1. Let $f(x, y) = x^3 + 2xy^2 - 3y + 10$. Write down, but do not solve, the equations for finding critical points for this function.

Ans 1.

\[ f_x = 3x^2 + 2y^2 = 0 \]
\[ f_y = 4xy - 3 = 0. \]
2. Below is a contour map of a function $g(x,y)$. Estimate $g_y(5,2)$. Explain your calculation briefly — a sentence or two or some indications on the contour map itself.

![Contour Map]

Ans 2. The usual approach is to estimate with an average slope read off the contours. For instance, $(5, 2)$ is on the contour line $f = 3$. Going up, one has to go to $y = 3 \frac{1}{3}$ to get to another shown contour line, $f = 4$. Thus $(4 - 3)/(3 \frac{1}{3} - 2) = 1/(4/3) = \frac{3}{4}$ is an approximation to $g_y(5, 2)$ and an acceptable answer.

But if you try the same approach with decreasing $y$, you get

$$g_y(5, 2) \approx (4 - 3)/(\frac{2}{3} - 2) = 1/(-4/3) = -\frac{3}{4}.$$  

If you do a 2-sided approximation, you get 0.

So in this case, it is actually better to use a different approach. If you move along the line $x = 5$ from $y = 0$ to $y = 4$, you find that $g(5, y)$ falls until $y = 2$ and then rises. That is, $(5, 2)$ is a min point along this slice of $g$, and therefore $g_y(5, 2)$ must be 0 (unless there is no partial at all there because of some jaggedness too small to see from the contour lines). This sort of qualitative approach (looking at the shape of the slice) usually gives less precise information than taking average slopes, but in the case of a min or max point, it is exact.
3. The table below shows calories burned per minute by someone of weight \( w \) rollerblading at speed \( s \). Call this function \( C = b(w, s) \).

a) Estimate two ways

\[
\frac{\partial C}{\partial s} \bigg|_{(160,10)}
\]

Explain your estimates briefly.

b) What are the units for your answers to a), e.g., ft/calorie?

<table>
<thead>
<tr>
<th>( w ) lbs</th>
<th>8 mph</th>
<th>9 mph</th>
<th>10 mph</th>
<th>11 mph</th>
</tr>
</thead>
<tbody>
<tr>
<td>120 lbs</td>
<td>4.2</td>
<td>5.8</td>
<td>7.4</td>
<td>8.9</td>
</tr>
<tr>
<td>140 lbs</td>
<td>5.1</td>
<td>6.7</td>
<td>8.3</td>
<td>9.9</td>
</tr>
<tr>
<td>160 lbs</td>
<td>6.1</td>
<td>7.7</td>
<td>9.2</td>
<td>10.8</td>
</tr>
<tr>
<td>180 lbs</td>
<td>7.0</td>
<td>8.6</td>
<td>10.2</td>
<td>11.7</td>
</tr>
<tr>
<td>200 lbs</td>
<td>7.9</td>
<td>9.5</td>
<td>11.1</td>
<td>12.6</td>
</tr>
</tbody>
</table>

Ans 3.  a) A right-sided estimate:

\[
\frac{b(160, 11) - b(160, 10)}{11 - 10} = \frac{10.8 - 9.2}{1} = 1.6.
\]

A two-sided estimate:

\[
\frac{b(160, 11) - b(160, 9)}{11 - 9} = \frac{10.8 - 7.7}{2} = 1.55.
\]

A left-sided estimate is fine too.

b) \( f(w, s) \) is in calories/minute. \( s \) is in mph (miles/hour). Thus \( f_s(w, s) \), being the limit of \( \Delta f/\Delta s \), is in cal/min per mi/hr, that is, (cal/min)/(mi/hr). That’s good enough. You can, however, “simplify” the units as follows:

\[
\frac{\text{cal}}{\min \text{ hr}} = \frac{\text{cal}}{\min} \cdot \frac{\text{hr}}{\text{mi}} = \frac{\text{cal}}{\text{mi}} \cdot \frac{\text{mi}}{\text{mile minutes}} = \text{calories per 60 miles}
\]

since \( \text{hr}/\text{min} = 1/60 \).