Special Problem H (corrected)

H. Recall, we used the identity \( \cos^2 x = \frac{1+\cos 2x}{2} \) to find that
\[
\int \cos^2 x \, dx = \frac{1}{2} x + \frac{1}{2} \sin 2x \,(+C).
\] (1)

We found the identity beginning with the double angle formula
\[
\cos 2x = \cos^2 x - \sin^2 x,
\] (2)
and working it around into the form
\[
\cos^2 x = 1 + \cos 2x.
\] (3)

a) Find an antiderivative for \( \cos^2(3t) \).

b) Find a formula for \( \sin^2 x \) starting from (2) by writing the number 1 in a special form and subtracting (2) from it.

c) Use b) to find \( \int \sin^2 x \, dx \).

d) Evaluate \( \int e^t \cos^2(e^t) \, dt \).

e) Ernestine suggested using integration by parts to find \( \int \cos^2 x \, dx \). We tried and got to
\[
\int \cos^2 x \, dx = \cos x \sin x + \int \sin^2 x \, dx
\]
and then gave up because subtracting \( \int \sin^2 x \, dx \) from both sides didn’t help. We were so close! Rewrite \( \sin^2 x \) as \( 1 - \cos^2 x \) and now take something to the other side. Continue to find a formula for \( \int \cos^2 x \, dx \).

f) The formula we get in e) is not (1). Prove that your formula is still correct, that is, it is identically equal in value to (1).

g) Use the quick trick discussed at the end of class to write down the value of \( \int_3^{3\pi} \cos^2 x \, dx \).

h) Use the same trick trick to write down the value of \( \int_{\pi}^{3\pi} \sin^2 x \, dx \).