I said in seminar that in 1 dimension the change of variable theorem holds even without assuming \( g \) is 1-to-1 (and thus without insisting that \( g' \) always has the same sign). The proof is in most calculus books, but I should have showed it right there, instead of waving my hands. Here’s the proof. As I noted, Munkres proved less because less is what can be proved in more dimensions.

The proof in 1-D follows from the Fundamental Theorem of Calculus (FTC).

**Theorem.** Suppose \( f, g, g' \) are continuous real functions (or at least continuous on the appropriate intervals). Then

\[
\int_{g(a)}^{g(b)} f(x) \, dx = \int_a^b f(g(x))g'(x) \, dx.
\]

Proof: Because \( f \) is continuous, it has an antiderivative \( F \) and also the FTC applies, yielding

\[
\int_{g(a)}^{g(b)} f(x) \, dx = F(x) \bigg|_{g(a)}^{g(b)} = F(g(b)) - F(g(a)).
\]

Furthermore, \( \frac{d}{dx}F(g(x)) = f(g(x))g'(x) \). Since \( f(g(x))g'(x) \) is continuous, by FTC

\[
\int_a^b f(g(x))g'(x) \, dx = F(g(x)) \bigg|_a^b = F(g(b)) - F(g(a)).
\]