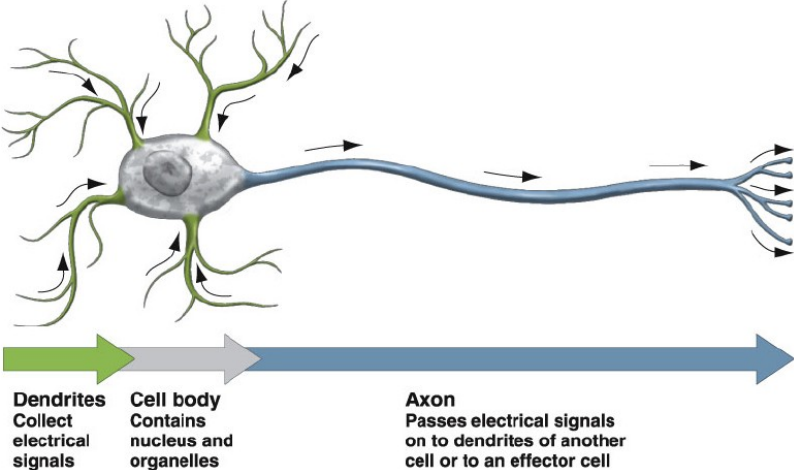


Hodgkin-Huxley Model of Action Potential

Neuron



Axon = Electrical Cable?

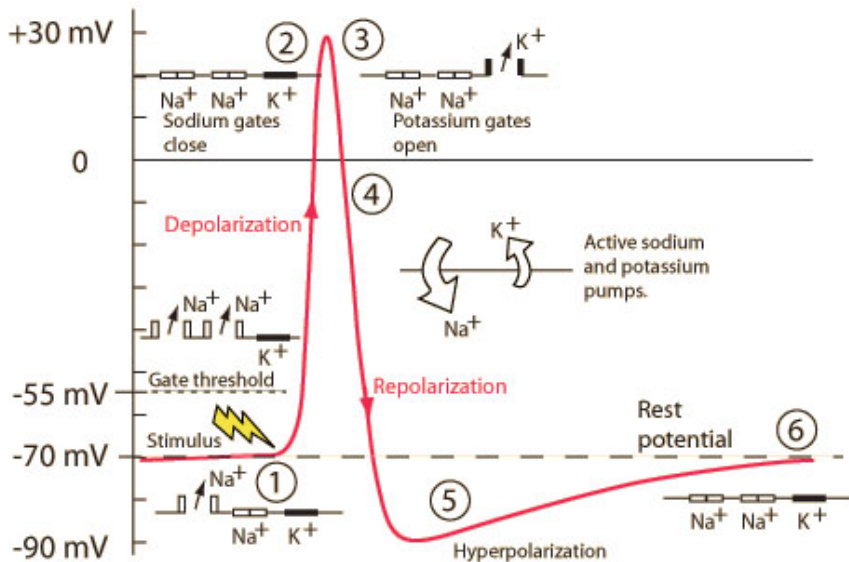
Resistance of

Nerve fiber	=	Copper wire
1m long		with a diameter of 0.6 mm
with a diameter of $1\mu\text{m}$		length 10x the distance to Saturn

Current is a flow of ions, not electrons

Direction is not longitudinal but transverse, passing into and out of the cell through ion channels in the membrane

