1. What do mathematicians call the following things?

   a) \[a, ar, ar^2, \ldots, ar^{16}\]

   b) \[3 + \frac{3}{4} + \frac{3}{16} + \cdots\]

Ans 1. a) A finite geometric sequence

   b) An infinite geometric series

2. a) Evaluate the expression in Problem 1, part b).

   b) Is this expression truly a sum? (One sentence explanation)

Ans 2. a) This expression is an infinite geometric series with \(a = 3\) and \(r = 1/4\). Since \(|1/4| < 1\), such a series has a value, namely,

   \[
   \frac{a}{1 - r} = \frac{3}{\frac{4}{4}} = 4.
   \]

   b) No, it’s a limit, because you can’t actually sum an infinite number of things.

3. A patient takes a 10mg dose of a drug at 8am every day. After 24 hours, the body breaks down all but 40% of a dose. In the long run

   a) What is the maximum amount in the patient’s body?

   b) What is the minimum amount?

Ans 3. a) \[
\frac{a}{1 - r} = \frac{10}{1 - .4} = 16\frac{2}{3}.\] (This is the limit of the amount in the body just after the \(n\)th dose, as \(n \to \infty\).)

   b) 40% of the answer in a), or \(6\frac{2}{3}\). (This is the limit of the amount just before the next dose, when all but 40% of the highest amount has degraded.) Or reason this way: The answer to b) must be exactly 10 less than the answer to a), because the patient will be back up to the maximum amount as soon as she takes her 10mg next dose.

4. If the going rate of interest is 5%, what is the present value of an annuity that pays you $1000 today, $1000 again a year from now, again 2 years from now, continuing for 10 payments total.
Ans 4. The present value is

\[ 1000 + \frac{1000}{1.05} + \frac{1000}{(1.05)^2} + \cdots + \frac{1000}{(1.05)^9} = \frac{a(1 - r^{10})}{1 - r}, \]

where \( a = 1000 \) and \( r = \frac{1}{1.05} \approx .9524 \). Notice that the last exponent on the left is 9, not 10. Using a calculator one obtains

\[ 1000 \left( \frac{1 - .6139}{.0476} \right) = 8107.82 \]

Slightly different answers are ok; they indicate that you rounded at intermediate steps instead of doing the calculation in one continuous sequence at the full accuracy of your calculator.

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