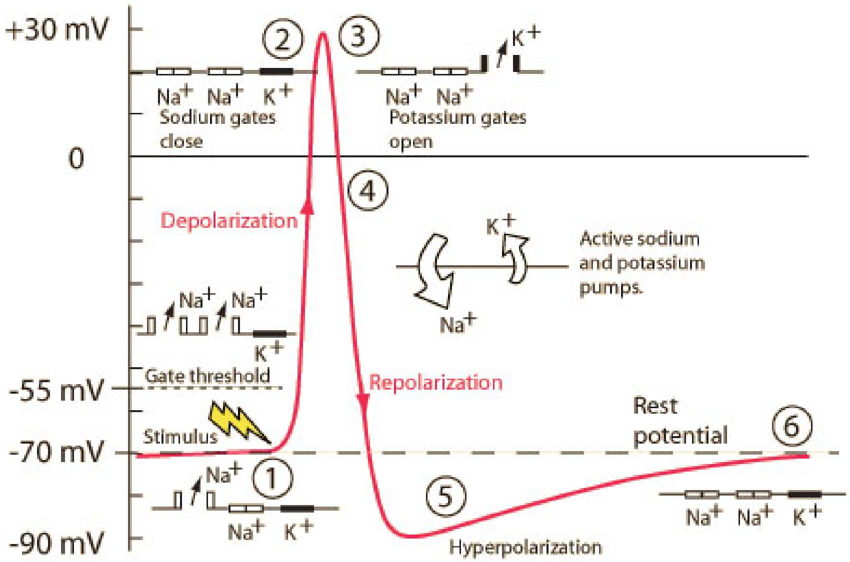
The Dynamics of Hodgkin-Huxley

Action Potential



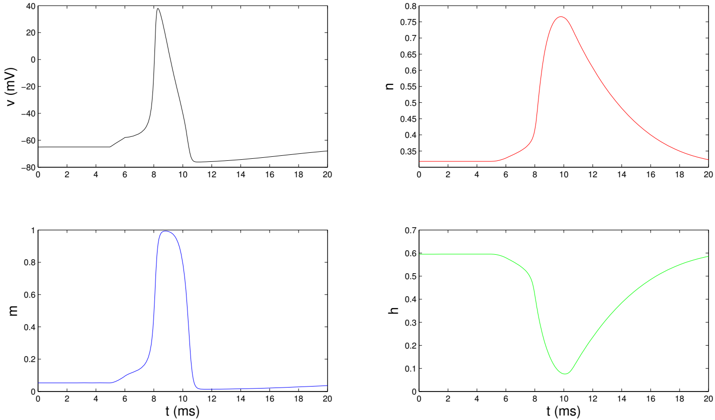
Hodgkin-Huxley Model

$$\frac{dV}{dt} = -\frac{1}{C}\left(\bar{g}_K n^4 (V - V_K) + \bar{g}_{Na} m^3 h (V - V_{Na}) + \bar{g}_L (V - V_L)\right)$$

$$\begin{aligned}
\tau_n(V)\frac{dn}{dt} &= -(n - n_\infty(V)) \\
\tau_m(V)\frac{dm}{dt} &= -(m - m_\infty(V)) \\
\tau_h(V)\frac{dh}{dt} &= -(h - h_\infty(V))
\end{aligned}$$

$$\tau_i(V) = \frac{1}{\alpha_i(V) + \beta_i(V)}, \quad i_\infty(V) = \frac{\alpha_i(V)}{\alpha_i(V) + \beta_i(V)}$$

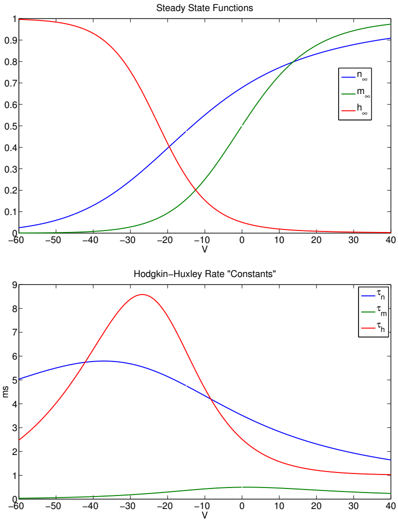
Simulations



Question

DESCRIBE THE BEHAVIOR OF n, m, h.