Action Potential



Modeling the Dynamics of Action Potential

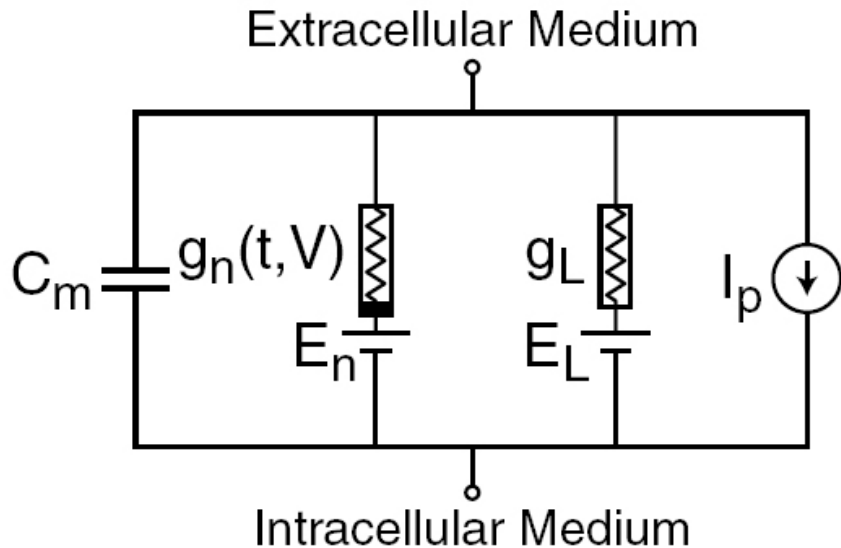
Past models

model constructed from knowledge of interactions
determine long term behavior

Hodgkin-Huxley

know the behavior
model to understand underlying mechanisms

Hodgkin-Huxley as Electrical Circuit



Circuit Terms

$q(t)$	the charge carried by particles in circuit at time t
$I(t)$	the current = $\frac{dq}{dt}$ rate of flow of charge in the circuit at time t
$V(t)$	the voltage difference in the electrical potential at time t
$R(V, t)$	the resistance property of material that impedes flow of particles
$g(V, t)$	the conductance, $\frac{1}{R(V, t)}$
C	the capacitance property of an element that physically separates charges

- ▶ Capacitor causes the difference in electrical potential (voltage)
- ▶ $g(V, t)$ = voltage gated ion channels
- ▶ C = lipid bilayer

Physical Relationships in a Circuit

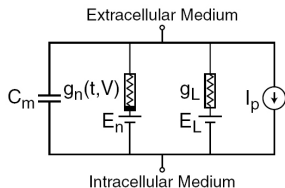
Ohm's Law - the voltage drop across a resistor is proportional to the current through the resistor; R is the factor of proportionality.

$$V_R(t) = I(t)R = \frac{I(t)}{g(V, t)}$$

Faraday's Law - the voltage drop across a capacitor is proportional to the electric charge; $\frac{1}{C}$ is the factor of proportionality.

$$V_C(t) = \frac{q(t)}{C}$$

Elements in Parallel



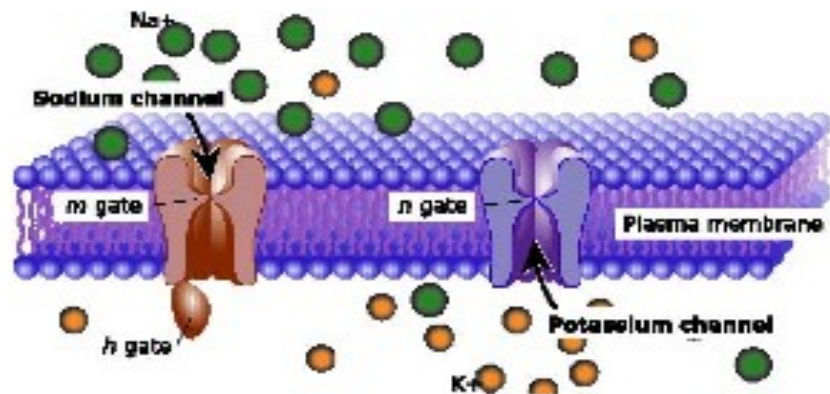
For elements in parallel,

the total current is equal to the sum of currents in each branch
voltage across each branch is the same

