satisfy $f(x, y) = C$ is called the level curve of at $C$, and an entire family of level curves is generated as $C$ varies over a set of numbers. By sketching members of this family in the $xy$ plane, you can obtain a useful representation of the surface $z = f(x, y)$.

For instance, imagine that the surface $z = f(x, y)$ is a "mountain" whose "elevation" at the point $(x, y)$ is given by $f(x, y)$, as shown in Figure 9.5a. The level curve $f(x, y) = C$ lies directly below a path on the mountain where the elevation is always $C$. To graph the mountain, you can indicate the paths of constant elevation by sketching the family of level curves in the plane and pinning a "flag" to each curve to show the elevation to which it corresponds (Figure 9.5b). This "flat" figure is called a topographical map of the surface $z = f(x, y)$.

**EXAMPLE 1.5**

Discuss the level curves of the function $f(x, y) = x^2 + y^2$.

**Solution**

The level curve $f(x, y) = C$ has the equation $x^2 + y^2 = C$. If $C = 0$ the point $(0, 0)$, and if $C > 0$, it is a circle of radius $\sqrt{C}$. If $C < 0$ no points that satisfy $x^2 + y^2 = C$. 

![Image](image-url)
consider the graph of a function $\mathbb{R}^2 \rightarrow \mathbb{R}$, namely, the point $(x, y, f(x, y))$ in $\mathbb{R}^3$ where $(x, y)$ is in the domain of $f$. Such a graph is shown in Fig. 24 as a surface lying over a rectangle in three-dimensional space. The intersection of the surface with the vertical plane determined by $y = b$ is a curve satisfying the conditions

$$z = f(x, y), \quad y = b.$$ 

Consider as a subset of 2-dimensional space the curve...
\( \frac{\partial f}{\partial u} (0, 0, 0) = \frac{1}{2} \).

\( \mathbb{R}^2 \rightarrow \mathbb{R} \) be a function whose graph is a surface in 3-dimentional space, and let \( u \) be a unit vector in \( \mathbb{R}^2 \), i.e., \( |u| = 1 \). Ar

![Diagram of the function's graph and directional derivative](image)

**Figure 1**

own in Fig. 1. The value of the directional derivative \( \frac{\partial f}{\partial u} \) is by definition
Let \( R^2 \to R \) be a function whose graph is a surface in 3-dimer Euclidean space, and let \( u \) be a unit vector in \( R^2 \), i.e., \( |u| = 1 \). An ex

![Figure 1](image)

is shown in Fig. 1. The value of the directional derivative \( \frac{\partial f}{\partial u} (x, y) \) is by definition

\[
\frac{\partial f}{\partial u} (x) = \lim_{t \to 0} \frac{f(x + tu) - f(x)}{t}.
\]

The distance between the points \( x + tu \) and \( x \) is given by

\[
| (x + tu) - x | = |tu| = |t|.
\]

Hence, the ratio

\[
\frac{f(x + tu) - f(x)}{t}
\]

is the slope of the line through the points \( f(x + tu) \) and \( f(x) \). It that the limit, \( (\frac{\partial f}{\partial u})(x) \), of the ratio is the slope of the tangent \( (x, f(x)) \) to the curve formed by the intersection of the graph of \( f \) plane that contains \( x \) and \( x + u \), and is parallel to the \( z \)-axis. This