How do i_∞ and τ_i affect the graphs of $n(t)$, $m(t)$, $h(t)$.

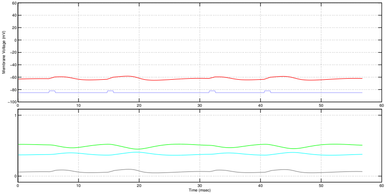


Questions

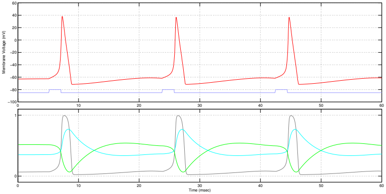
WHAT CAUSES THE SPIKE TO OCCUR?

Injecting Current - Perturbations to the System

Small currents will cause the voltage to return to the resting potential.



Large currents will initiate the bursting dynamic.



Time Scales

SLOW VARIABLES ARE IN QUASI STEADY STATE

Fast System

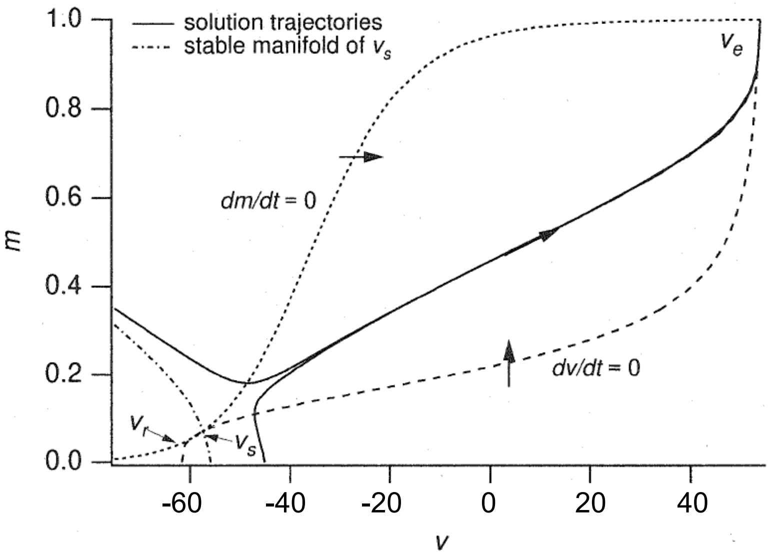
THIS SYSTEM ONLY MODELS THE DEPOLARIZATION PHASE (INCREASE IN MEMBRANE POTENTIAL)

$$\frac{dV}{dt} = -\frac{1}{C}\left(\bar{g}_K n_0^4 (V - V_K) + \bar{g}_{Na} m^3 h_0 (V - V_{Na}) + \bar{g}_L (V - V_L)\right)$$

$$\tau_m(V)\frac{dm}{dt} = -(m - m_\infty(V))$$

THIS SYSTEM DOES NOT MODEL THE ENTIRE ACTION POTENTIAL

Phase plane of fast system



Full Model

$$\frac{dV}{dt} = -\frac{1}{C}\left(\bar{g}_K n^4 (V - V_K) + \bar{g}_{Na} m^3 h (V - V_{Na}) + \bar{g}_L (V - V_L)\right)$$

$$\tau_n(V)\frac{dn}{dt} = -(n - n_\infty(V))$$
$$\tau_m(V)\frac{dm}{dt} = -(m - m_\infty(V))$$
$$\tau_h(V)\frac{dh}{dt} = -(h - h_\infty(V))$$

