they may get the idea that they can start at one end of the chalkboard and work all the way to the other end before erasing anything.

A good way to slow a professor down is to ask questions. If the only reason you ask a question is to slow her down, then ask her to wait instead of asking a question – professors need to know your real concerns. But if you really are puzzled, then a question is the right thing: it lets the professor know what is hard, it wakes you up, it gives more time for note-taking, and you may get a helpful answer.

What if you completely understand everything? Is it OK to take no notes? Well, you should do something, for otherwise you may fall asleep and miss something you don’t understand. Try to keep one step ahead of the professor; see if you can anticipate what she will say next. Or, create a harder version of the same problem and solve that. Or take out your text and read the next section. Or (heretical thought!) write a letter to your lover, or your mother. In any event, stay awake and keep one ear on the class, in case the subject changes to something you don’t understand.

Section 3. Tests and Homework: General Advice

A. The Purpose of Tests and Homework

To write well it is valuable to know why your instructor has made you write. So

Tests are for humiliation and homework for busywork.

Wrong.

Tests are to give grades, and homework is to make you learn the stuff.

Closer. But if we gave tests just to assign grades, multiple choice would be enough. We want you to know your stuff and show your stuff. Knowing your stuff means understanding it as well as being able to get answers. The only way to show that you understand is to make your reasoning clear. In short, communication is as important as understanding.

Note: On tests as well as homework, we faculty also hope you will learn things. If a question is nicely constructed, and you are concentrating, you may see things in a way you hadn’t quite before.

B. Are computations enough on tests and homework?

Mathematicians aren’t wordy, and why should you be under timed test conditions or when you have lots of homework to do? So, may you skip the words and just show the calculations?
The general answer is No. Mathematicians are also interested in reasons. They always know the crucial steps in their work, make these explicit, and expect others to do the same. Unless you are asked to merely “state” your answer, the work on your papers should make clear that you know what you are doing and why it works.

If a problem has a standard solution method, it is enough to follow that method without comment. That’s brief. However, if a problem involves some subtlety, or if it is nonstandard, or it is standard but you are solving it in a nonstandard way (probably not a good idea), you should explain what you are doing. Fortunately, you can still be brief. The key principle is: You must give some notice of any special reasoning, but the notice can be condensed – if you choose your words, symbols and sketches judiciously.

Graders. Your routine homework is likely to be graded by another student, typically a graduate student at universities. In some cases, even your tests will be graded by an assistant. Have pity on student graders. They are underpaid and have their own studies to attend to. They are also not psychics. If they cannot begin to make sense of your solution in 30 seconds, don’t be surprised if they give you a 0 and move on.

C. If You’re Stuck

Especially on tests, students sometimes just write *something*, without understanding, in hopes that it is right or at least will look reasonably good and win some credit. Unfortunately, mathematics attempted without understanding usually makes no sense at all, and looks laughable or pathetic to a knowledgeable reader.

It is better to write nothing than to write nonsense. Or, just write “I’m stuck”, or “I don’t know”. (Knowing when you don’t know is the first step to wisdom.) You might indicate *very briefly* what approaches you tried and why you know they are wrong. Be brief because you won’t get much credit; your time is best spent elsewhere. In any event, never write down anything you doubt is correct without saying so.

See §2.4.A for a related problem: what to write if you don’t know how to do a key step.

D. Wrong Solutions and Multiple Solutions

We all make mistakes. If you are taking a test, nervousness can induce unusually many. The longer you go on without detecting a mistake, the more wasted time and effort. Mistakes early in a solution are particularly harmful this way. So it’s wise to think a long time before writing, to cut off as many false starts as possible.

If you start on a problem and your solution gets more and more complicated (e.g., very messy calculations, or lots and lots of special cases) you have probably made a mistake. Instructors
don’t (intentionally) give messy problems on timed tests, and they don’t give them frequently on homework either. Most likely you’ve made some basic error. Indicate that you are stopping work on this approach because it is suspiciously complicated, and start over. Or go on to another problem.

(Sometimes homework problems are messy on purpose, to illustrate that real-world problems can be messy, or to make the point that a few simple ideas can help you cut through a mess and keep on track. In such cases, however, the instructor usually makes clear in advance that the problem is messy, and you may have calculator and computer tools available that you might not have for tests.)

What is not all right is to leave both the mistakes and a correct solution sitting on the paper with no indication which is which!

One way to avoid this No-No is to erase all mistakes. But sometimes you make repeated mistakes, or do a whole page of calculation which turns out to be mistaken. Why waste time erasing? Also, part of your calculation might still prove useful later even if the whole has errors.

The alternative, then, is to cross out. Cross out a whole page if need be.

But don’t cross out a large section and then squeeze the correct work onto a microdot in the corner. Don’t pepper your paper with crossouts, so that the reader must jump from place to place between them, looking for the correct stuff and guessing its logical order. If you have to cross out that much, fine, but start fresh and write the correct solution from scratch below it, or on another page. At the very least, number the various parts in order and put arrows between them.