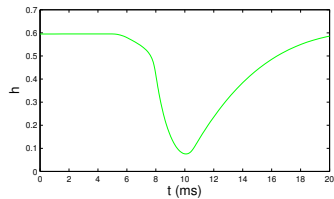
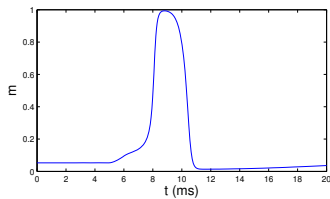
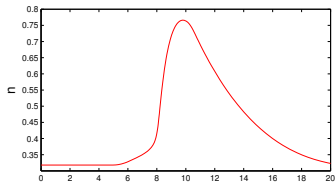
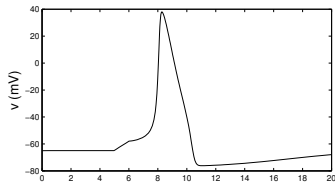
$$I(t) = I_1(t) + I_2(t) + I_3(t) = V(g_1 + g_2 + g_3)$$

$$V(t) = \frac{q(t)}{C}$$

Gates



Simulations



Hodgkin-Huxley Model

$$\frac{dV}{dt} = -\frac{1}{C} (\bar{g}_K n^4 (V - V_K) + \bar{g}_{Na} m^3 h (V - V_{Na}) + \bar{g}_L (V - V_L))$$

$$\tau_n(V) \frac{dn}{dt} = -(n - n_\infty(V))$$

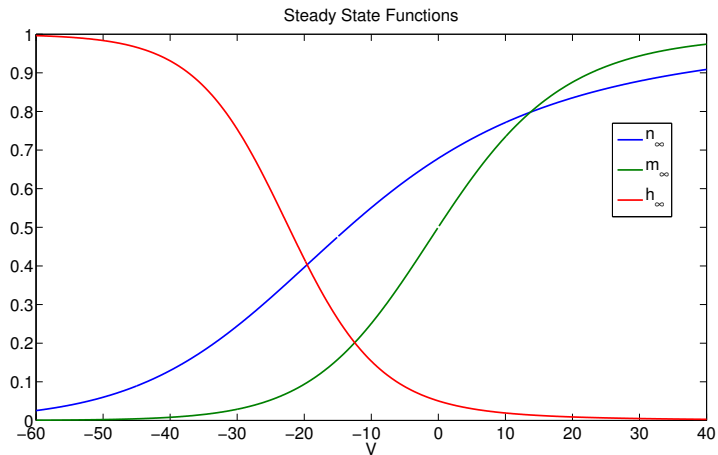
$$\tau_m(V) \frac{dm}{dt} = -(m - m_\infty(V))$$

$$\tau_h(V) \frac{dh}{dt} = -(h - h_\infty(V))$$

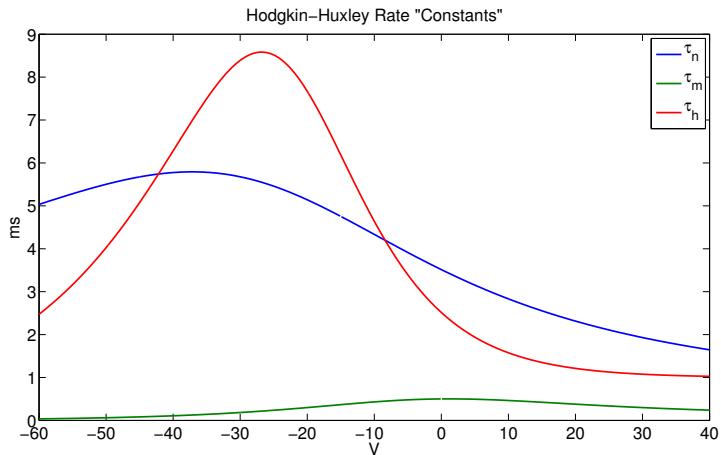
$$\tau_i(V) = \frac{1}{\alpha_i(V) + \beta_i(V)} \text{ - time scales}$$

$$i_\infty(V) = \frac{\alpha_i(V)}{\alpha_i(V) + \beta_i(V)} \text{ - steady state values}$$

Steady State Values



Time scales



Discussion Questions

1. Identify the biological events that explain the dynamics on "Solution" slide.
2. Interpret the "steady state" graphs.
3. Interpret the "time scales" graphs.
4. Identify the 'fast' and 'slow' variables.
5. Explain the bursting dynamic using the time scales identified.