Class Assignment
Week 2

Growth of Segmental Organisms

The following hypothetical situation arises in organisms such as certain filamentous algae and fungi that propagate by addition of segments. The rates of growth and branching may be complicated functions of densities, nutrient availability, and internal reserves. However, we present here a simplified version of this phenomenon. A segmental organism grows by adding new segments at intervals of 24 hours in several possible ways.

1. A terminal segment can produce a single daughter with frequency p, thereby elongating its branch.

2. A terminal segment can produce a pair of daughters (dichotomous branching) with frequency q.

3. a next-to-terminal segment can produce a single daughter (lateral branching with frequency r.

The question to be addressed is how the numbers of segments change as this organism grows.

Questions

1. Draw a picture depicting the three scenarios above.

2. Let

$$\begin{aligned} a_n &= \textit{number of terminal segments,} \\ b_n &= \textit{number of next-to-terminal segments,} \\ s_n &= \textit{total number of segments.} \end{aligned}$$

   Assume that all daughters are terminal segments; that all terminal segments participate in growth ($p + q = 1$) and thereby become next-to-terminal segments in a single generation; and that all next-to-terminal segments are thereby displaced and can no longer branch after each generation. Write equations for these three variables.

3. Show that the equations for a_n and b_n can be combined to give

$$a_{n+1} - (1 + q)a_n - ra_{n-1} = 0,$$

   and explain the equation.

4. If initially there is just one terminal segment, how many terminal segments will there be after 10 days? What will be the total number of segments be after 4 days?