**Example.** Maximize $P(x, y) = x^{2/3}y^{1/3}$ subject to the budget constraint $100x + 100y = 378,000$, and $x, y \geq 0$.

**Solution.** There is no interior. We are on the line $x + y = 3780$, that is, $y = 3780 - x$. Therefore we are reduced to the one-variable problem: maximize

$$g(x) = P(x, 3780-x) = x^{2/3}(3780 - x)^{1/3}$$

for $x$ in the interval $[0, 3780]$.

So compute $g'(x)$, set it to 0, find the solutions for $x$, and compare $g(x)$ at these points to the values at the endpoints $x = 0, 3780$.

Actually, the value of $g$ at the endpoints is 0, as it should be (if you don’t have at least some of each factor of production, you ain’t gonna produce anything.) So if there is just one point $x$ where $g'(x) = 0$, it will surely be the max if $g(x) > 0$.

Next, using the product rule and inside it the chain rule, we compute

$$g'(x) = \frac{2}{3}x^{-1/3}(3780 - x)^{1/3} + x^{2/3}\left(\frac{1}{3}(3780 - x)^{-2/3}(-1)\right)$$

$$= \frac{2}{3}x^{-1/3}(3780 - x)^{1/3} - \frac{1}{3}x^{2/3}(3780 - x)^{-2/3}.$$ 

Set $g'(x) = 0$ and solve for $x$:

$$\frac{2}{3}x^{-1/3}(3780 - x)^{1/3} = \frac{1}{3}x^{2/3}(3780 - x)^{-2/3}$$

$$2x^{-1/3}(3780 - x)^{1/3} = x^{2/3}(3780 - x)^{-2/3} \quad \text{[mult by 3]}$$

$$2(3780 - x)^{1/3} = x(3780 - x)^{-2/3} \quad \text{[mult by } x^{1/3}]$$

$$2(3780 - x) = x \quad \text{[mult by } (3780-x)^{2/3}]$$

$$7560 - 2x = x$$

$$7560 = 3x$$

$$x = 2520.$$ 

Thus $y = 3780 - 2520 = 1260$ and the maximum of $P$ is

$$P(2520, 1260) = (2520)^{2/3}(1260)^{1/3} \approx 2000.12.$$