Instructions. Do as many problems as you can. The point count to the left indicates how much time I think you should spend on a problem; do not spend a lot of time on a problem worth few points. As on the Honors exam, you may use any standard result as long as that result is not essentially what you are being asked to prove. If you do use a standard result, make sure you clearly identify the result and verify that the hypotheses are satisfied.

Part I. Do problems 5, 7, 8, 9 from the Honors Exam (but do the easier problems below first). Each of the Honors problems is worth 15 points.

Part II.

(5) A. Find the area of the parallelogram in $\mathbb{R}^4$ whose corners are the points $0, u = (1, 2, 3, 4), v = (4, 3, 2, 1)$ and $u + v$.

(5) B. Let $M$ be a manifold with boundary $N = \partial M$. $N$ is itself a manifold. What is its boundary? What fact is this fact dual to? (No explanation necessary.)

(10) C. Prove: if $S$ is rectifiable, then every bounded continuous function on $S$ is Riemann integrable, i.e., no extended integrals necessary. (This follows pretty quickly from some standard definitions and theorems.)

(15) D. In class, we proved oh(Oh) = oh. Now devise and prove a theorem about Oh(oh). Begin by defining Oh and oh in proper notation, and then properly stating your theorem.

E. Let $M$ be an $m \times n$ matrix. As in Munkres, let $\|x\|$ be the usual Euclidean norm of $x$, and let $|x|$ be the max norm on $x$ and $|M|$ be the max norm on $M$.

In this problem we define the operator norm $\|M\|$ on $M$, namely,

$$\|M\| = \max\{\|Mx\| : \|x\| = 1\}.$$ 

The max is obtained because $x \rightarrow \|Mx\|$ is a continuous function on a compact set in $\mathbb{R}^n$.

(5) a) Prove: for all $x \in \mathbb{R}^n$, $\|Mx\| \leq \|M\|\|x\|$.

(7) b) Let $K$ be a $k \times m$ matrix. State and prove a relationship between $\|KM\|$, $\|K\|$ and $\|M\|$. (Note: Recall from Munkres that $|KM| \leq m|K||M|$.)

(8) c) Find and prove a relationship between $|M|$ and $\|M\|$.
F.  a) Define 2-tensor and alternating 2-tensor.

b) If $T : V \to W$ is a linear transformation, and $\phi$ is a 2-tensor on $W^2$, define the induced 2-tensor $T^* \phi$.

c) Prove, showing the details, that $T^* \phi$ really is a 2-tensor.

d) Prove: if $\phi$ is alternating, so is $T^* \phi$.

G. Let $M$ be a $k$-manifold in $\mathbb{R}^n$ with boundary $\partial M$. Suppose two boundary patches $\alpha, \alpha'$ of $M$ (that is, patches with domain in $H^k$) overlap positively.

a) Prove that the restrictions of $\alpha, \alpha'$ to patches for the manifold $\partial M$ also overlap positively.

b) What theorem is part a) a key part towards proving?