1. The bounds on a region in polar coordinates are $0 \leq \theta \leq \pi$ and $1 \leq r \leq 2$. Quickly sketch the region in the $xy$-plane.

Ans 1.

![Region Sketch]

2. Consider $D = \int_{0}^{2} \int_{\sqrt{4-x^2}}^{\sqrt{4-x^2}} x^2 + y^2 \, dy \, dx$.

   a) Rewrite the integral in polar form.

   Ans 2. a) $\int_{-\pi/2}^{\pi/2} \int_{0}^{r} r^2 \, r \, dr \, d\theta$.

   b) $\int_{-\pi/2}^{\pi/2} d\theta \int_{0}^{2} r^3 \, dr = \frac{\pi}{4} \cdot 16 = 4\pi$.

3. Use your answer to Problem 2a to quickly convert the following to cylindrical coordinates. Do not evaluate.

   $\int_{0}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{2} dz \, dy \, dx$.

   Ans 3. $\int_{-\pi/2}^{\pi/2} \int_{0}^{2} r^2 \, dz \, dr \, d\theta$.

4. Consider the unit ball centered at the origin; let $D$ be the part with $x \geq 0$, $y \geq 0$, $z \geq 0$, in other words, the positive octant. Evaluate

   $\iiint_D z \, dV$
Ans 4.

\[
\int_{0}^{\pi/2} \int_{0}^{\pi/2} \int_{0}^{1} (\rho \cos \phi)(\rho^2 \sin \phi) d\rho \, d\phi \, d\theta
\]

\[
= \int_{0}^{\pi/2} d\theta \int_{0}^{\pi/2} \sin \phi \cos \phi \, d\phi \int_{0}^{1} \rho^3 \\
= \frac{\pi}{2} \left( \frac{1}{2} \sin^2 \phi \right) \bigg|_{0}^{\pi/2} \frac{1}{4} \quad [u = \sin \phi, \, du = \cos \phi \, d\phi] \\
= \frac{\pi}{2} \cdot \frac{1}{2} \cdot \frac{1}{4} = \frac{\pi}{16}.
\]

(10) Bonus. The integral in Problem 3 computes a volume. Of what? Sketch or describe the figure.

Ans Bonus. One-half (the half with $x \geq 0$) of the inverted cone of height 2 with lateral surface $z = r = \sqrt{x^2 + y^2}$. In the figure below, think of the positive $x$-axis as coming out of the paper. Then we want the half of the cone that is “above” the paper.

![Image of a cone](image-url)