Sometimes you come up with several solutions and you don’t know which is right. Try to find the right one and cross out all the others. Don’t leave all your solutions sitting on your paper without comment, hoping your instructor will pick out the right one. At the least indicate that you are unsure which solution is correct and say which one is most likely correct.

Rarely on a test, more frequently on homework, you may choose to give several different correct solutions. Mathematicians like to see this, especially when they didn’t ask for it. Be sure to state that you are deliberately giving several solutions, and indicate where each one begins and ends.

E. Scratchwork

How much scratchwork should you do? If a problem looks complicated, start with scratchwork in a separate place. Do enough so that you know you have the answer and can write out a complete solution. Then write the clean answer you will hand in. This step should not consist of copying over, because your rough form can be much more abbreviated than something you expect someone else to read.

If the problem is fairly simple, say, $\int_1^3 (dx/x^2)$, start right in on your final copy. Any mistakes can be crossed out.
If you do separate scratchwork on a test question, label it (give the problem number, and your name if it is on a separate sheet) and hand it in. Instructors will often consult it if they are baffled by your answer.

F. How Much Simplification Should You Do?

It happens that
\[
\int_{0}^{1} x \sqrt{1 - x^2} \, dx = -\frac{1}{3} \left( \left( \frac{3}{4} \right)^{3/2} - 1^{3/2} \right) = -\frac{1}{3} \left( \left( \frac{3}{4} \right)^{3/2} - 1 \right),
\]  
(3)
as you can show by substituting \( u = 1 - x^2 \) and \( du = -2x \, dx \). But the answer looks ugly. You could go on as follows:
\[
-\frac{1}{3} \left( \left( \frac{3}{4} \right)^{3/2} - 1 \right) = \frac{1}{3} \left( 1 - \frac{3\sqrt{3}}{8} \right) \approx .1168.
\]  
(4)
Should you do these extra simplifications?

Probably not, unless your instructor wants numerical answers. Even if you do need numerical answers, it may be easier (depending on what calculator or computer program you have at your fingertips) to go directly from the lefthand side of (4) to the numerical value, or to use some other intermediate expression. The point is: the purpose of the problem may be to see if you are knowledgeable about “integration by substitution”. If so, the instructor won’t care about the form of the answer. Of course, check with the instructor to find out.

Never do a simplification in your head, no matter how obvious, if it obscures your method. For instance, if you are asked to evaluate \( \int_{0}^{2} x \, dx \), write
\[
\int_{0}^{2} x \, dx = \frac{x^2}{2} \bigg|_{0}^{2} = \frac{2^2}{2} - \frac{0^2}{2} = 2;
\]  
do not write
\[
\int_{0}^{2} x \, dx = \frac{x^2}{2} \bigg|_{0}^{2} = 2,
\]  
(5)
even if it is obvious to you that \( \frac{2^2}{2} - \frac{0^2}{2} = 2 \). The trouble with (5) is that it is not clear that your answer 2 is obtained by substituting 2 and then 0 for \( x \) in \( x^2/2 \). If you are answering homework or a test question, your grader cannot be sure you understand the method. If you are writing an explanation for another student, the student will not learn the method from what you have written.

G. Keep your work

Your homework solutions are a valuable part of your notes. So are your tests that you get back. Both of these things can be used to study for later tests, or to look at during a later course. Therefore, strive to make your work clear enough that you can make sense of it much later. Even
homework that is assigned but not collected (and you should do it) should be written clearly for this purpose.

**Section 4. Examples of Homework and Test Solutions**

This section consists of examples, labeled as Problem 1, 2, 3, . . . . Each problem is followed by one or more solutions and commentary. Some of these problems are probably very elementary for readers of this book, but their simplicity will allow us to concentrate on the quality of the solutions. Problems with longer commentary are placed later.

An excellent, short, additional reference on how to write homework is Price [20]. It applies just as well to tests.

**Problem 1.** You get on a turnpike at mile marker 38 and get off at mile marker 79.

a) How many miles did you travel on the turnpike?

b) How many mile markers did you see?

The problem asks two questions, suggesting (correctly) that the answers are not the same. Indeed, the answers are:

a) \(79 - 38 = 41\)

b) \(41 + 1 = 42\)

Since the problem hints at some subtlety, it is best to give some explanation for both parts. I suggest:

a) When you get off the turnpike, you are 79 miles from its start. You skipped the first 38 miles. So you went \(79 - 38 = 41\) miles.

b) When exiting, you see marker 79. You missed the first \(\frac{37}{3}\) markers. So you saw \(79 - 37 = 42\) markers. Or:

You saw a marker at the end of each mile you traveled, plus one at the beginning of your first mile, so \(41 + 1 = 42\).

**Telegraphic Prose.** You can shorten the above answer to Problem 1 by writing

a) When get off tpk, 79 mi from start. Skipped first 38. So went \(79 - 38 = 41\).

b) Exit marker 79. You miss first \(\frac{37}{3}\). So see \(79 - 37 = 42\).

You can save half your time with such telegraphic prose, and it makes a difference for people who write slowly and with discomfort. (The number of us is going to increase, now that penmanship has died out in the schools.) The essential point on tests and most homework is for you to show that you understand both why and how. Even abbreviated prose can show this, if done well.