HOW CAN WE USE TIME SCALES TO REDUCE THE MODEL AND PRESERVE ALL DYNAMICS

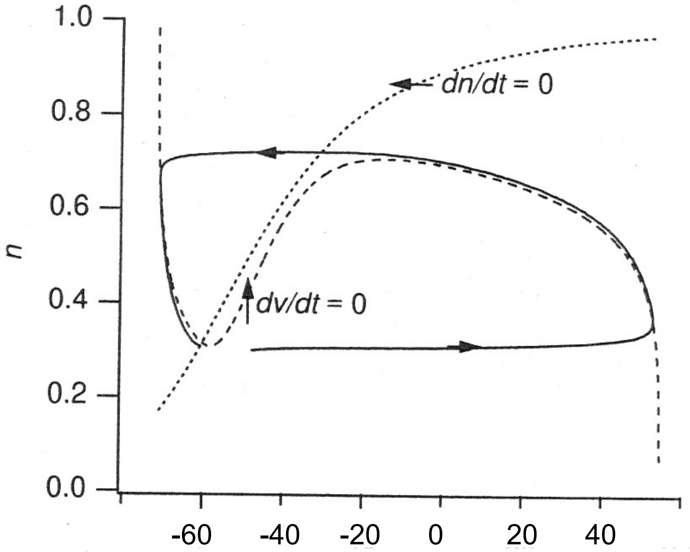
Fast-Slow Model

$$\frac{dV}{dt} = -\frac{1}{C}\left(\bar{g}_K n^4 (V - V_K) + \bar{g}_{Na} m_\infty^3(V)(K - n)(V - V_{Na}) + \bar{g}_L (V - V_L)\right)$$

$$\tau_n(V)\frac{dn}{dt} = -(n - n_\infty(V))$$

WHAT DO THE DYNAMICS LOOK LIKE?

Fast-Slow Phase Plane



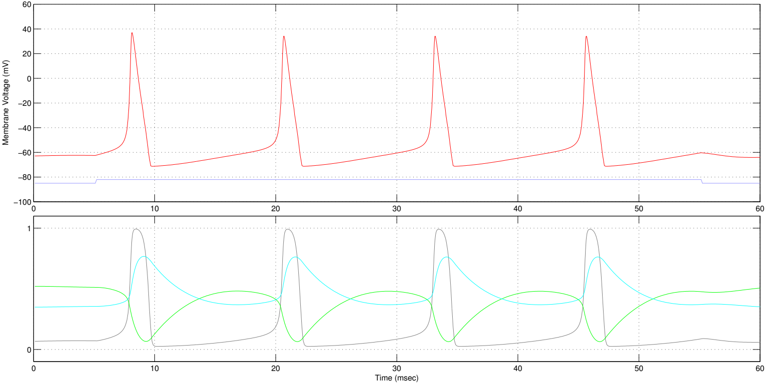
Constant Current

$$\frac{dV}{dt} = -\frac{1}{C}\left(\bar{g}_K n^4 (V - V_K) + \bar{g}_{Na} m^3 h (V - V_{Na}) + \bar{g}_L (V - V_L)\right) + I_{app}$$

$$\begin{aligned}
\tau_n(V)\frac{dn}{dt} &= -(n - n_\infty(V)) \\
\tau_m(V)\frac{dm}{dt} &= -(m - m_\infty(V)) \\
\tau_h(V)\frac{dh}{dt} &= -(h - h_\infty(V))
\end{aligned}$$

WHAT ARE THE DYNAMICS WITH A CONSTANT CURRENT?

Constant Current



periodic solutions!

Discussion Questions

1. Would changing the level of injected current change the peak of the spike?
2. What aspects of the phase plane create the bursting dynamic?
3. How would dynamics change if the nullclines were shifted? How many steady states? Stability? Bursting?
4. If injecting a constant current, would the periodic solution be neutrally stable, a stable limit cycle, or an unstable limit cycle?
5. Do you think there there exists a I_c such that if $I_{app} < I_c$, the membrane potential will remain constant?
6. Write a general form for a bursting model (2 variable system) with a periodic solution.

Homework - due Thurs

Generalized Fitzhugh-Nagumo Equations

$$\begin{aligned}\epsilon\frac{dv}{dt} &= f(v,w) + I \\ \frac{dw}{dt} &= g(v,w)\end{aligned}$$

Let $I = 0$, $\epsilon = 0.01$ and

$$\begin{aligned}f(v,w) &= v(v-\alpha)(1-v) - w \\ g(v,w) &= v - 0.5w\end{aligned}$$

Analyze the model completely. Use phase portraits for $\alpha = .1$ and $\alpha = -.1$ to assist in the analysis.