\Big)^{100}$ is an approximation to, whether it is high or low, and by what percent. Then use a calculator to check if you are right. As in class, take the ratio of the approximation to the truth.

3. Eq (1) gives the "first and second order" corrections when $(1+\frac{a}{n})^n$ is used to approximate e^a. What is the third-order correction? If you are tired of manipulating power series by hand to answer such questions, check the Mathematica file "Rate Conv", which I have put on the Math 6B Maurer server.